Transient Response For The Advection Dispersion Problem To Approach From Unsteady To Steady State

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Abstract— Transient response of one dimensional advective dispersive transport problem to approach from unsteady to steady state is determined. Explicit Finite-Difference numerical technique is used to find the numerical solution of transport equations. Also MATLAB code is developed for the numerical solution. The effect of different parameter changes with respect to spatial distribution at a particular time has been compared and time required from unsteady to steady state with respective parameter change is determined. As dispersion coefficient and decay constant enhances, the concentration of solute enhances but time required to approach from unsteady to steady state enhances in case of dispersion coefficient where as reduces in case of decay constant. Also as velocity enhances, the solute concentration enhances and time of approach reduces. Effect of Peclet number on solute concentration with respect to spatial and temporal variation also analyzed.

Keywords— Solute transport; Finite Difference Method; Concentration Profile; Dispersion Coefficient; Decay constant; Peclet number; Spatial Moment; Temporal Moment.

INTRODUCTION

In recent days, groundwater resources have become increasingly polluted by the production of reactive and non reactive contaminants from industrial and/or household waste, infiltration of pesticides and fertilizers from agricultural areas and leakage of organic pollutants from petrol stations, refineries, pipelines etc. Once ground water infected it becomes difficult to improve its quality. In the present study the mathematical modelling of ground water solute transport in saturated porous media with respect to different real life problem were done. This is due to the fact that many of the geo environmental engineering problems have direct or indirect impact on the groundwater flow and solute transport. So its management and remediation becomes a essential work for the engineers. Previously several mathematical models were developed by scientists for the solute transport problems. Many of the models used to describe the transport of solutes in the subsurface are based on the convection-dispersion equation as given by Bear(1972, 1979).Broadly the type of models are divided in to two types-non reactive and reactive solute transport. Yetes (1990) developed an analytical solution for describing the 1-D transport of dissolved substances in heterogeneous porous media with a distance-dependent dispersion relationship. The model in this used assumes that the dispersion coefficient is a linear function of the space dimension and that this linear dependence is a direct consequence of the heterogeneous nature of the porous medium. The solution has been obtained for a constant concentration and for constant flux boundary conditions using both an exact and numerical inversion of the Laplace-transformed transport equation. Batu and Genuchten (1990) developed first and third type boundary conditions in 2-D solute transport modelling .It presents a general analytical solution for convective-dispersive solute transport in a 2-D semi infinite porous medium. Fresh et al (1998) executed column experiment to known the effect of water content on solute transport in a porous medium containing reactive micro-aggregates. The water content of porous media may affect the transport behavior of conservating & sorbing solutes. An accurate 2D simulation of advective-diffusive-reactive transport was proposed by Stefanovic and Stefan (2001). It presents an accurate numerical algorithm for the simulation of 2D solute/heat transport by unsteady advection-diffusion-reaction. Atul Kumar et al (2009) developed analytical solutions of one-dimensional advection–diffusion equation with variable coefficients in a finite domain for two dispersion problems. In the first one, temporally dependent solute dispersion along uniform flow in homogeneous domain is studied. In the second problem the velocity is considered spatially dependent due to the in-homogeneity of the domain and the dispersion is considered proportional to the square of the velocity. The velocity is linearly interpolated to represent small increase in it along the finite domain. This analytical solution is compared with the numerical solution in case the dispersion is proportional to the same linearly interpolated velocity. Fedi (2010) proposed a new analytical solution for the 2D advection–dispersion equation in semi-Infinite and laterally bounded domain. An analytical solution for 2-d Solute transport (Singh et al,2010) in Finite Aquifer with Time-dependent Source Concentration was developed. Using Hankel Transform Technique, an analytical solution is derived for 2-d solute transport in a homogeneous isotropic aquifer with time-dependent source concentration. Chen and Liu (2011) proposed generalized analytical solution for 1-D advection-dispersion equation in finite spatial domain with arbitrary time-dependent inlet boundary condition. The generalized analytical solution of the equation is derived by using the Laplace transform. Result shows an excellent agreement between the analytical and numerical solutions. Sharma and Srivastava (2011) proposed numerical analysis of virus transport through a 2-D heterogeneous porous media. Virus transport through two-dimensional heterogeneous porous media at field scale is simulated using an advective dispersive virus transport equation with first-order adsorption and inactivation constant. An increasing exponential dispersivity function has been used to account for heterogeneity of the porous media. Implicit finite-difference numerical technique is used to get the solution of two-dimensional virus transport equation for virus concentration in suspension. A reactive transport through porous media using finite-difference and finite-volume methods developed by Sharma et al(2012). Finite-volume method (FVM) and implicit finite-difference method (FDM) have been used to solve multi-process non-equilibrium (MPNE) transport equation for reactive solute transport through porous media. Sharma et al (2013) developed stochastic numerical method for analysis of solute transport in fractured porous media. They developed stochastic two dimensional numerical models for the solute transport through fractured rock, treating the matrix diffusion coefficient as a stochastic process and evaluated the effect of the variance of log diffusion and integral scale on mean travel distance, spreading behavior and effective dispersion coefficient for the solute in the fracture. Sharma et al (2014) proposed finite volume model (FVM) for reactive transport in fractured porous media with distance- and time-dependent dispersion. In this study, the behavior of temporal and spatial concentration profiles with distance- and time-dependent dispersion models is investigated. Also Hulagabali et al(2014) suggested contaminant transport modeling through saturated porous media using Finite Difference and Finite Element Methods. It presents an alternative numerical method to model the two dimensional contaminant transport through saturated porous media using a finite difference method (FDM) and finite element method (FEM).Also a MATLAB code was developed to obtain the numerical solution. CTRAN/W also used for modeling of contaminant transport which is based on the finite element method. Results of the FDM and FEM are compared and it is found that they agree well.

In this work the transport of solute in porous media is modeled for one direction. According to it the Advective Diffusive Equation is developed and its approximate solution was found out by explicit Finite Difference Method. Also a computer programming in MATLAB is developed for the explicit scheme. The model is used to simulate the field experimental data of spatial and temporal moments. The time required from unsteady to steady state with respective parameter change is also determined.

GOVERNING EQUATION

The one-dimensional solute transport in homogeneous, saturated porous media is governed by the following advective-diffusion- equation(ADE) with first-order reaction.

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} - U\frac{\partial C}{\partial x} - kC \qquad (1)$$

Where C= concentration (moles/m^3), D=dispersion coefficient (m^2/h), U=Q/A=velocity of water (m/h), Q=flow rate (m^3/h), A=tank's c/s area (m^2), k =the first order decay coefficient (h^{-1}), L=length of the flume (m) and t =time (h).

In this investigation, an explicit finite-difference numerical technique has been used to get the solution of solute transport equations Eqs. (1).The explicit finite-difference formulation of above equation can be written as:

$$\frac{C_i^{l+1} - C_i^l}{\Delta t} = D\frac{C_{i+1}^l - C_i^l + C_{i-1}^l}{(\Delta x)^2} - U\frac{C_{i+1}^l - C_{i-1}^l}{2\Delta x} - kC_i^l \qquad (2)$$

Here, the subscript i-designates the grid point along x-direction, l time factor and Δx grid size in x direction and Δt is the time step.

APPLICATION OF MODEL

A elongated flume with a single entry and single exit point as shown in fig.(1) is considered. The water is flowing to the flume from a tank with some velocity to some given distance. Prior to time (t) =0, the tank is filled with water that is devoid of the reactive solute. At time=0, the reactive solute is injected into the flume's inflow at a constant level of C_{in}. The flume is well mixed with the solute vertically and laterally. The reactive solute is subject to first-order decay.

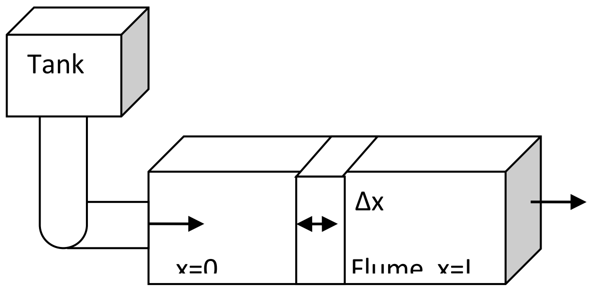

Figure-1 Flume of length L with an overhead tank with a single entry and exit point.

Following initial and boundary conditions have been used:

Initial condition

$C=C_{in}$ $0<x<L$ at t=0 (3a)

Boundary condition

$QC_{in}=QC_0-DA\frac{dC_0}{dx}$ at x=0

and $\frac{\partial C}{\partial x}=0$ at x=L (3b)

where C_0=the concentration at x=0. An explicit finite-difference numerical technique has been used to get the solution of solute transport equations Eqs. (1),(3a) and(3b).

RESULT AND DISCUSSION

The ADE of one dimensional solute transports was solved. The solute transport in x direction is discritised. The graphical presentation of normalised concentration according to different parameter change with respect to space and time was shown below which are drawn by application of MATLAB.

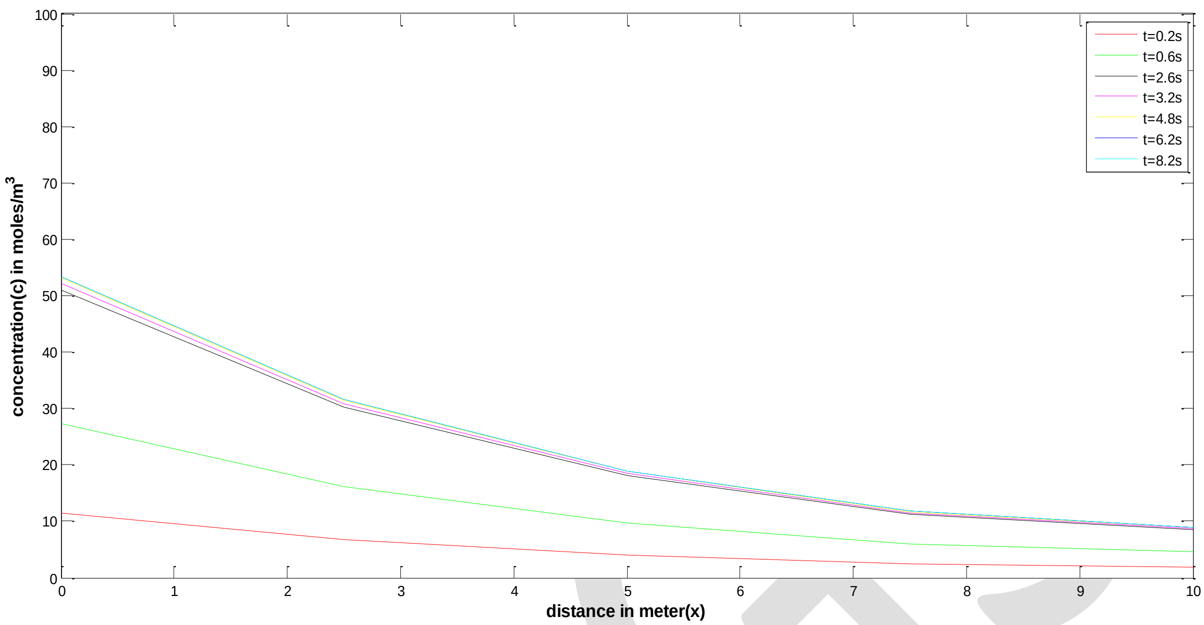

Figure-2 Spatial distribution of concentration profiles at different time interval in hour to attained to steady state .(U=0.5m/h, D=2m^2/h, k=0.2h^{-1})

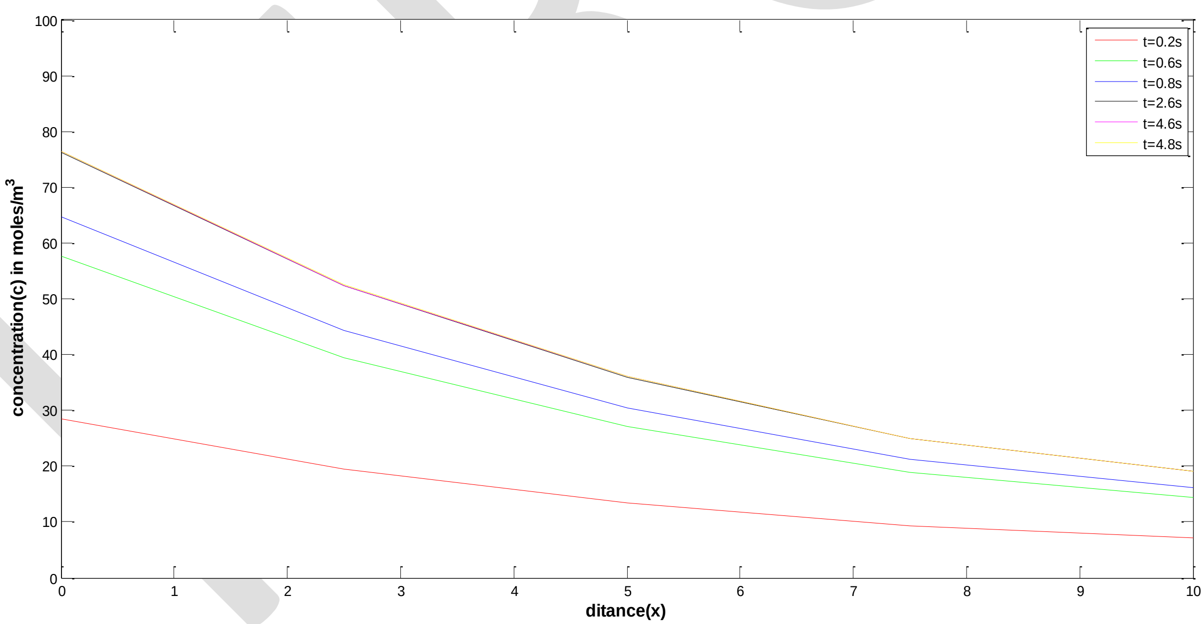

Figure-3 Spatial distribution of concentration profiles at different time interval in hour to attained to steady state (U=1m/h, D=2m^2/h, k=0.2h^{-1})

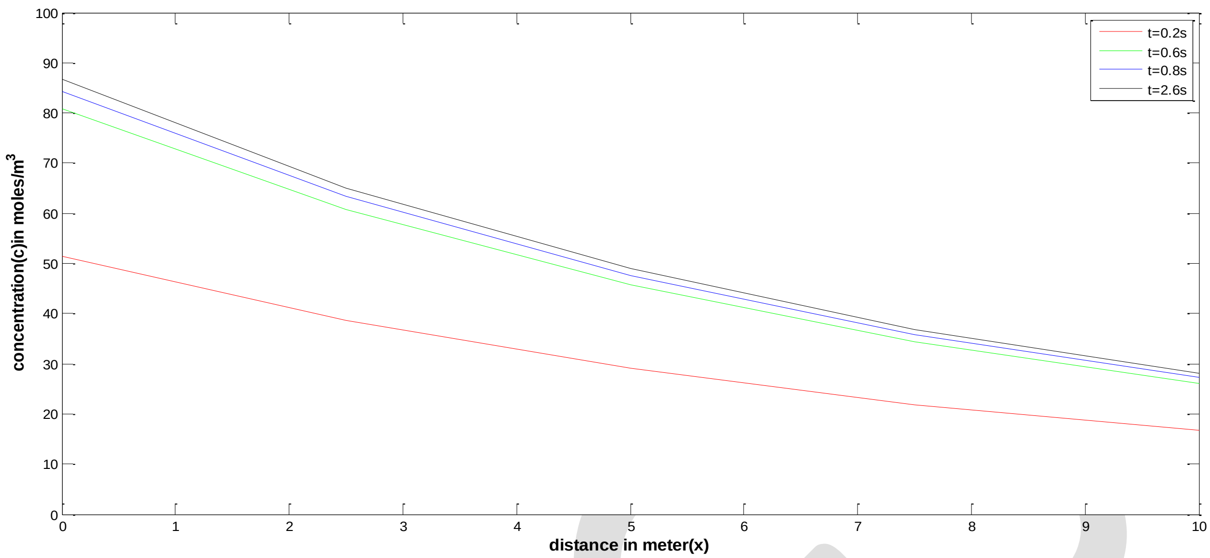

Figure-4 Spatial distribution of concentration profiles at different time interval in hour to attained to steady state (U=1.5m/h, D=2m^2/h, k=0.2h^{-1})

Comparing fig.2, 3 and 4 it is observed that as velocity (U) increases the concentration profile value also increases in both x and y direction but the time required to attain from unsteady state to steady state decreases. If we compared the concentration profile at steady state in the three figure we get 53.18, 76.40 & 86.60 in y direction with velocity value of 0.5, 1 and 1.5m/s respectively. Hence if velocity increases the concentration value also increases. Also we can get time interval of 8.2, 4.8 and 2.6s at the steady state with the velocity value of 0.5, 1 and 1.5m/s respectively which conclude that as velocity increases the time required to attain from unsteady state to steady state decreases. As time passes we also get ascending concentration profiles which merges gradually with steady state profile and seen as one solid line.

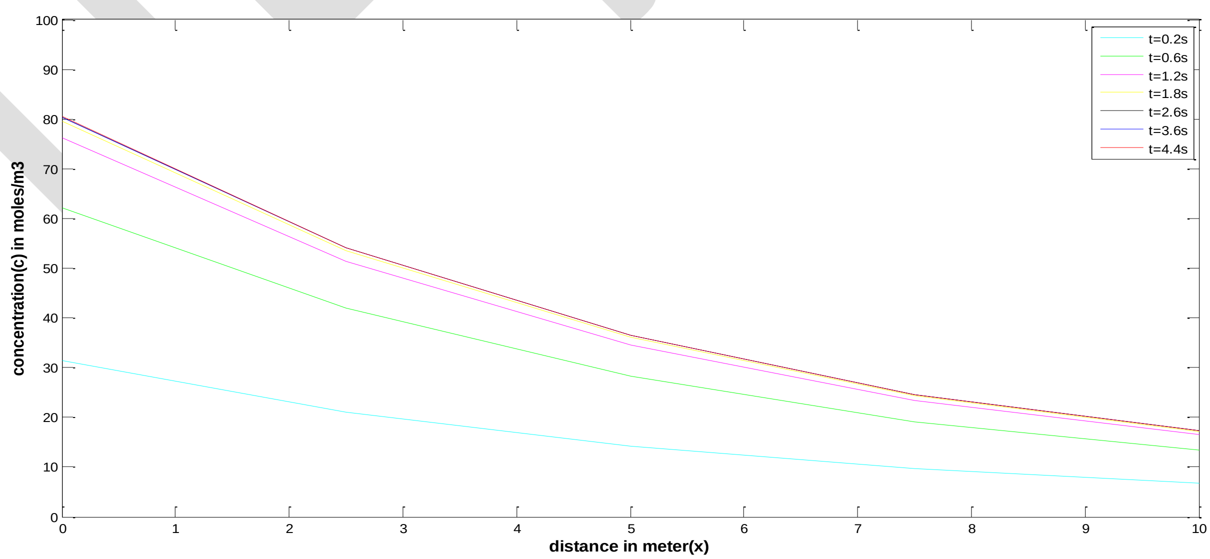

Figure- 5 Spatial distribution of concentration profiles at different time interval in hour to attained to steady state (U=1m/s, D=1.5m^2/h, k=0.2h^{-1})

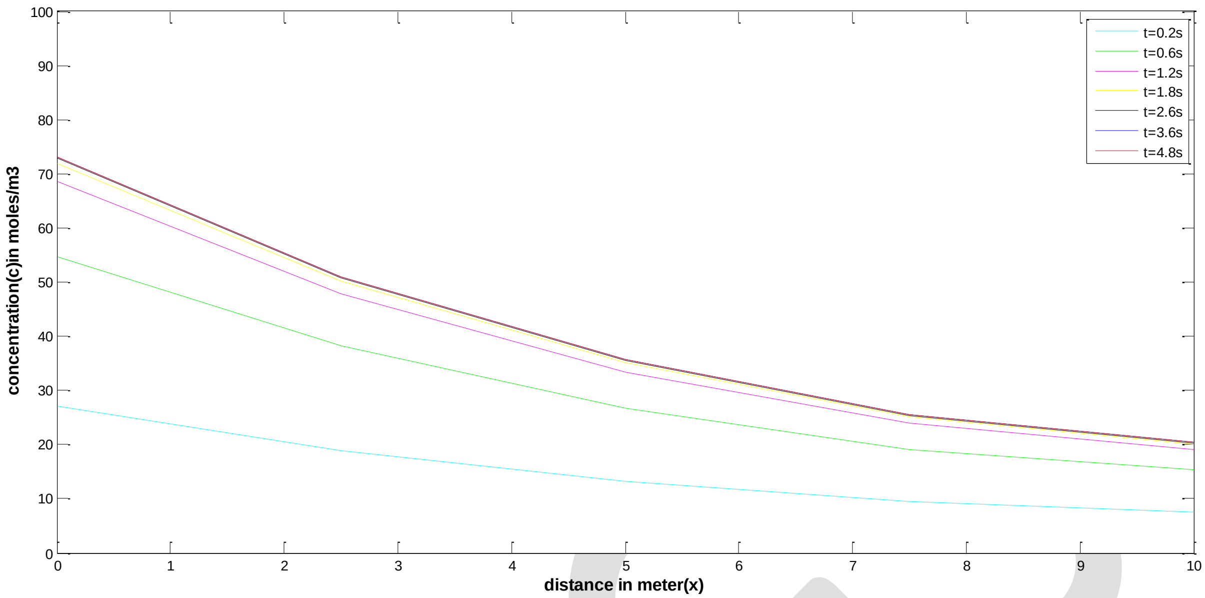

Figure-6 Spatial distribution of concentration profiles at different time interval in hour to attained to steady state (U=1m/s, D=2.5m^2/h, k=0.2h^{-1})

Comparing fig.3, 5 and 6 we get ascending profiles as time increases and at steady state they merge. It is observed that as dispersion coefficient (D) increases the time required to attained from unsteady state to steady state increases. Also if we compared the concentration profile at steady state in the three figure we get that if dispersion coefficient increases the concentration value also decreases and become flatter and flatter.

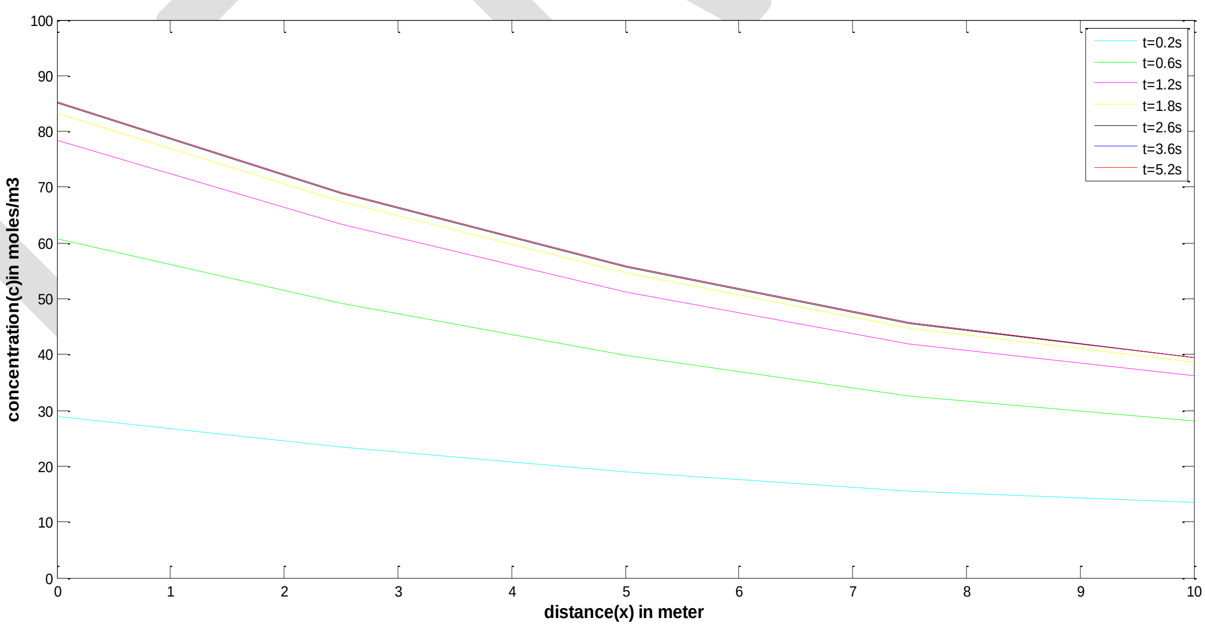

Figure-7 Spatial distribution of concentration profiles at different time interval in hour to attained to steady state (U=1m/s, D=2m^2/h, k=0.1h^{-1})

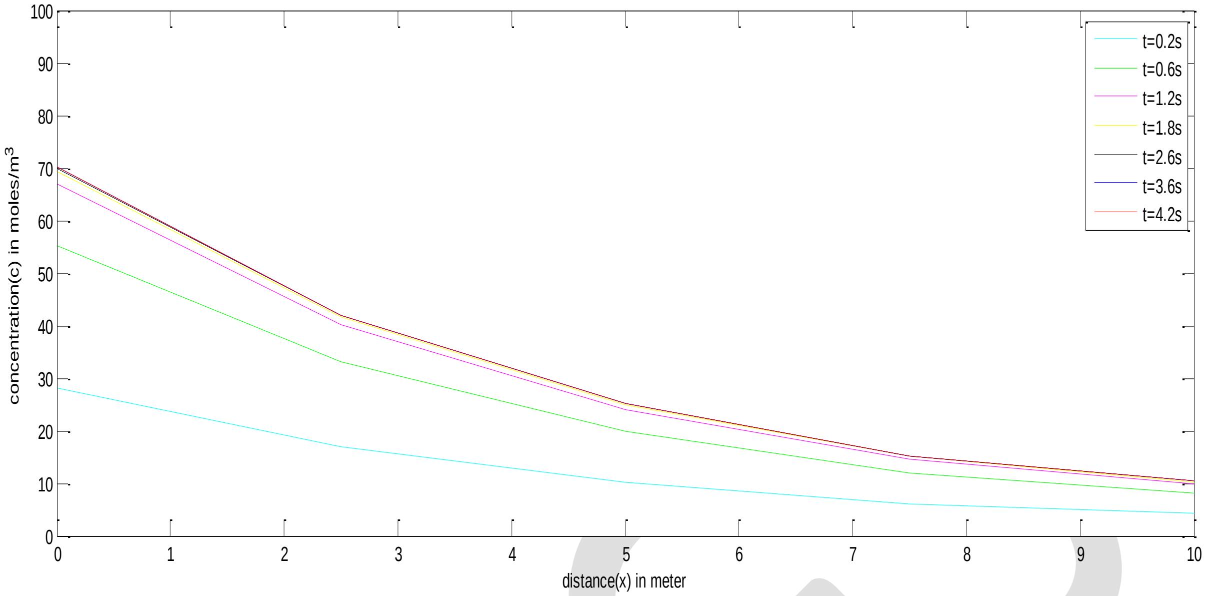

Figure-8 Spatial distribution of concentration profiles at different time interval in hour to attained to steady state (U=1m/s, $D=2m^2/h$, $k=0.3h^{-1}$)

Comparing fig.3, 7 and 8 we also get ascending profiles as time passes. It is observed that as decay constant increases the time required to attain from unsteady state to steady state decreases. Also if we compared the concentration profile at steady state in the three figure we get that if decay constant increases the concentration value also decreases.

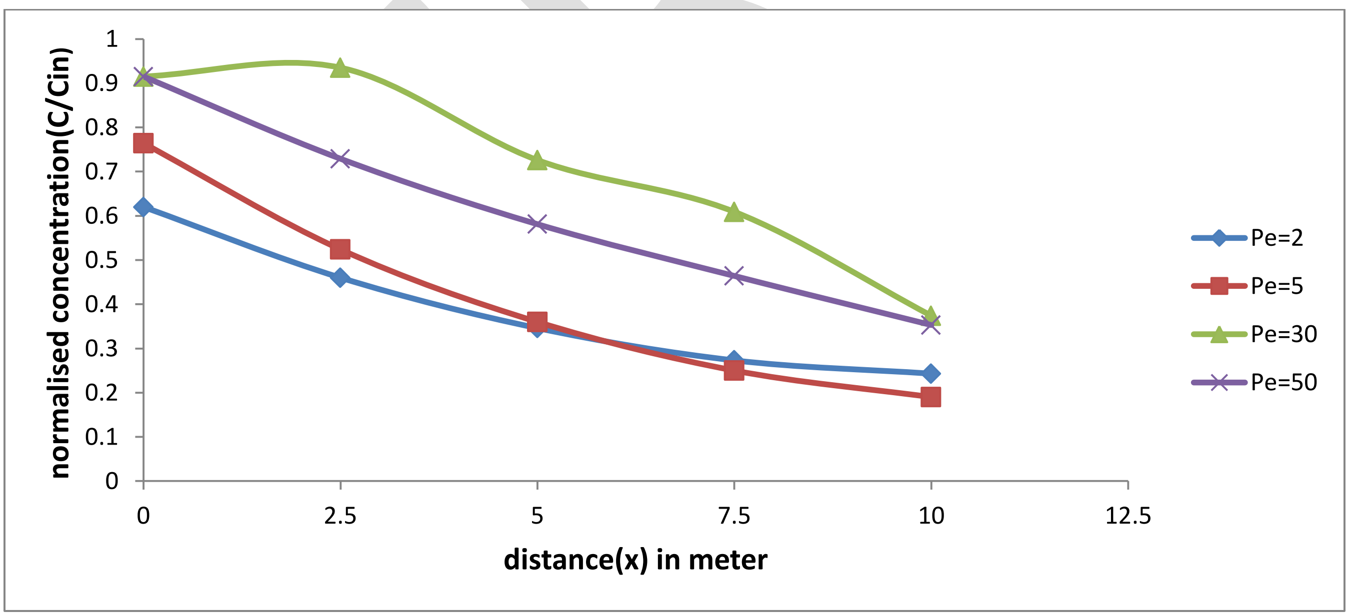

Figure-9 Spatial distribution of concentration profiles for different value of Peclet number after time 4hr ($k=0.2h^{-1}$)

Fig. 9 shows the spatial distribution of solute concentration profile at a time of 4 hour at different value of column peclet number (Pe=Ul/D where l is a reference length).We get that as Peclet no. increases the concentration value increases. But more increase in Peclet no gives an unpredicted value. To reduce the numerical error, we kept value of both the grid Peclet number($U*\Delta x/D$) and Courant number($U*\Delta t/\Delta x$) less than one. Here grid size Δx=2.5 cm and Δt=0.2h is taken.

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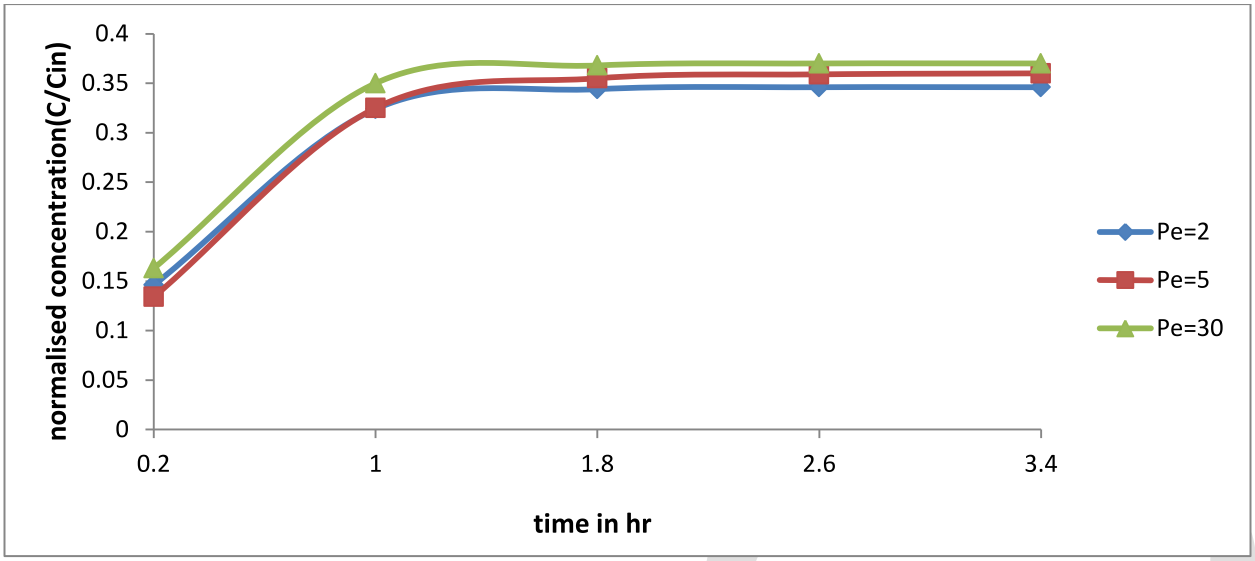

Figure-10 Temporal distribution of concentration profiles for different value of Peclet number at distance 5m.

Fig 10 shows that as time increases the solute concentration increases at early time but as time increases it becomes steady. As time increases the profile increases rapidly but after some time it becomes straight and slope becomes zero. So temporal variation is reverse of spatial variation.

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CONCLUSION

An explicit finite-difference technique has been used to get the solution of solute transport model for the case of unsteady state in porous media at field scale. According to parameter changes velocity is directly proportional to the concentration of solute where as decay constant and dispersion coefficient are inversely proportional to the concentration of solute in case of spatial variation. The time period required to attain from unsteady state to steady state decreases as velocity increases. Also it is observed that as dispersion coefficient increases the time required to attained from unsteady state to steady state increases but as decay constant increases the time required to attain from unsteady state to steady state decreases.

The effect of Peclet number on spatial and temporal solute concentration in porous media is studied. The numerical calculation minimizes the error when Peclet number taken as less than one. The concentration profile has descending nature with respect to spatial variation and ascending and remains constant after a particular time with respect to temporal variation.

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