

Diversity Analysis of Coded OFDM in Frequency Selective Channels

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Abstract— For broadband wireless communication systems, Multi-Input Multi-Output (MIMO) techniques have been incorporated with Orthogonal Frequency Division Multiplexing (OFDM).. Beamforming is a multi-input multi-output technique utilizing the channel knowledge both at the transmitter and the receiver. Multiple beamforming uses more than one subchannel to improve the capacity. For frequency selective channels, to achieve Inter Symbol Interference and achieve spatial diversity combine beamforming with OFDM. Also by adding channel coding spatial diversity and multipath diversity can be achieved The diversity analysis of BICMB-OFDM-SG is limited to $R_c SL \leq 1$ where L is the number of channel taps, S is the number of parallel streams transmitted at each subcarrier and R_c is the code rate. In this paper precoding technique is employed to overcome this limitation. Also precoding provides better performance.

Keywords— MIMO systems, Beamforming, diversity methods, subcarrier multiplexing.

INTRODUCTION

High spectral efficiency and performance for a given bandwidth can be achieved by Multiple-Input Multiple-Output (MIMO) systems. In flat fading MIMO channels, single beamforming carrying only one symbol at a time achieves full diversity but spatial multiplexing without channel coding results in the loss of the full diversity order. Bit-Interleaved Coded Multiple Beamforming (BICMB) overcomes the performance degradation.

If the channel is in frequency selective fading, Orthogonal Frequency Division Multiplexing (OFDM) can be used to combat the Inter-Symbol Interference (ISI) caused by multipath propagation. Along with this for MIMO channels beamforming achieves multipath diversity and spatial diversity. Advantage of OFDM is that it has high spectral efficiency. By adding channel coding multipath diversity can be achieved. Both spatial diversity and multipath diversity can be achieved by adding channel coding. The subcarrier grouping technique is employed to provide multi-user compatibility. Bit-Interleaved Coded Multiple Beamforming Orthogonal Frequency Division Multiplexing with Subcarrier Grouping (BICMB-OFDM-SG) technique exploits these properties. For broadband wireless communication BICMB-OFDM be an important technique. In this paper, the diversity analysis of BICMB-OFDM-SG with precoding is carried out.

SYSTEM MODEL

Consider a BICMB-OFDM-SG system employing N_t transmit and N_r receive antennas, a convolutional code of code rate R_c , and transmitting S parallel data streams.

First, generate the binary message and it is encoded using convolution encoder of code rate R_c . Trellis Structure is used to create required code rate, for a high rate punctured code a perforation matrix is combined. From the information bits this generates the bit codeword c . The code word is given to bit interleaver. For burst error-correction, interleaving is widely used. Here random bit interleaver is used.

In digital communication and storage systems, to improve the performance of forward error correcting codes interleaving is used frequently. Many communication channels in now a days are not memory less. So errors typically occur in bursts rather than independently. It fails to recover the original code word, if the number of errors within a code word exceeds the error-correcting code's capability. Interleaving creates a more uniform distribution of errors by shuffling source symbols across several code words. Interleaving in multi carrier communication also provide frequency diversity e.g., to mitigate frequency-selective fading or narrowband interference.

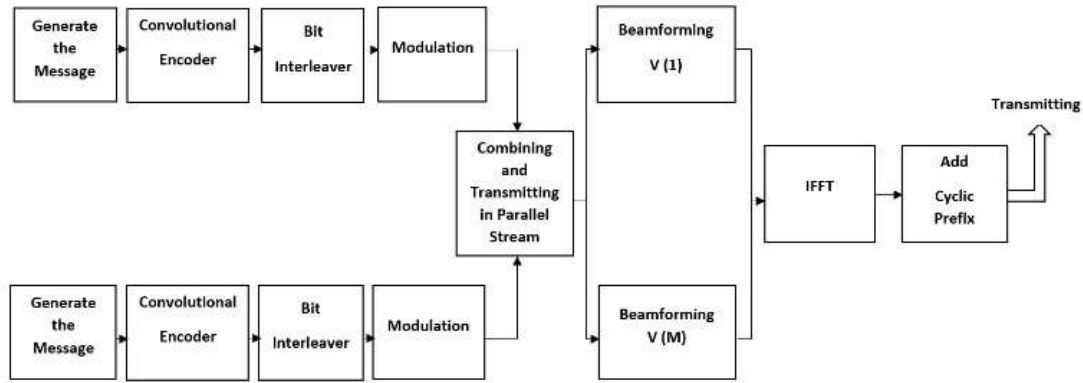


Fig. 1: Bit Interleaved Coded Multiple Beamforming with Subcarrier Grouping transmitter side

The interleaved bit sequence is then modulated, here Quadrature Amplitude Modulation (QAM) is used. Let the number of streams transmitted for each subcarrier be $S \leq \min\{N_t, N_r\}$ where N_t and N_r be the number of transmit and receive antennas. The symbol sequence is transmitted through M subcarriers. Hence, an $S \times 1$ symbol vector $x_k(m)$ is transmitted through the m^{th} subcarrier at the k^{th} time instant with $m = 1, \dots, M$. Inverse Fourier Transform is applied to the sequence. Then Cyclic Prefix is added to the sequence. The length of Cyclic Prefix (CP) is $L_{cp} \geq L$ with L denoting the number of channel taps. Cyclic prefix is employed by OFDM to combat ISI caused by multipath propagation. It is then transmitted.

Let the quasi-static flat fading MIMO channel observed at the m^{th} subcarrier be $H(m)$. The frequency selective fading MIMO channel is assumed to be Rayleigh fading channel. For each subcarrier the Singular Value Decomposition beamforming is carried out. The beamforming matrices at the m^{th} subcarrier are determined by SVD of $H(m)$,

$$H(m) = U(m)\Lambda(m)V^H(m),$$

where the $U(m)$ matrix has order of $N_r \times N_r$ and the $V(m)$ matrix of $N_t \times N_t$ are unitary, and the $N_r \times N_t$ matrix $\Lambda(m)$ is diagonal rectangular matrix. When S streams are transmitted at the same time, $U(m)$ and $V(m)$ are chosen as beamforming matrices at the receiver and transmitter at the m^{th} subcarrier, respectively. The multiplications with beamforming matrices are carried out for each subcarrier.

The system input-output relation for the m^{th} subcarrier at the k^{th} time instant is

$$y_{k,s}(m) = \lambda_s(m)x_{k,s}(m) + n_{k,s}(m)$$

with $s = 1, \dots, S$, where $y_{k,s}(m)$ and $x_{k,s}(m)$ are the s^{th} element of the $S \times 1$ received symbol vector $y_k(m)$ and the transmitted symbol vector $x_k(m)$ respectively, and $n_{k,s}(m)$ is the additive white Gaussian noise with zero mean.

In the receiver side the received data contains added white noise. Cyclic prefix is being removed from the received data. Then Fourier transform is applied to the sequence. Using the Beamforming matrix $U(m)$, information is retrieved. The data is then separated to different encoders. Each encoding section consists of QAM demodulator, Random Bit De-interleaver, and Viterbi decoder. The output obtained from the decoder is the recreated message.

Finally, the Viterbi decoder, which applies the soft-input Viterbi decoding to find the information bit sequence \hat{b} the message from the codeword \hat{c} with the minimum sum weight.

The Maximum Likelihood (ML) bit metrics at the receiver for $c_k = b \in \{0, 1\}$ as

$$\Delta(y_{k,s}(m), c_{k'}) = \min_{x \in X_{c_{k'}}^1} |y_{k,s}(m) - \lambda_s(m)x|^2$$

and makes decisions according to

$$\hat{c} = \arg \min_c \sum_{k'} \Delta(y_{k,s}(m), c_{k'})$$

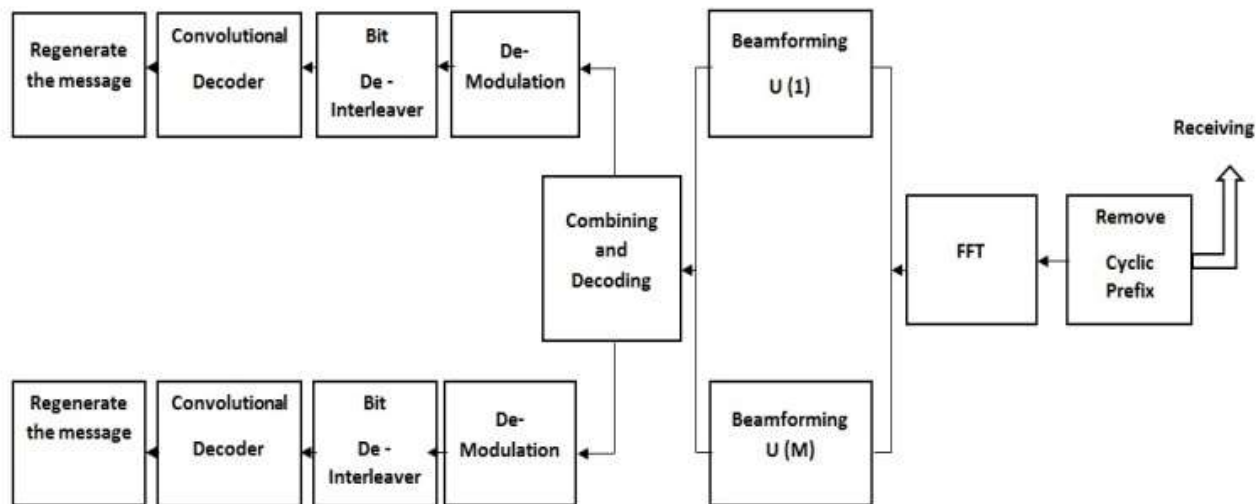


Fig. 2: Bit Interleaved Coded Multiple Beamforming with Subcarrier Grouping receiver side

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In practice, number of subcarriers M is always much larger than number of taps L . There exists correlation among subcarriers. Due to subcarrier correlation it will cause performance degradation. Subcarrier grouping is done to overcome the performance degradation. The subcarrier grouping technique is to transmit information through multiple group of subcarriers through multiple streams. The advantages of using OFDM are multi-user interference elimination, complexity reduction and Peak-to-Average Ratio (PAR) reduction.

For $L < M$, although there exists among subcarriers some subcarriers could be uncorrelated numbers. There are $G = M/L$ groups of uncorrelated subcarriers. So transmit multiple streams of bit codewords through these G different groups of uncorrelated subcarriers

The diversity of BIMB-OFDM_SG is limited to $R_c SL \leq 1$. In the case of $R_c SL > 1$, there always exists at least an error path with no errored bits transmitted through the first subchannel of a subcarrier. Proof of the limitation is explained in [2].

BICMB-OFDM-SG WITH PRECODING

The precoding technique can be applied to each subcarrier. As compared to BICMBOFDM- SG in Fig. 1, two more precoding blocks are added along with the channel coding, bit interleaver, and modulation. At the receiver side post decoding is done. Using this precoding technique it is able to overcome the criteria BICMB-OFDM-SG of $R_cSL \leq 1$. So it provides better performance with the same transmission rate and offers multi-user compatibility.

Fig. 3 and fig. 4 represents the structure of BICMB-OFDM-SG with precoding. In MIMO antenna communications precoding is a generalization of beamforming to support multi-stream transmission.

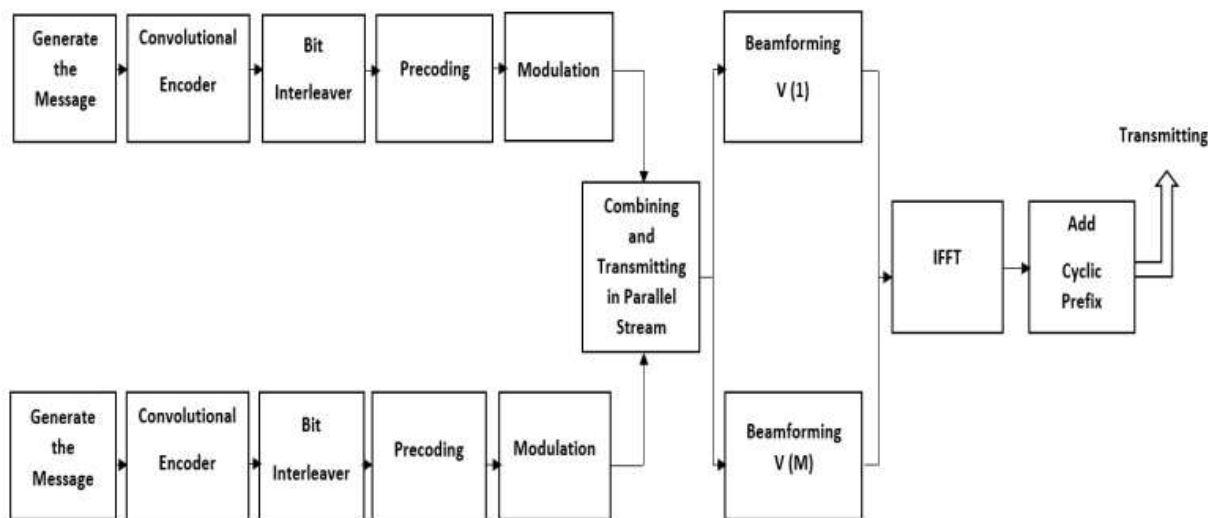


Fig.3: BICMB-OFDM_SG with precoding transmitter side

In precoding data from the interleaver is being previously coded before transmitting. The precoded data is then combined, beamformed and transmitted. At the receiving side after receiving the data, the postcoding technique is done. Information obtained after postcoding is given to demodulator, deinterleaver and decoder.

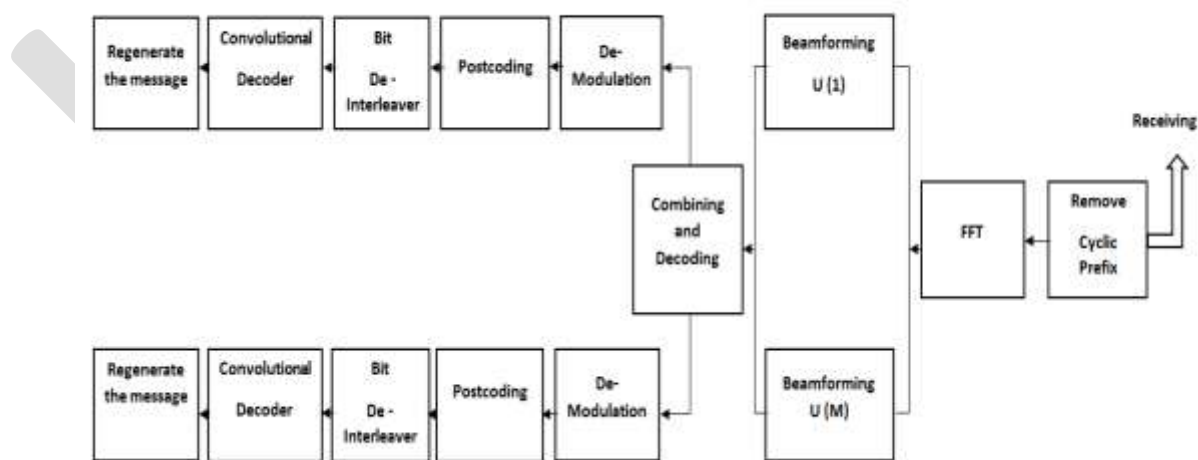


Fig. 4: BICMB-OFDM_SG with precoding receiver side

SIMULATION RESULTS

The simulation is done on MATLAB 2010. To verify the diversity analysis, $M = 64$ BICMB-OFDM-SG with $L = 1$ using 16-QAM are considered for simulations. The number of employed subchannels for each subcarrier is assumed to be the same. The generator polynomials in octal for the convolutional codes with $R_c = \frac{1}{2}$ is (5, 7) respectively. The length of CP is $L_{cp} = 8$. Different streams with $S=1$ and $S=2$ is being considered.

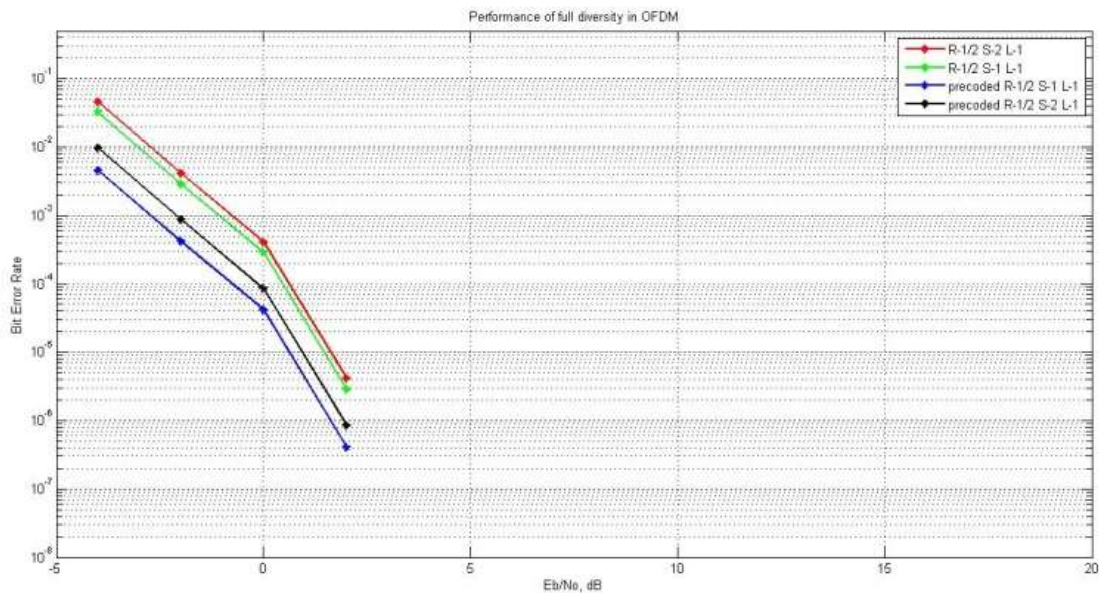


Fig. 5: BER vs. SNR for BICMB-OFDM-SG with and without precoding over equal power channel taps.

CONCLUSION

For frequency selective fading MIMO channels BICMB-OFDM-SG combines MIMO and OFDM to achieve spatial diversity, multipath diversity, spatial multiplexing, and frequency multiplexing. For broadband wireless communication it is an important technique. The comparison of BICMB-OFDM-SG and BICMB-OFDM-SG with precoding is carried out in this paper.

A sufficient and necessary condition in BICMB-OFDM-SG for achieving full diversity was $R_c S L \leq 1$. So using this precoding technique it is able to overcome the limitation and provides better result. Precoding also provide multi-user compatibility. So it is very important technique in practical application. Further improvement in diversity analysis can be done by using Low Density Parity Codes (LDPC) instead of Convolutional Codes.

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