

FINITE ELEMENT MODELLING OF COMPOSITES USING PIEZOELECTRIC MATERIAL

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ABSTRACT

This paper deals with the Deflection control of beam and frame like structures with distributed piezoelectric (PVDF) actuator layers bonded on top and bottom surfaces of the beam. The patches are located at the different positions on the frame to determine the better control effect. The study is demonstrated through simulation in MATLAB for various voltage controllers. The entire structure is modeled using the concept of piezoelectric theory, Finite Element Method (FEM) using lumped mass approach. The numerical simulation shows that the sufficient deflection control can be achieved by the proposed method.

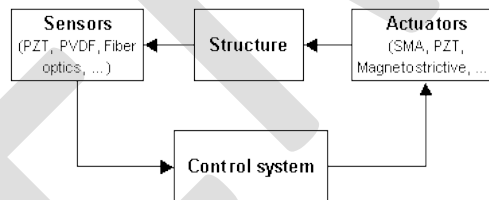
Keywords: smart structure, piezoelectric material, PVDF patch, FEM, deflection control, bimorph beam, portal frame

1. INTRODUCTION

Smart structures is a rapidly advancing field with the range of support and enabling technologies having significant advances, notable optics and electronics. The definition of smart structure was a topic of controversy from the late 1970 to 1980.

In this workshop Smart structure is defined as “A system or material which has built in intrinsic Sensor, actuator and control mechanism whereby it is capable of sensing a stimulus, responding to it in a predetermined manner and extent, in a short time and reverting to its original state as soon as the stimulus is removed.”

Smart structure contains a host structure, a sensor to gauge its internal state, an actuator to affect its internal state and, a controller whose purpose is to process the sensors and appropriately send signals to actuators.



2. DIFFERENT TYPES OF SMART MATERIALS

Smart materials and structures are load carrying components that contain arrays of sensors and actuators embedded in such a way that overall mechanical properties are not adversely affected. The host material is usually a polymer, although in principle a flexible structure built from metals and ceramics are also possible. The incorporation of actuators in the material enables a structure to optimally respond to the environment in which it performs. These actuators must therefore enable changes of shape, modifications of elastic modulus. The common types of materials used are:

- Piezoelectric materials
- Shape memory alloys
- Electrorheological fluids
- Magnetostrictors
- Electrostrictors

These materials are being configured in novel ways to accomplish adaptive changes of smart structure. In this report, the control of deflection using piezoelectric actuator is presented.

Use of Piezoelectric material

Piezoelectric are the materials that convert electrical energy to mechanical motion. Actuator devices made from these materials deliver small but accurate displacements with fast response time. Compared with electromagnetic actuators, they are more compact, consume less power, and are less prone to overheating. For these reasons these actuators have many commercial applications such as optical tracking devices, sonar transducers, and impact dot matrix printers. Piezoelectric materials are basically divided into two group Piezo-Ceramics and Piezo-Polymers. In case of a piezo-ceramic, the most common commercial piezo-polymer is Barium Titanate (BaTiO₃), Lead Titanate (PbTiO₃), Lead Zirconate ((PbZrO₃) Lead metaniobate (PbNb₂O₆) and Lead (plumbum) Zirconate Titanate (PZT) [Pb (ZrTi) O₃]. Among these materials last Lead (plumbum) Zirconate Titanate (PZT) becomes the dominant piezoelectric ceramic material for transducer due to its high coupling coefficient (0.65).

In case of a piezo-polymer, the most common commercial piezo-polymer is polyvinylidene Fluoride (PVDF). It is made up of long chains of the repeating monomer (- Ch₂ – Cf₂ -) each of which has an inherent dipole moment. Both PZT and PVDF are usually produced in the thin sheets with film of metal deposited on the opposite surface to form electrodes.

1. ANALYSIS OF A CANTILEVER BEAM

In this paper, a simple smart system with PVDF actuator is tested for deflection analysis. A bimorph cantilever beam consisting of two identical PVDF beams laminated together with opposite polarities is

The opposite forces developed at the free end due to piezoelectric material form a couple of moment M as shown in figure. Hence, by using the actuation law of a piezoelectric material the deflection variation for a bimorph cantilever beam with end moment M formed due to embedded PVDF material is calculated as

$$w(x)_{elec} = \frac{e_{31} V}{E} \left(\frac{x}{t}\right)^2 \quad (1)$$

$$w(l)_{elec} = \frac{e_{31} V}{E} \left(\frac{l}{t}\right)^2 \quad \text{at } x=l$$

In addition to this electric field if a mechanical load 'P' is applied on the beam as shown in fig 2 below,

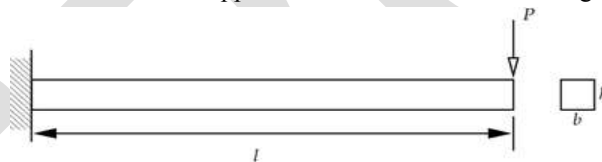


Fig 2

Then the deflection formed due to this applied mechanical load P at the end is

$$W(x)_{mech} = P \left(\frac{x^3}{6EI} - \frac{x^2 l}{2EI} \right) \quad (2)$$

The tip deflection in this case when $x=L$ is

$$W(L)_{mech} = - \left(\frac{Pl^3}{3EI} \right)$$

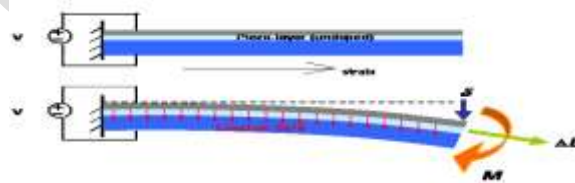


Fig1

considered as shown in figure 1. The PVDF patches are poled in such a way that it produces strain perpendicular to poling direction.

Total deflection corresponding to Applied Mechanical and Electrical load

Now when both mechanical and electrical loads are applied, the total deflection will be sum of the deflections due to mechanical load and electrical load, which given by

$$W(x)_{total} = w(x)_{mech} + w(x)_{elec}$$

$$W(x)_{total} = P\left(\frac{x^3}{6EI} - \frac{x^2 l}{2EI}\right) + \frac{e_{31} V}{E} \left(\frac{x}{t}\right)^2 \quad (3)$$

Maximum deflection occurs when $x=l$.Substituting in above equation we get

$$w(l)_{total} = w(l)_{mech} + w(l)_{elec}$$

$$w(l)_{total} = -\left(\frac{Pl^3}{3EI}\right) + \frac{e_{31} V}{E} \left(\frac{l}{t}\right)^2 \quad (4)$$

The deflection variations along the length of the beam with respect to the applied voltages is shown using commercial FE software (MATLAB). The graph corresponding to eq.(3) is plotted using MATLAB and the deflection variations due to applied voltages on the beam are shown in figure 3 below

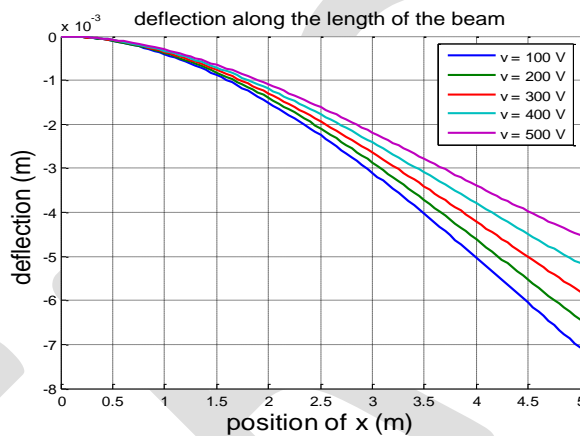


Fig 3

From the above graph which is from eq(3) we see that as the voltage is increased , it reduces the net downward deflection due to the mechanical load .Hence, It is clear that *the* presence of electrical load helps to eliminate the deflection of the cantilever beam due to mechanical load.

The voltage necessary to force the total deflection equal to zero is given by the eq(4) as $V = \frac{pl}{e_{31}bt}$

2. ANALYSIS OF A SIMPLY SUPPORTED BEAM

As discussed earlier, in the same way if two identical PVDF patches are laminated together with opposite polarities on a simply supported bimorph beam as shown in fig (4). the total deflection along the length of the beam due to applied mechanical load and the electrical load is determined as follows

Using the actuation law,The transverse displacement $w(x)$ of a simply supported beam with a moment M formed at both ends due to PVDF material as shown in fig(4) for the section AC is given by

$$w(x)_{elec} = -\frac{2e_{31}V}{Et^2} \left(\frac{x^3}{3l} - \frac{x^2}{2} + \frac{lx}{4}\right) \quad eq(5)$$

$$w(l)_{elec} = -\frac{e_{31} V}{12E} \left(\frac{l}{t}\right)^2 \quad \text{at } x=l/2$$

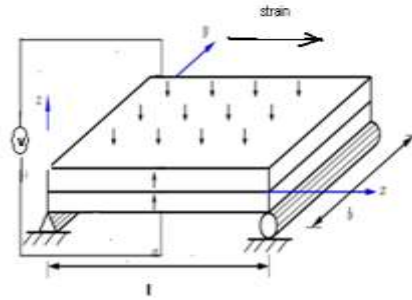


Fig 4

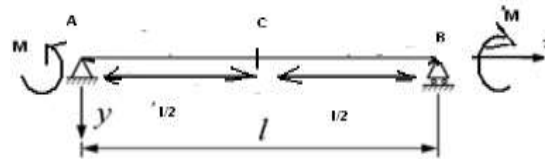


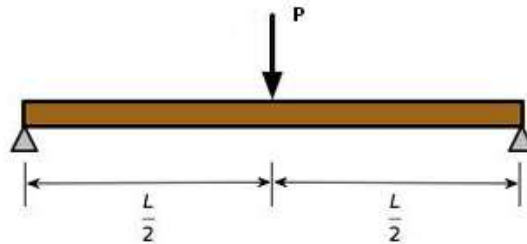
Fig 5

Similarly, the transverse displacement $w(x)$ of a simply supported beam with a moment M formed at both ends due to PVDF material as shown in fig (b) for the section **CB** is given by

$$w(x)_{elec} = -\frac{2e_{31}V}{Et^2} \left(-\frac{x^3}{3l} + \frac{x^2}{2} - \frac{lx}{4} + \frac{l^2}{12} \right) \quad \text{eq(6)}$$

$$w(l)_{elec} = -\frac{e_{31}V}{12E} \left(\frac{l}{l} \right)^2 \quad \text{at } x=l/2$$

In addition to the electric field if mechanical load (p) is applied



Deflection of a simply supported beam subjected to concentrated load P at the centre for the section **AC** is

$$w(x)_{mech} = \left(-\frac{Px^3}{12EI} + \frac{Pl^2x}{16EI} \right) \quad \text{eq(7)}$$

The deflection of the section AC when $x=l/2$ is $w(l)_{mech} = \left(\frac{Pl^3}{48EI} \right)$

Similarly, the deflection of a simply supported beam subjected to concentrated load P at the Centre for the section **CB** is

$$w(x)_{mech} = \left(\frac{Px^3}{12EI} - \frac{Plx^2}{4EI} + \frac{3Pl^2x}{16EI} - \frac{Pl^3}{48EI} \right) \quad \text{eq(8)}$$

The deflection of the section AC when $x=l/2$ is $w(l)_{mech} = \left(\frac{Pl^3}{48EI} \right)$

Total deflection corresponding to Applied Mechanical and Electrical load

$$w(x)_{total} = w(x)_{mech} + w(x)_{elec}$$

The total deflection of the section **AC** is

$$w(x)_{\text{total}} = \left(-\frac{Px^3}{12EI} + \frac{Pl^2x}{16EI} \right) - \frac{2e_{31}V}{Et^2} \left(\frac{x^3}{3l} - \frac{x^2}{2} + \frac{lx}{4} \right) \quad \text{eq(9)}$$

The total deflection of the section **CB** is

$$w(x)_{\text{total}} = \left(\frac{Px^3}{12EI} - \frac{Plx^2}{4EI} + \frac{3Pl^2x}{16EI} - \frac{Pl^3}{48EI} \right) - \frac{2e_{31}V}{Et^2} \left(-\frac{x^3}{3l} + \frac{x^2}{2} - \frac{lx}{4} + \frac{l^2}{12} \right) \quad \text{eq(10)}$$

Maximum deflection occurs at $x=l/2$. Substituting $x=l/2$ in **eq (9&10)** the total deflection along the length of the beam AB is given as

$$w(l)_{\text{total}} = \left(\frac{Pl^3}{48EI} \right) - \frac{e_{31}V}{12E} \left(\frac{l}{t} \right)^2 \quad \text{eq(10a)}$$

Basically the electrical deflection opposes the mechanical deflection, so when we increase the voltage V it will decrease the total deflection to a such a level that we can actually choose the voltage which reduces the total deflection to maximum extent.

The graph corresponding to eq. (9&10) is plotted using MATLAB and the deflection variations due to applied voltages on the beam are shown in figure below. From the above graph which is from eq (9 & 10) we see that as the voltage is increased, it reduces the net downward deflection due to the mechanical load. Hence, it is clear that the presence of electrical load helps to totally eliminate the deflection of the simply supported beam due to mechanical load.

The voltage necessary to force the total deflection equal to zero is given by the **eq (10a)** as $V = \frac{3pl}{4e_{31}bt}$.

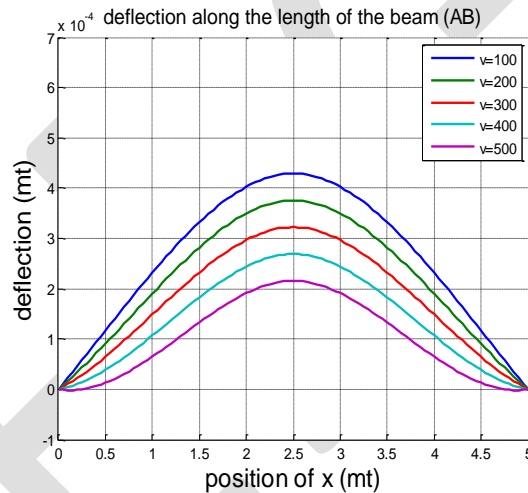


Fig 6

3. Analysis of a Piezoelectric Bimorph Portal frame

In this chapter a determinate portal frame is considered and in the same way two identical PVDF patches are laminated together with opposite polarities are placed on all the legs of a portal frame individually then the deflection at a point C is determined and the deflection variations are plotted using MATLAB.

3.1 Analysis of a portal frame with piezoelectric smart material on AB

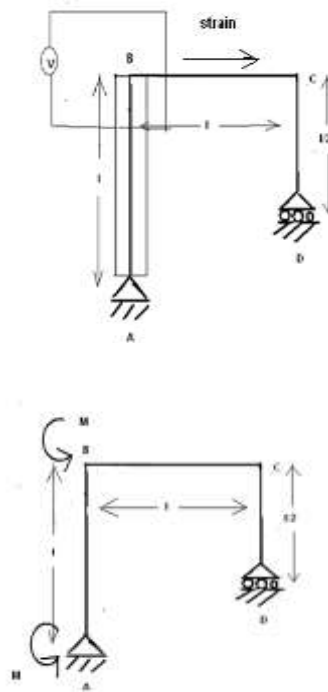


Fig 7

A portal frame ABCD is considered frame which consists of two identical PVDF beams laminated together with opposite polarities on AB as shown in fig(7). The PVDF patches are poled in such a way that it produces strain perpendicular to poling direction.

Using actuation law, The displacement variation in a bi-morph piezoelectric portal frame $w(x)$ with a moment M at both ends of AB formed due to PVDF patch as shown in fig (b) for the section CD is

$$w(x)_{elec} = -\frac{2e_{31}V}{Et^2} \left(-\frac{x^2}{2} + \frac{lx}{2} \right) \quad \text{eq(11)}$$

$$w(l)_{elec} = -\frac{e_{31}V}{4E} \left(\frac{l}{t} \right)^2 \quad \text{at } x=l/2$$

In addition to the electric field if mechanical load(p) is applied on a frame shown in fig 8

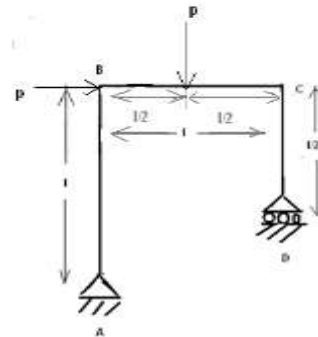


Fig 8

Deflection of a portal frame subjected to concentrated load P at the centre of BC and sway force P at B for the section CD is

$$w(x)_{mech} = \left(-\frac{Px^3}{6EI} + \frac{Pl^2x}{8EI} \right) \quad \text{eq(12)}$$

The deflection of the section CD when $x=l/2$ is $w(l)_{mech} = \left(\frac{Pl^3}{12EI} \right)$

Now when both mechanical and electrical loads are applied, the total deflection will be sum of the deflections due to mechanical load and electrical load, which given by

$$w(x)_{total} = w(x)_{mech} + w(x)_{elec}$$

The total deflection of the section **CD** when the piezoelectric smart material is on **AB** is

$$w(x)_{total} = \left(-\frac{Px^3}{6EI} + \frac{Pl^2x}{8EI} \right) - \frac{2e_{31}V}{Et^2} \left(-\frac{x^2}{2} + \frac{lx}{2} \right)$$

eq(13)

Maximum deflection on **CD** occurs at $x=l/2$. Substituting $x=l/2$ in **eq (13)** the total deflection along the length of the section **CD** is given as

$$w(l)_{total} = \left(\frac{Pl^3}{12EI} \right) - \frac{e_{31}V}{4E} \left(\frac{l}{t} \right)^2$$

eq(14)

the deflection variations with respect to applied voltages are plotted using MATLAB. The graph corresponding to eq.(13) plotted using MATLAB and the deflection variations due to applied voltages when a piezoelectric material is embedded on **AB** are shown in figure below

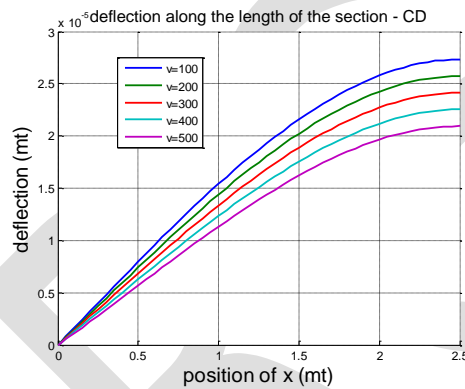


Fig 9

From the above graph which is from eq(13) we see that as the voltage is increased, it reduces the net downward deflection due to the mechanical load. Hence, it is clear that the presence of electrical load helps to totally eliminate the deflection of the portal frame **CD** due to mechanical load.

The voltage necessary to force the total deflection equal to zero is given by the **eq (14)** as

$$V = \frac{pl}{e_{31}bt}$$

5.2. Analysis of a portal frame with piezoelectric smart material on **BC**

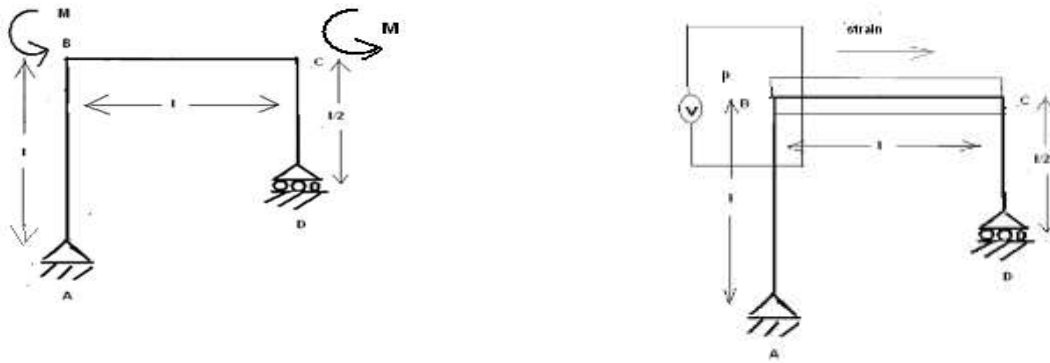


Fig10

when a PVDF patch is placed on BC shown in the figure the total deflection at the point C due to applied mechanical load and the electrical load is determined as follows

Using actuation law, The displacement variation in a bi-morph piezoelectric portal frame $w(x)$ with a moment M at both ends of BC formed due to PVDF patch as shown in fig (10) for the section CD is

$$w(x)_{elec} = -\frac{2e_{31}V}{Et^2}(-x^2 + lx) \quad \text{eq(15)}$$

$$w(l)_{elec} = -\frac{e_{31}V}{2E}\left(\frac{l}{t}\right)^2 \quad \text{at } x=l/2$$

In addition to the electric field if mechanical load(p) is applied on a frame shown in fig8 earlier

Deflection of a portal frame subjected to concentrated load P at the centre of BC and sway force P at B for the section CD is

$$w(x)_{mech} = \left(-\frac{Px^3}{6EI} + \frac{Pl^2x}{8EI}\right) \quad \text{eq(17)}$$

The total deflection of the section CD when the piezoelectric smart material is on BC is $w(x)_{total} = \left(-\frac{Px^3}{6EI} + \frac{Pl^2x}{8EI}\right) - \frac{2e_{31}V}{Et^2}(-x^2 + lx)$ eq(18)

Maximum deflection on CD occurs at $x=l/2$. Substituting $x=l/2$ in eq (16) the total deflection along the length of the section CD is given as

$$w(l)_{total} = \left(\frac{Pl^3}{12EI}\right) - \frac{e_{31}V}{2E}\left(\frac{l}{t}\right)^2 \quad \text{eq(19)}$$

The graph corresponding to eq.(16) plotted using MATLAB and the deflection variations due to applied voltages when a piezoelectric material is embedded on BC is shown in figure 11 below

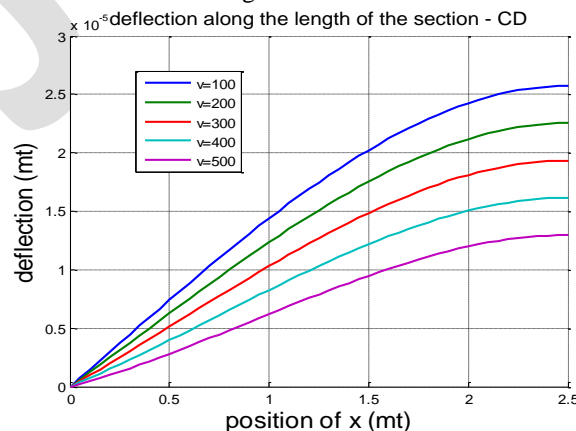


Fig11

The voltage necessary to force the total deflection equal to zero is given by the eq (17) as

$$V = \frac{pl}{2e_{31}bt}$$

5.3 Analysis of a portal frame with piezoelectric smart material on CD

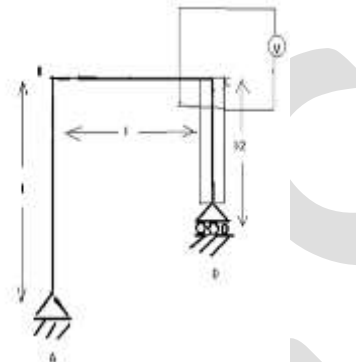
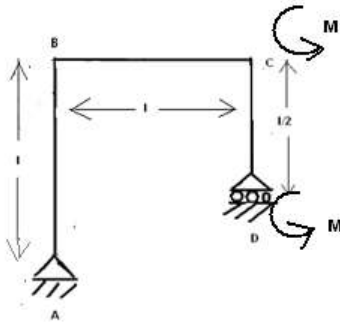


Fig12

When a PVDF patch is placed on CD shown in the figure the total deflection at the point C due to applied mechanical load and the electrical load is determined as follows Using actuation law, The displacement variation in a bi-morph piezoelectric portal frame $w(x)$ with a moment M at both ends of CD formed due to PVDF patch as shown in fig (12) for the section CD is

$$w(x)_{elec} = -\frac{2e_{31}V}{Et^2} \left(-\frac{x^2}{2} + \frac{lx}{2} \right) \quad \text{eq(18)}$$

$$w(l)_{elec} = -\frac{e_{31}V}{4E} \left(\frac{l}{t} \right)^2 \quad \text{at } x=l/2$$

as discussed earlier if the same mechanical load is applied shown in figure 8 then the deflection due to the applied mechanical load is given in the eq(17).

Hence the total deflection due to the applied mechanical and electrical loads is as follows

$$w(x)_{total} = \left(-\frac{Px^3}{6EI} + \frac{Pl^2x}{8EI} \right) - \frac{2e_{31}V}{Et^2} \left(-\frac{x^2}{2} + \frac{lx}{2} \right) \quad \text{eq(19)}$$

Maximum deflection on CD occurs at $x=l/2$. Substituting $x=l/2$ in eq (19) the total deflection along the length of the section CD is given as

$$w(l)_{total} = \left(\frac{Pl^3}{12EI} \right) - \frac{e_{31}V}{4E} \left(\frac{l}{t} \right)^2 \quad \text{eq(20)}$$

The graph corresponding to eq.(16) plotted using MATLAB and the deflection variations due to applied voltages when a piezoelectric material is embedded on CD is shown in figure below

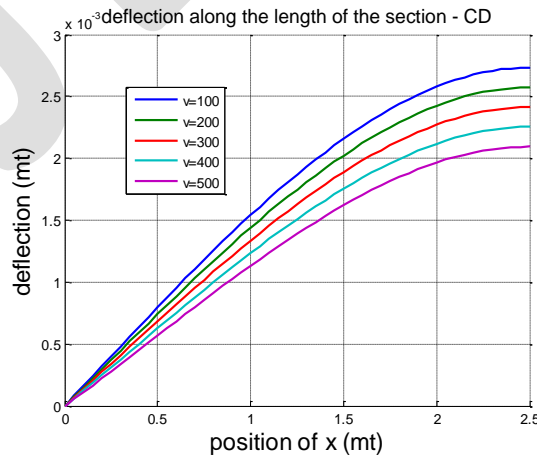


Fig 13

The voltage necessary to force the total deflection equal to zero is given by the eq(20) as

$$V = \frac{pl}{e_{31}bt} .$$

CONCLUSION:

Present work deals with the mathematical formulation and the computational analysis for the deflection control of a structure with piezoelectric smart material. By this thesis it is acceptable that using a piezoelectric material in beams and also in a frame, deflection can be controlled at different voltages.

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