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LP- SASAKIAN MANIFOLDS WITH SOME CURVATURE PROPERTIES

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ABSTRACT

The object of the present paper is to study the extended generalised φ-recurrent LP-Sasakian manifolds. Also the existence of such manifold is ensured by an example.

KEYWORDS: LP-Sasakian Manifold, Generalised Recurrent LP-Sasakian Manifold, Extended Generalized φ- Recurrent LP-Sasakian Manifold, Quasi-Constant Curvature.

1. INTRODUCTION

In 1989, K. Matsumoto ([1]) introduced the notion of LP-Sasakian manifolds. Then I. Mihai & R. Rosca ([3]) introduced the same notion independently & obtained many interesting results. LP-Sasakian manifolds are also studied by U. C. Dey, K. Matsumoto & A. A. Shaikh ([4]), I. Mihai, U. C. De & A. A. Shaikh ([2]) & others ([5], [6], [7]).

The notion of local symmetry of Riemannian manifolds has been weakened by many authors in several ways to a different extent. In [8] Takahasi introduced the notion of locally φ-symmetric Sasakian manifolds as a weaker version of local symmetry Riemannian manifolds. In [9], De et al studied the φ-recurrent Sasakian manifold. In [12], Al-Aqeel et al studied the notion of generalized recurrent LP-Sasakian maniofold. Generalised recurrent manifold is also studied by Khan [14] in the frame of Sasakian manifold. Recently, Jaiswal et al [11] studied generalised φ-recurrent LP-Sasakian manifold. Motivated from the work of Shaikh & Hui, we propose to study extended generalized φ-recurrent LP-Sasakian manifold. The paper is organised as follows

In section 2, we give brief account of LP-Sasakian manifolds. In section 3, we study generalised φ-recurrent LP-Sasakian manifolds & obtained that the associated vector field of the 1-forms are co-directional with the unit timelike vector field ξ. Section 4 is concerned with extended generalised φ-recurrent LP-Sasakian manifolds & found that such a manifold is generalised Ricci recurrent provided the 1-forms are linearly dependent, whereas every generalized φ-recurrent LP-Sasakian manifold is generalised Ricci recurrent. Among others, we have also proved that such a manifold is of quasi-constant curvature & the unit timelike vector ξ is harmonic. In section 5, the existence of extended generalised φ-recurrent LP-Sasakian manifold is ensured by an example.

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2. LP SASAKIAN MANIFOLDS

An n-dimensional differentiable manifold M is said to be an LP-Sasakian manifold ([6],[7],[8]), if it admits a (1,1) tensor field φ, a unit timelike contravariant vector field ξ, and a 1-form η and a Lorentzia metric g which satisfy the relations:

$$\eta(\xi) = -1,\ g(X, \xi) = \eta(X),\ \varphi^2 X = X + \eta(X)\xi, \tag{2.1}$$

$$g(\varphi X, \varphi Y) = g(X, Y) + \eta(X)\eta(Y),\ \nabla_X \xi = \varphi X, \tag{2.2}$$

$$(\nabla_X \varphi)(Y) = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{2.3}$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g. It can be easily seen that in an LP-Sasakian manifold, the following relations hold:

$$\varphi\xi = 0,\ \eta(\varphi X) = 0,\ rank\ \varphi = n - 1. \tag{2.4}$$

Again, if we put

$$\Omega(X,Y) = g(X, \varphi Y),$$

for any vector field X,Y then the tensor field $\Omega(X,Y)$ is a symmetric (0,2) tensor field ([3],[7]). Also, since the vector field η is closed in an LP-Sasakian ([2], [4]) manifold, we have

$$(\nabla_X \eta)(Y) = \Omega(X, Y),\ \Omega(X, \xi) = 0, \tag{2.5}$$

for any vector field X & Y.

Let M be an n-dimensional LP-Sasakian manifold with structure (φ, ξ, η, g). Then the following relations hold ([7]):

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.6}$$

$$\eta(R(X, Y)Z) = g(Y, Z)\eta(X) - g(X,Z)\eta(Y), \tag{2.7}$$

$$S(X, \xi) = (n - 1)\eta(X), \tag{2.8}$$

$$S(\varphi X, \varphi Y) = S(X, Y) + (n - 1)\eta(X)\eta(Y), \tag{2.9}$$

$$\begin{aligned}(\nabla_W R)(X,Y)\xi &= 2[\Omega(Y, W)X - \Omega(X, W)Y] - \varphi R(X, Y)W \\ &- g(Y, W)\varphi X + g(X,W)\varphi Y - \\ &2[\Omega(X, W)\eta(Y) - \Omega(Y, W)\eta(X)]\xi \\ &- 2[\eta(Y)\varphi X - \eta(X)\varphi Y]\eta(W),\end{aligned} \tag{2.10}$$

$$g((\nabla_W R)(X, Y)Z, U) = -g((\nabla_W R)(X, Y)U, Z), \tag{2.11}$$

for any vector field X,Y,Z,U on M where R is the curvature tensor of the manifold.

3. GENERALISED Φ RECURRENT LP-SASAKIAN MANIFOLDS

Definition3.1. An LP-Sasakian manifold is called generalised φ-recurrent, if its curvature tensor R satisfies the condition:

$$\varphi^2((\nabla_W R)(X,Y)Z) = A(W)R(X, Y)Z + B(W)[g(Y, Z)X - g(X, Z)Y], \tag{3.1}$$

where A and B are two non-zero l-forms and these are defined as

A (W)= g (W,ρ), B (W)= g (W,σ),

where ρ, σ are the vector fields associated to the 1-form A & B respectively. If the 1-form B vanishes identically, then the equn. (3.1) becomes

$$\varphi^2 ((\nabla_W R)(X, Y)Z) = A(W) R(X, Y)Z, \qquad (3.2)$$

and such manifold is known as φ-recurrent LP-Sasakian manifold which is studied by Al-Aqeel, De & Ghosh [13].

Theorem3.1. *Every Generalised φ-recurrent LP-Sasakian manifold (M^n,g) (n > 3) is generalised Ricci recurrent.*

Proof: Using (2.1) in (3.1) & then taking inner product in both sides by U, we have

$$g((\nabla_W R)(X, Y)Z, U) + \eta((\nabla_W R)(X, Y)Z)\,\eta(U)$$

$$= A(W)\, g(R(X, Y)Z, U) + B(W)\,[g(Y, Z)\, g(X, U) - g(X, Z)\, g(Y, U)] \qquad (3.3)$$

Let {e_i, *i* = 1, 2,..., n} be an orthonormal basis at any point P of the manifold M. Setting X=U=e_i, in (3.3) & taking summation over i, 1< i < n, we get

$$(\nabla_W S)(Y,Z) + \sum_{i=1}^{n} \eta((\nabla_W R)(e_i, Y)Z)\,\eta(e_i) = 0.$$

$$= A(W)S(Y,Z) + (n-3)B(W)g(Y,Z). \qquad (3.4)$$

In view of (2.9) & (2.10), the expression

$$\sum_{i=1}^{n} \eta((\nabla_W R)(ei, Y)Z)\,\eta(ei) = 0. \qquad (3.5)$$

By virtue of (3.5), (3.4) yields

$$(\nabla_W S)(Y, Z) = A(W)\, S(Y, Z) + (n - 3)\, B(W)\, g(Y, Z), \qquad (3.6)$$

for all W, Y, Z. This completes the proof.

Corollary 3.1. *Every generalised φ-recurrent LP-Sasakian manifold (M^n,g) (n > 3) is an Einstein manifold.*

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Proof: Replacing Z by ξ in (3.6) & using (2.8), we obtain

$$(n-1)\Omega(W, Y) - S(Y, \varphi W) = (n-1)\, A(W)\, \eta(Y) + (n-3)\, B(W)\, \eta(Y). \qquad (3.7)$$

Replacing Y by φY in (3.7) & then using (2.2) & (2.9), we get

$$S(Y, W) = (n-1)\, g(Y, W), \qquad (3.8)$$

for all Y & W. This completes the proof.

Theorem.3.2. *In a generalized φ-recurrent LP-Sasakian manifold (M^n,g) (n > 3), the Ricci tensor S along the associated vector field of the 1-form A is given by*

$$S(Z, \rho) = (½)[rA(Z) + (n - 3)(n - 4) B(Z)]. \quad (3.9)$$

Proof: Contracting over Y & Z in (3.6), we get

$$dr(W) = A(W)\, r + (n - 3)(n - 2) B(W), \quad (3.10)$$

for all W.

Again, contracting over W & Y in (3.6), we have

$$(1/2)\, dr(Z) = S(Z,\rho) + (n - 3) B(Z). \quad (3.11)$$

By virtue of (3.10) & (3.11), we get (3.9).This proves the theorem.

Theorem.3.3. *In a generalised φ-recurrent LP-Sasakian manifold (M^n,g) (n > 3), the associated vector field corresponding to the 1-forms A & B are co-directional with the unit timelike vector field ξ.*

Proof: Setting Z=ξ in (3.9) & using (2.8), we get

$$\eta(\rho) = \left[\frac{(n-3)(n-4)}{2(n-1)-r}\right]\eta(\sigma). \quad (3.12)$$

This completes the proof.

4. EXTENDED GENERALIZED Φ-RECURRENT LP-SASAKIAN MANIFOLDS

Definition 4.1.([12]). An LP-Sasakian manifold is said to be extended generalised φ-recurrent, if its curvature tensor R satisfies the condition

$$\varphi^2((\nabla_W R)(X, Y)Z) = A(W)\, \varphi^2(R(X,Y)Z) + B(W)\, [g(Y, Z)\, \varphi^2(X) - g(X, Z)\, \varphi^2(Y)], \quad (4.1)$$

where A and B are two non-zero 1-forms and these are defined as

$A(W) = g(W, \rho)$, $B(W) = g(W, \sigma)$

and ρ, σ are vector fields associated to the 1-form A & B respectively.

Theorem 4.1. *Let (M^n,g) (n > 3) be an extended generalised φ-recurrent LP-Sasakian manifold. Then such a manifold is a generalised Ricci recurrent LP-Sasakian manifold if the associated 1-forms are linearly dependent & the vector fields of the associated 1-forms are of opposite directions.*

Proof: Using (2.1) in (4.1) & then taking inner product on both sides by U, we have

$g((\nabla_W R)(X, Y)Z, U) + \eta((\nabla_W R)(X, Y)Z)\, \eta(U)$

$= A(W)\, [g(R(X, Y)Z, U) + \eta(R(X, Y)Z)\, \eta(U)]$

$+ B(W)\, [g(Y, Z)\, g(X, U) - g(X, Z)\, g(Y, U)$

$+\{g(Y, Z)\eta(X) - g(X, Z)\ \eta(Y)\eta(U)\}]$. (4.2)

Let $\{e_i, i=1,2,...,n\}$ be an orthonormal basis at any point P of the manifold M. Setting $X=U=e_i$, in (4.2) & taking summation over i, $1 < i < n$, we get

$(\nabla_W S)(Y,Z)+ \sum \eta((\nabla_W R)(e_i, Y)Z)\ \eta(e_i)=0.$

$= A(W)\ [S(Y, Z)+ \eta(R)\ \xi(Y, Z)]$

$+B(W)\ [(n - 2)\ g(Y, Z) - \eta(Y)\ \eta(Z)].$ (4.3)

In view of (2.9) & (2.10), the expression

$$\sum_{i=1}^{n} \eta((\nabla_W R)(e_i, Y)Z)\ \eta(e_i)=0. \quad (4.4)$$

By virtue of (2.7) & (4.4), (4.3) yields

$(\nabla_W S)(Y,Z) = A(W)\ S(Y,Z) + (n - 2)\ B(W)\ g(Y,Z)$

$-[A(W)+ B(W)]\eta(Y)\ \eta(Z).$ (4.5)

If the associated vector fields of the 1-forms are of opposite directions, i.e., $A(W) + B(W) = 0$, then (4.5) becomes

$(\nabla_W S)(Y,Z) = A(W)\ S(Y, Z) + (n - 2)\ B(W)\ g(Y, Z).$ (4.6)

This completes the proof.

Theorem 4.2. *Every extended generalised φ-recurrent LP-Sasakian manifold (M^n, g) $(n > 3)$ is an Einstein manifold.*

Proof: Setting $Z=\xi$ in (4.5) & then using (2.2) & (2.8), we get

$(n - 1)\ \Omega(W, Y) - S(Y, \varphi W)=[nA(W) + (n - 1)\ B(W)]\eta(Y).$ (4.7)

Replacing Y by φY in (4.7) & using (2.2), (2.4) & (2.9), we obtain

$S(Y, W) = (n - 1)\ g(Y, W),$ (4.8)

for all Y, W. This completes the proof.

Theorem 4.3. *In an extended generalised φ-recurrent LP-Sasakian manifold (M^n, g) $(n > 3)$, the timelike vector field ξ is harmonic provided the vector fields associated to the 1-forms are codirectional.*

Proof: In an extended generalised φ-recurrent LP-Sasakian manifold (M^n,g) $(n > 3)$, the relation (4.2) holds. Replacing Z by ξ in (4.2), we have

$(\nabla_W R)(X, Y)\ \xi = A(W)\ R(X, Y)\ \xi + B(W)[\eta(Y)\ X - \eta(X)\ Y]$

$= [A(W) + B(W)]\ [\eta(Y)\ X - \eta(X)\ Y].$ (4.9)

By virtue of (2.10) & (4.9), we have

$$\varphi R(X, Y)W = [A(W) + B(W)][\eta(X)Y - \eta(Y)X]$$
$$+2[\Omega(Y, W)X - \Omega(X, W)Y] - \varphi R(X, Y)W$$
$$-g(Y, W)\varphi X + g(X, W)\varphi Y$$
$$-2[\Omega(X, W)\eta(Y) - \Omega(Y, W)\eta(X)]\xi$$
$$-2[\eta(Y)\varphi X - \eta(X)\varphi Y]\eta(W). \quad (4.10)$$

Taking inner product in both sides of (4.10) by φU & then using (2.2), we obtain

$$\acute{R}(X, Y, W, U) = [A(W) + B(W)][\Omega(Y, U)\eta(X) - \Omega(X,U)\eta(Y)]$$
$$+2[\Omega(Y, W)\Omega(X, U) - \Omega(X, W)\Omega(Y, U)]$$
$$-g(Y, W)g(X, U) + g(X, W)g(Y, U)$$
$$+2[g(X, W)\eta(Y)\eta(U) - g(Y, W)\eta(X)\eta(U)$$
$$+g(Y,U)\eta(W)\eta(X) - g(X, U)\eta(W)\eta(Y)], \quad (4.11)$$

where $\acute{R}(X,Y, W,U) = g(R(X,Y)W,U)$.

Contracting over X & U in (4.11), we get

$$S(Y, W) = 2[\psi\,\Omega(Y,W) - g(\varphi Y,\varphi W)] - \psi[A(W)+ B(W)]\eta(Y)$$
$$-(n-3)g(Y, W) - 2(n - 2)\eta(Y)\eta(W), \quad (4.12)$$

where $\psi = Tr.\varphi$.

Next setting $Y=\xi$ in (4.12), we get

$$\psi[A(W) + B(W)] = 0, \quad (4.13)$$

which yields $\psi = 0$, because the vector fields associated to the 1-forms are codirectional. Consequently, ξ is harmonic. This completes the proof.

Theorem 4.4. *Every extended generalised φ-recurrent LP-Sasakian manifold (M^n,g) $(n > 3)$ is η--Einstein, if the vector fields associated to the 1-forms are codirectional.*

Proof: Since in an generalised φ-recurrent LP-Sasakian manifold (M^n,g) $(n > 3)$, the timelike vector field ξ is harmonic i.e., $\psi = 0$ for $A(W) \neq - B(W)$, it follows from (4.12) that

$$S(Y, W) = -(n-1)g(Y, W) - 2(n - 1)\eta(Y)\eta(W), \quad (4.14)$$

which proves the theorem.

Definition 4.2. An LP-Sasakian manifold (M^n,g) $(n > 3)$ is said to be a manifold of quasi-constant curvature, if its cuvature tensor $\acute{R}$ of type (0,4) satisfies:

$$\acute{R}(X, Y, W, U) = a[g(Y, W)\, g(X, U) - g(X, W)\, g(Y, U)]$$

$$+b[g(Y, W)\, \eta(X)\, \eta(U) - g(X, W)\, \eta(Y)\, \eta(U)$$

$$+g(X, U)\, \eta(W)\, \eta(Y) - g(Y, U)\, \eta(W)\, \eta(X)], \tag{4.15}$$

where a & b are scalars of which a, b$\neq$ 0 & $\acute{R}(X, Y, W, U)= g(R(X, Y)W, U)$.

The notion of a manifold of quasi-constant curvature was first introduced by Chen & Yano [10] in 1972 for a Riemannian manifold.

Theorem 4.5. *An extended generalised φ-recurrent LP-Sasakian manifold (M^n, g) (n > 3) is a manifold of quasi-constant curvature with associated scalars a=−1, b= −2, if & only if*

$$[A(W) + B(W)]\,[\Omega(Y, U)\, \eta(X) - \Omega(X, U)\, \eta(Y)]$$

$$= 2\,[\Omega(X, W)\, \Omega(Y, U) - \Omega(Y, W)\, \Omega(X, U)], \tag{4.16}$$

holds for all vector fields X, Y, U, W on M.

Proof: In an extended generalised φ-recurrent LP-Sasakian manifold (M^n, g) (n > 3), the relation (4.11) is true. If the manifold of under consideration is of quasi-constant curvature with associated scalars a=−1, b=−2, then the relation (4.16) follows from (4.11).

Conversely, if in an extended generalised φ-recurrent LP-Sasakian manifold, the relation (4.16) holds, then it follows from (4.11) that the manifold is of quasi-constant curvature with associated scalars a=−1, b=−2. This proofs the theorem.

Theorem 4.6. *Let (M^n,g) (n > 3) be an extended generalised φ-recurrent LP-Sasakian manifold. Then the associated vector fields of the 1-form are related by*

$$\eta(\rho) = \left[\frac{(n-2)(n-3)}{2(n-1)-r}\right]\eta(\sigma).$$

Proof: Changing X,Y,W cyclically in (4.2) & adding them, we get by virtue of Bianchi's identity that

$$A(W)\,[R(X, Y)Z + \eta(R(X, Y)Z)\,\xi] + B(W)[g(Y, Z)X - g(X, Z)Y + g(Y, Z)\,\eta(X)\,\xi - g(X, Z)\,\eta(Y)\,\xi]$$

$$+A(X)\,[R(Y, W)Z + \eta(R(Y, W)Z)\,\xi] + B(X)\,[g(W, Z)Y - g(Y, Z)W + g(W, Z)\,\eta(Y)\,\xi - g(Y, Z)\,\eta(W)\,\xi]$$

$$+A(Y)\,[R(W, X)Z + \eta(R(W, X)Z)\,\xi] + B(Y)\,[g(X, Z)W - g(W, Z)X + g(X, Z)\,\eta(W)\,\xi - g$$

$$(W, Z)\,\eta(X)\,\xi] = 0. \tag{4.17}$$

Taking inner product in both sides of (4.12) by U13& then contracting over Y & Z, we obtain

$$A(W)\,[S(X, U) + (n-2)\,\eta(X)\,\eta(U)] + A(X)\,[S(U, W) + (n-2)\,\eta(W)\,\eta(U)]$$

$$+ (n-2)\,B(W)\,[g(X, U) + \eta(X)\,\eta(U)] - (n-2)\,B(X)\,g(\varphi W, \varphi U)$$

$$= \acute{R}(W, X, U, \rho). \tag{4.18}$$

Again, contracting over X & U in (4.13), we get

$S(W,\rho)=(1/2)(r-n+2)A(W)-(1/2)(n-2)^2B(W)-(1/2)(n-2)\eta(W)[\eta(\rho)+\eta(\sigma)]$. (4.19)

Setting $W=\xi$, we obtain

$$\eta(\rho) = \left[\frac{(n-2)(n-3)}{2(n-1)-r}\right]\eta(\sigma). \quad (4.20)$$

This completes the proof.

5. EXISTENCE OF GENERALIZED Φ-RECURRENT LP-SASAKIAN MANIFOLDS

Ex 5.1. We consider a 3-dimensional manifold $M = \{(x,y,z)\varepsilon R^3\}$, where (x,y,z) are the standard coordinates of R^3. Let $\{e_1,e_2,e_3\}$ be linearly independent global form of M, given by

$e_1 = e^z (\partial/\partial x)$, $e_2= e^{z-ax} (\partial/\partial y)$, $e_3 = \partial/\partial z$, where a is non-zero constant.

Let g be the Lorentzian metric defined by

$g(\partial/\partial x,\partial/\partial x) = e^{-2z}$, $g(\partial/\partial y,\partial/\partial y) = e^{2(ax-z)}$, $g(\partial/\partial z,\partial/\partial z) = -1$.

$g(\partial/\partial x,\partial/\partial y) = 0$, $g(\partial/\partial y,\partial/\partial z) = 0$, $g(\partial/\partial z,\partial/\partial x) =0$.

Let η be the 1-form defined by $\eta(U) = g(U,e_3)$, for any $U \varepsilon \chi(M)$. Let φ be the (1, 1) tensor field defined by

$\varphi(e^z \partial/\partial x) = -e^z \partial/\partial x$, $\varphi(e^{z-ax}\partial/\partial y) = -e^{z-ax} \partial/\partial y$, $\varphi(\partial/\partial z) = 0$.

Then using the linearity of φ and g, we have

$\eta(\partial/\partial z) = -1$, $\varphi^2U = U + \eta(U) e_3$, $g(\varphi U,\varphi W) = g(U,W) + \eta(U)\eta(W)$,

for any $U, W \varepsilon \chi(M)$.

Thus for $\partial/\partial z=\xi$, (φ,ξ,η,g) defines a Lorentzian paracontact structure on M.

Let ∇ be the Levi-Civita connection with respect to the Lorentzian metric g and R be the curvature tensor. Then we have,

$[e_1, e_2] = -ae^z e_2$, $[e_1, e_3] = -e_1$, $[e_2, e_3] = -e_2$.

Taking $e_3 = \xi$ and using Koszul formula for the Lorentzian metric g, we can easily calculate

$\nabla_{e1} e_1 = -e_3$, $\nabla_{e2} e_1 = ae^z e_2$, $\nabla_{e3} e_1 = 0$,

$\nabla_{e1} e_2 = 0$, $\nabla_{e2} e_2 = -ae^z e_1 - e_3$, $\nabla_{e3} e_2 = 0$,

$\nabla_{e1} e_3 = -e_1$, $\nabla_{e2} e_3 = -e_2$, $\nabla_{e3} e_3 = 0$.

From the above, it can be easily seen that (φ,ξ,η,g) is an LP-Sasakian structure on M. Consequently $M^3(\varphi,\xi,\eta,g)$ is an LP-Sasakian manifold. Using the above relations, we can easily calculate the non- vanishing components of the curvature tensor as follows:

$R(e_2, e_3) e_3 = -e_2$, $R(e_2, e_3) e_2 = -e_3$, $R(e_1, e_3) e_3 = -e_1$

$R(e_1, e_3)\, e_1 = -e_3$, $R(e_1, e_2)\, e_1 = -(1 - a^2e^{2z})\, e_2$, $R(e_1, e_2)\, e_2 = (1 - a^2e^{2z})\, e_1$,

and the components which can be obtained from these by the symmetry properties. Since $\{e_1,e_2,e_3\}$ forms a basis, any vector field $X,Y,Z\ \varepsilon\ \chi(M)$ can be written as:

$X = a_1e_1+ b_1e_2+ c_1e_3$, $Y = a_2e_1+ b_2e_2+ c_2e_3$, $Z = a_3e_1+ b_3e_2+ c_3e_3$, where $a_i,b_i,c_i\ \varepsilon\ R^+$; $i = 1, 2,3$.

This implies that

$R(X, Y)\, Z = le_1 + me_2 + ne_3$,

where $l = (a_1b_2 - a_2b_1)(1-a^2 e^{2z})b_3-(a_1 c_2+a_2 c_1)c_3$,

$m = (a_1b_2 - a_2b_1)(1-a^2 e^{2z})a_3+(b_1 c_2-b_2 c_1)c_3$,

$n = (a_1c_2 - a_2c_1)a_3+ (b_1 c_2-b_2 c_1)b_3$,

$G(X, Y)\, Z = pe_1 + qe_2 + re_3$,

where $p = (b_1b_2 - c_2c_3)a_1- (b_1 b_3-c_1 c_3)a_2$,

$q = (a_2a_3-c_2c_3)b_1- (a_1 a_3-c_1 c_3)b_2$,

$r = (a_2a_3+b_2b_3)c_1- (a_1 a_3+b_1 b_3)c_2$.

By virtue of the above, we have

$(\nabla_{e1} R)(X,Y)\, Z = -(1e_3 + ne_1)$,

$(\nabla_{e2} R)(X, Y)\, Z = -ae^z me_1 + (ae^z l- n)\, e_2 - me_3$,

$(\nabla_{e3} R)(X, Y)\, Z = 2a^2e^{2z} (a_1b_2 - a_2b_1)(a_3e_2-b_3 e_1)$.

Hence, $\varphi^2((\nabla_{e1} R)(X, Y)\, Z) = -ne_1$,

$\varphi^2((\nabla_{e2}R)(X, Y)\, Z) = -ae^zme_1 + (ae^zl - n)\, e_2$,

$\varphi^2((\nabla_{e3} R)(X, Y)\, Z) = 2a^2e^{2z} (a_1b_2 - a_2b_1)(a_3e_2-b_3 e_1)$,

$\varphi^2(R(X,Y)\, Z) = 1e_1 + me_2$,

$\varphi^2(G(X, Y)\, Z) = pe_1 + qe_2$.

Let us choose the non-vanishing 1-forms as

$A(e_1) = nq/(lq+mp)$; $B(e_1) = -mn/(lq+mp)$;

$A(e_2) = [pn-(lp+mq)ae^z]/ (lq+mp)$; $B(e_2) = [ln-(l^2-m^2)ae^z] /(lq+mp)$

$A(e_3) = [2a^2e^z(a_1 b-a_2 b_1)(a_3p-b_3q)]/(lq+mp)$;

$B(e_3) = -[2a^2e^z(a_1 b_2-a_2 b_1)(a_3l+b_3m)]/(lq+mp)$

Thus,we have

$\varphi^2((\nabla_{e_i} R)(X,Y)Z) = A(e_i)\varphi^2(R(X,Y)Z)+B(e_i)\varphi^2(G(X,Y)Z); i=1,2,3.$

Consequently, the manifold under consideration is an extended generalized φ-recurrent LP-Sasakian manifold.

This leads to the following:

Theorem 5.1. *There exists an extended generalised φ-recurrent LP Sasakian manifold which is not generalised φ- recurrent LP-Sasakian manifold.*

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