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DESIGN OPTIMIZATION OF TRIPOD TRUSS: SLP APPROACH

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ABSTRACT

The efficiency of sequential linear programming technique in optimizing nonlinear constrained structural optimization problems is studied in this paper considering tripod truss structure as a case study. The problem is formulated for minimum weight considering localized buckling stress, Euler buckling stress and direct compressive stress as constraints. The axial force in each of the members of the truss due to payload is estimated using vector mechanics. The structure is optimized considering mean diameter and payload height as design variables. The weight of the truss got reduced by 20.51%.The optimum values of design variables obtained are compared with the values obtained using graphical method. The optimum values of objective functions obtained using both the approaches are in reasonable agreement with a mere 5.17% variation.

Keywords: Sequential Linear programming, mean diameter, height, buckling stress.

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1. INTRODUCTION

The optimization of nonlinear multi variable constrained problems can be broadly addressed using four approaches. The heuristic search methods (eg: box method), methods of feasible search direction (Rosen, zoutendijk's...etc), sequential linear and quadratic methods, using sequential unconstrained minimization techniques (Interior, exterior penalty methods and Augmented Lagrange methods).

Rao (2009) presented in detail various nonlinear constrained optimization techniques, their relative advantages and limitations. The sequential linear programming has the following advantages over other methods. Unlike box method, SLP doesn't insist that the starting design vector should be a feasible design vector.

The rate of convergence in most of the methods based on feasible search direction depend on the choice of initial starting design vector and step length as the gradient value of the function evaluated at the starting design vector and step length influences the successive design vector. In case of SLP (Sequential linear programming), the nonlinear problem is solved as a series of LP (Linear programming) problems without relying on random search direction and step length. This ensures faster convergence compared to gradient (feasible direction) methods.

Penalty function and Augmented Lagrange approaches cannot be applied independently to many structural design problems as it is very difficult or sometimes nearly impossible to express design variables explicitly in terms of penalty parameters upon partial differentiation. These penalty methods have to be applied in conjunction with any of the nonlinear unconstrained methods.

This makes the process complex, highly iterative involving large computational time and effort. On the other hand SLP (Sequential Linear programming) is computationally simple requiring less computational time and effort. SLP also known as cutting plane algorithm was first introduced by Cheney and Goldstein and later improved by Kelly.

Deb (2009) presented in detail with examples, the Frank-Wolfe method which is another SLP technique. It also works on the principle of linearization of objective function and constraints and solves a sequence of LPPs to arrive at optimum. However, it relies on the parameter $\alpha \in (0,1)$ for generation of successive points in a unidirectional search approach. The major limitation of this method is that in highly

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nonlinear problems, the search is limited to a small neighborhood of the start point. The present problem is modeled as a multi variable nonlinear constrained optimization problem.

Local buckling stress, Euler's buckling stress and direct compressive stress are considered as constraints to the optimization problem. Schafer and Asce (2002) presented various empirical models for localized buckling of thin walled columns and struts depending upon end conditions, t/w or t/d ratio and section geometry.

Mamaghani (2004) studied the influence of ratio parameter (t/d), slenderness ratio, residual stress on the ultimate strength of concrete filled thin steel columns. Bradford, Hy and Uy (2002) established slenderness limits for various circular thin walled steel tubes by giving the localized buckling stress its due importance.The problem so formulated with above mentioned constraints and variables is optimized using Kelly's SLP approach and graphical method of optimization.

2. FORMULATION OF THE PROBLEM

A tripod truss with the following specifications is considered as the case study problem. Elastic modulus (E)= 207x109 N/m2, Density (ρ)=7800 kg/m3, payload (p)=111kN and yield stress (σy)=414x106 N/m2 and Poisson's ratio υ=0.3.The geometry of the tripod truss is shown in Figure 1. The truss is made of three identical members of hollow circular section arranged in the manner shown. The coordinate positions A, B, C and D of the truss are estimated from the geometry of the figure.

The axial forces in each of the members of the truss AD, CD and BD are estimated as follows.

$$\bar{F_A} = \left(\frac{1.558i + 0.9j - hk}{\sqrt{\left(1.558^2 + 0.9^2 + h^2\right)}} \right) x(F_A) \quad (1)$$

$$\bar{F_B} = \left(\frac{-1.558i + 0.9j - hk}{\sqrt{\left(1.558^2 + 0.9^2 + h^2\right)}} \right) x(F_B) \quad (2)$$

$$\bar{F_C} = \left(\frac{0i - 1.8j - hk}{\sqrt{\left(0^2 + 1.8^2 + h^2\right)}} \right) x(F_C) \quad (3)$$

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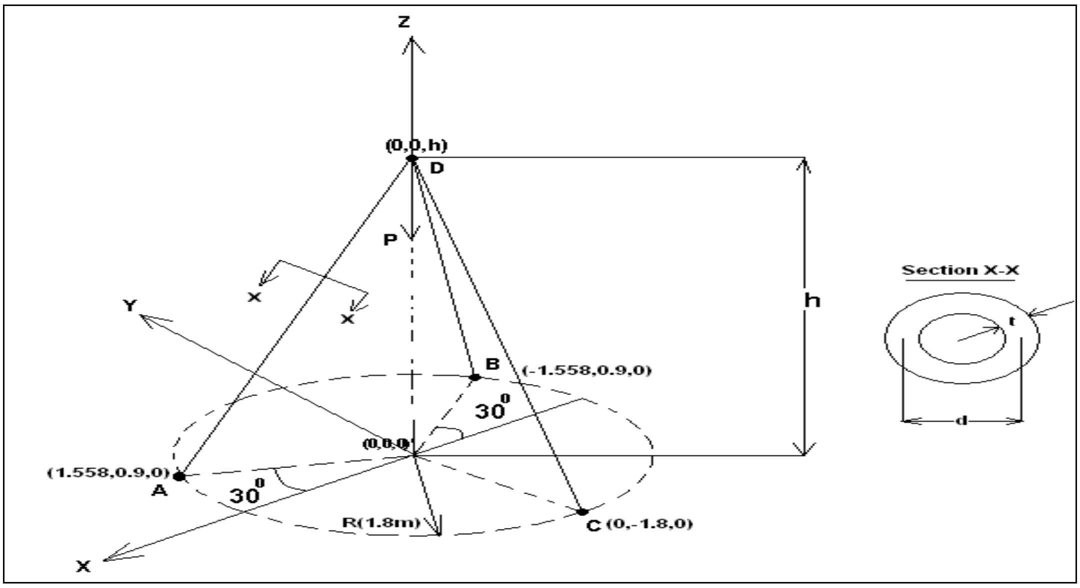

Figure 1: Geometry of tripod truss

The resultant is formed as algebraic sum of equations 1 to 3 and $\bar{F}$ =111kN.

$\bar{R} = \bar{F}_A + \bar{F}_B + \bar{F}_C + \bar{F}$ For equilibrium of system of forces, the resultant R=0

$$\bar{R}=0=\left[\frac{1.558(F_A)}{\sqrt{(3.24+h^2)}}-\frac{1.558(F_B)}{\sqrt{(3.24+h^2)}}-\frac{0(F_C)}{\sqrt{(3.24+h^2)}}\right](i)$$

$$+\left[\frac{0.9(F_A)}{\sqrt{(3.24+h^2)}}+\frac{0.9(F_B)}{\sqrt{(3.24+h^2)}}-\frac{1.8(F_C)}{\sqrt{(3.24+h^2)}}\right](j)$$

$$+\left[\frac{-h(F_A)}{\sqrt{(3.24+h^2)}}-\frac{h(F_B)}{\sqrt{(3.24+h^2)}}-\frac{h(F_C)}{\sqrt{(3.24+h^2)}}-111\right](k)$$

For $\bar{R} = 0$, each of i, j, k equal to zero

Equating each i, j, k equal to zero we have $F_A = F_B = F_C = \left(\frac{37}{h}\right)\left(\sqrt{(3.24+h^2)}\right)x10^3$

i.e., $F_A = \left(\frac{37}{x_1}\right)\left(\sqrt{(3.24+x_1^2}\right)x10^3$

The objective of minimizing weight of truss as function of design variables mean diameter (d) and height of the truss (h) is expressed as:

$$Min(\mathrm{w}) = Min(f(x)) = 3(\rho g \pi\, \mathrm{dt}\, \ell)$$

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$$= 3(\rho \pi g\, \mathrm{dt}\left(\sqrt{(h^2 + 1.8^2)}\right)) = 3664(\sqrt{(x_1^2 + 1.8^2)}(x_2) \qquad (4)$$

For $X = \begin{pmatrix} x_1 = h \\ x_2 = d \end{pmatrix}$

The Euler and local buckling stresses and direct compressive stresses are expressed as:

$$\sigma_e \cong \left(\frac{\pi^2 EI}{A\ell_e^2}\right) \leq 414x10^6 \qquad (5)$$

$$(\ell_e = \ell)\ \ hinged\ ends) \text{ and } \ell = \sqrt{(h^2 + 1.8^2)}$$

$$where\ \ A = \pi dt = (\pi(x_2)(0.00508)) = 0.016x_2$$

$$I = \left(\frac{\pi}{64}\right)\left(d_0^4 - d_i^4\right) = \left(\frac{\pi}{64}\right)(d_0^2 + d_i^2)(d_0^2 - d_i^2)$$

$$= \left(\frac{\pi}{16}\right)\left(\frac{(d_0 + d_i)(d_0 - d_i)}{4}\right)(d_0^2 + d_i^2)$$

$$= \left(\frac{\pi}{16}\right)(dt)\left((d+t)^2 + (d-t)^2\right)$$

$$= \left(\frac{\pi}{16}\right)(dt)\left(2(d^2 + t^2)\right) = \left(\frac{\pi}{8}\right)\left(x_2(0.00508)\right)\left((x_2{}^2 + 0.00508^2)\right)$$

$$Therefore, \sigma_e \cong \left(\frac{\pi^2 (207x10^9)\pi\left(x_2\right)(0.00508)(x_2{}^2 + 0.00508^2)}{\pi\left(x_2\right)(8)(0.00508)(x_1{}^2 + 1.8^2)}\right) \leq 414x10^6$$

$$i.e;\ \sigma_e \cong 617x_2^2 + 0.016 - x_1^2 - 3.24 \leq 0 \cong -x_1^2 + 617x_2^2 - 3.22 \leq 0$$

The localized buckling is given by Schafer's empirical relation [Ref no:3] as:

$$\sigma_{lb} \cong \left(\frac{k\pi^2 E}{(12)(1-\upsilon^2)}\right)\left(\frac{t}{d}\right)^2 \leq 414x10^6\ (Schafer's\ \ empirical\ relation) \qquad (6)$$

$$k = 0.43\ for\ hollow\ circular\ columns$$

$$\cong \left(\frac{2.076x10^6}{x_2{}^2}\right) \leq 414x10^6$$

$$\sigma_{lb} \cong 0.005 - x_2{}^2 \leq 0$$

The direct compressive stress may be expressed as:

$$\sigma_c = \left(\frac{F_A}{A}\right) \cong \left(\frac{F_A}{\pi dt}\right) \leq 414x10^6 \qquad (7)$$

$$\cong \left(\frac{\left(\frac{37}{h}\right)\left(\sqrt{(h^2 + 3.24)}\right)x10^3}{\pi dt}\right) \leq 414x10^6$$

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$$\cong (2318399)\left(\frac{\sqrt{(x_1^2+3.24)}}{(x_1x_2)}\right)\leq 414x10^6$$

$$\cong 0.0056\left(\sqrt{(x_1^2+3.24)}\right)-\left(x_1x_2\right)\leq 0$$

Therefore, the problem of minimizing the weight of the tripod truss structure for the given set of design variables subject to various stress constraints stated can be expressed as follows:

$$Min(f(x)) = 3664(\sqrt{(x_1^2+1.8^2)}(x_2)$$

$$for\ X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Subject to

$$g_1\left(x\right)\cong\sigma_e\cong -x_1^2+617x_2^2-3.22\leq 0$$

$$g_2(x)\cong\sigma_{lb}\cong 0.005-x_2{}^2\leq 0$$

$$g_3\left(x\right)\cong\sigma_c\cong 0.0056\left(\sqrt{(x_1^2+3.24)}\right)-\left(x_1x_2\right)\leq 0$$

3. SEQUENTIAL LINEAR PROGRAMMING TECHNIQUE

The flow chart shown in Figure 2 illustrates the working of Sequential Linear programming technique. In case of sequential linear programming technique, the starting design vector need not be feasible. However, for the present problem, a feasible design vector is chosen satisfying all the constraints for a possible reduction in the number of iterations. The starting design vector for the present problem is (x1=0.9m, x2=0.08m). The linearized objective function and constraints based on the starting design vector for the formulation of initial simplex table are given by equations 8 to 11.

$$f^1(X)\cong 131x_1+7374x_2-118 \quad (8)$$

$$g_1^1(X)\cong -1.8x_1+98.72x_2-6.36\leq 0 \quad (9)$$

$$g_2^1(X)\cong -0.077x_1-0.9x_2+0.0806\leq 0 \quad (10)$$

$$g_3^1(X)\cong -0.16x_2+0.0114\leq 0 \quad (11)$$

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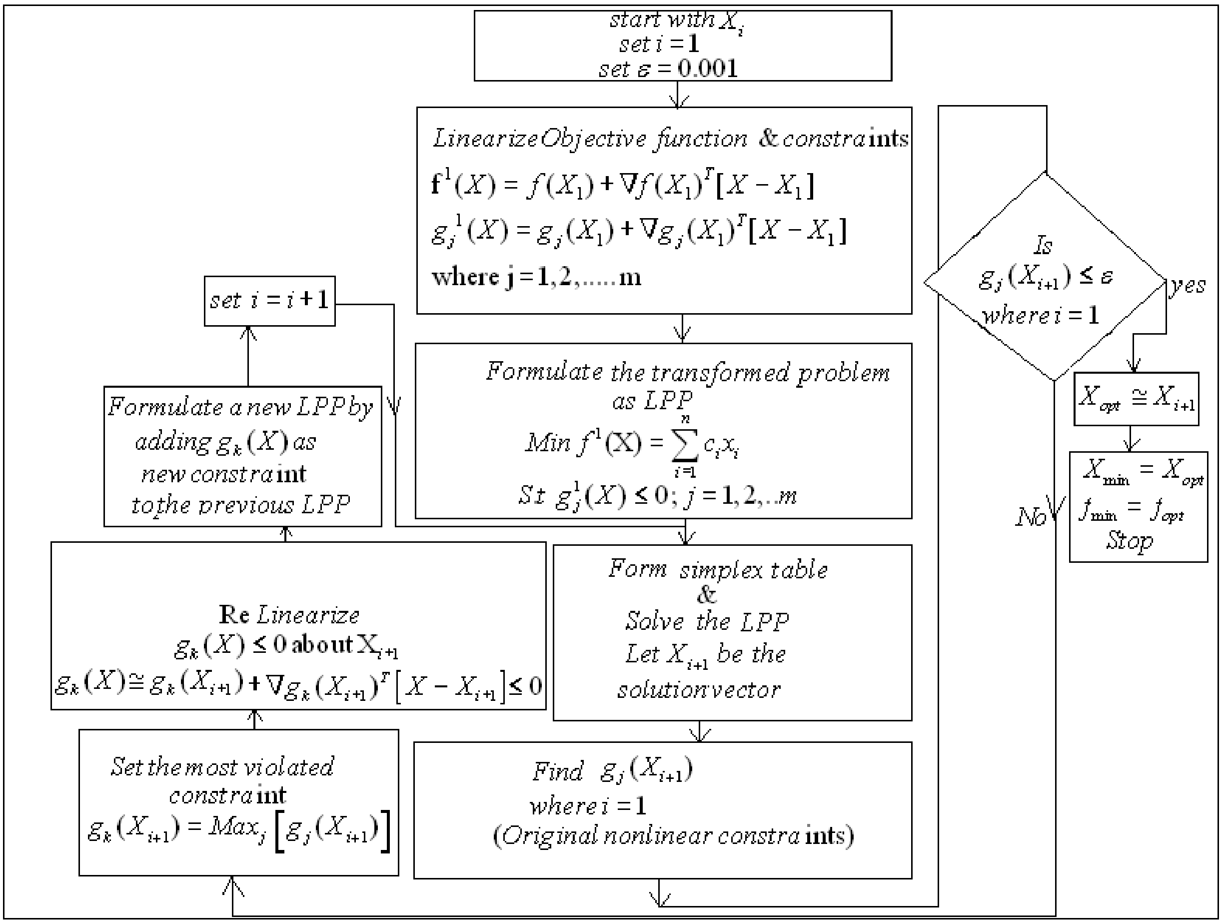

Figure 2: Flow Chart for Sequential Linear Programming technique

4. RESULTS AND DISCUSSION

The linearized objective function and constraints given by equations 8 to 11 are optimized by forming LPP and deploying two phase simplex scheme. Table 1 presents the results of the final optimized simplex. From the table, it can be seen that the optimum value of the objective function is 469.31 N at x1=0.3 meters and x2=0.0701meters.

A 20.51% reduction in weight of the truss is observed from a starting value of 590N for an initial design vector of X= (0.9m, 0.08m). The values of the optimum design variables satisfied the original nonlinear constraints and the need for relinearization of constraints did not arise for the particular problem as the condition gj(Xi+1)≤ε is satisfied for all j=1 to 3, for the chosen value of ε=0.001.

This can be attributed to the fact that the initial design vector X=(0.9m, 0.072m) chosen is feasible, in spite of the fact that SLP doesn't insist for a feasible starting design vector. The SLP program generated a local optimum in the close

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neighborhood of the initial design vector. Figure 3 presents the optimum values of the nonlinear objective function and constraints from graphical approach.

From the graph shown in Figure 3, Xopt= (0.22m, 0.067m), which yielded a value of 445N to the objective function. The optimum values of objective functions from SLP and graphical approaches differed by 5.17%. The variation in the optimum values of objective function obtained using the two approaches can be explained using Figure 4.

From Figure 4, it can be observed that the points c, e and f fall outside the feasible space and point "a" corresponds to the actual optimum lying on the boundary of feasible region. Each stage of linearization produces only an approximate linear function which may not satisfy all the constraints given by gj(X). As seen from Figure 4, to move close to the point "a", a series of linearization steps are required which in turn depend upon the order of nonlinearity, convexity of the function and the chosen value of starting design vector. Therefore, a small positive quantity ε is chosen as the convergence criterion to minimize the iterative steps. The value of the parameter "ε" chosen influenced the variation in the results.

Table 1: Optimum Simplex table from two phase simplex method

PHASE-II FINAL OPTIMUM TABLE

Table		c_j	-131	-7374	0	0	0
Basic Var	c_b	x_b Solution	x_1	x_2	s_1	s_2	s_3
x_2	-7374	0.0701124	0	1	-1.7347E-18	0	-6.25
x_1	-131	0.3001452	1	0	-0.55555555	0	-342.777777
s_2	0	0.012047656555	0	0	-0.042777777	1	-32.0188888
Max Z		-469.31447	0	0	72.77777	0	90991.3888

FROM TABLE OPTIMUM SOLUTION MIN Z=469.31 X1=0.3 X2=0.0701

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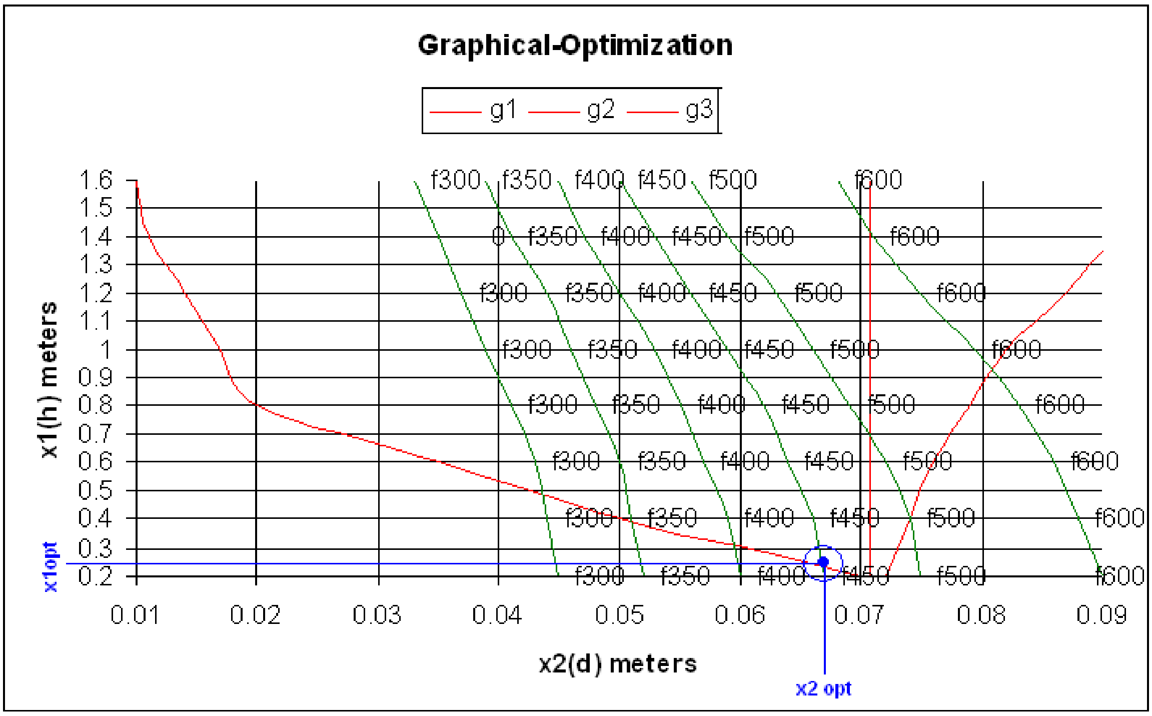

Figure 3: Optimum solution for the nonlinear constrained problem using Graphical approach

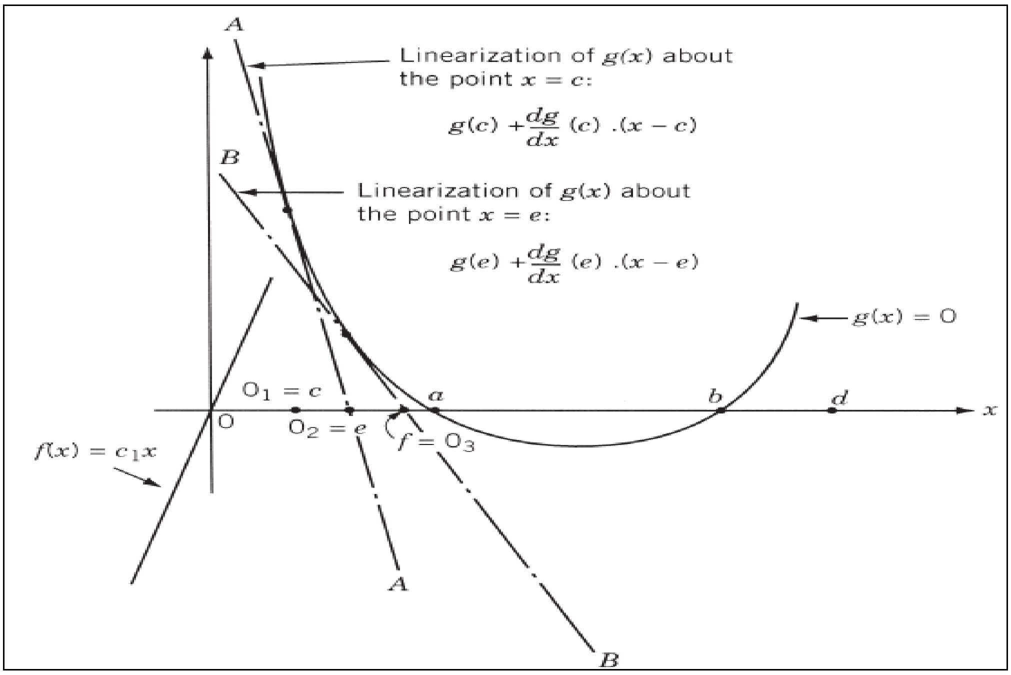

Figure 4: Graphical representation of SLP approach:
Source: Rao, 2009

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5. CONCLUSIONS

The following conclusions are drawn from the present work:

- The efficacy of Sequential linear programming technique in optimizing nonlinear constrained structural engineering problems is studied in this paper.
- A 20.51% reduction in weight of the truss is found using SLP approach.
- The design variable x_1(Height of the truss h) predominantly influenced the optimum value of the objective function.
- The optimum value of design variable x_2 (Mean diameter) did not oscillate much from the starting feasible value due to the linear restriction imposed on the design variable ($x_2 \leq 0.072$).
- The value of the chosen convergence parameter "ε", influenced the variation in results obtained from the two approaches (SLP and graphical).

REFERENCES

BRADFORD, M. A.; HY, L.; UY, B. (2002) Slenderness limits for circular steel tubes. **Journal of Constructional Steel Research**, v. 58, p. 243-252.

DEB, K. (2009) **Optimization for Engineering Design**: Algorithms and examples, PHI Pvt ltd.

MAMAGHANI, I. H. P. (2004) Seismic design and retrofit of thin walled steel tubular columns, **13th world conference on earth quake Engineering**, august, p.1-15.

RAO, S. S. (2009) **Engineering Optimization-Theory and Practice**, 4th edition, John-Wiley & sons.

SCHAFER, B. W.; ASCE, M. (2002) Local, distortional and Euler buckling of thin walled columns, **Journal of structural Engineering**, March, p. 289-299.