MISINTERPRETATION OF THE HAGENBACH-BISCHOFF QUOTA

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Abstract:

The present study deals with the electoral quota suggested by E. Hagenbach-Bischoff in his works, such as Die Frage der Einführung einer Proportionalvertretung statt des absoluten Mehres (1888) and Die Verteilungsrechnung beim Basler Gesetz nach dem Grundsatz der Verhältniswahl (1905). In the study, it is shown the Hagenbach-Bischoff quota is the same as Droop quota, so the correct formula that should be associated with name of E. Hagenbach-Bischoff is Q = [V/(S+1)]+1 or its mathematical equivalent (in the formula V represents total valid votes and S is total number of seats to be filled in the election and brackets [ ] denote the floor function, which rounds a real number down to the next integer).

Keywords:

Hagenbach-Bischoff quota, Droop quota, Electoral quota.

Hagenbach-Bischoff quota, which is used to calculate the "price" of a seat in electoral systems based on proportional representation, is interpreted in a variety of ways, correct definitions of the quota occurring very rarely. It is difficult to say what causes such explanatory discrepancies, particularly when Hagenbach-Bischoff's studies (esp. *Die Verteilungsrechnung biem Basler Gesetz nach dem Grundsatz der Verhältniswahl*) explicate the calculation of the quota with mathematical exactness, also offering various examples for illustration, which leave no room for misinterpretation.

In scientific literature (e.g. Cox, 1997, p. 57, Lebeda 2008, p. 80, Chytilek et al. 2009, p. 36 and 192, Krejčí, 2006, p. 68), the Hagenbach-Bischoff quota is commonly associated with the formula Q = V/(S+1) (in the formula *V* represents total valid votes and *S* is total number of seats to be filled in the election). The above mention formula do not provide any answers to how the result of a calculation is to be rounded off if the result is fractional or whether the result is to be rounded off at all, which has led to various (mis)interpretations of calculations: a) the result is not rounded off at all (e.g. Cox, 1997, Lebeda, 2008, Krejčí, 2006); b) the result is rounded up (e.g. Lijphart, 1990, 1994); c) the result is rounded off in a standard way (e.g. Act No. 333/2004 Coll., on Elections to the National Council of the Slovak Republic; Act No. 331/2003 Coll., on Elections to the European Parliament); d) the result is rounded down (e.g. Act No. 123/1920 Coll. in Czechoslovakia). Most of the authors listed above point out that such a calculation may lead to the allocation of more seats than the number of seats actually available. Although it is true[1], it throws a negative light on E. Hagenbach-Bischoff, who is thus indirectly taken for a person who

[1] Using electoral quota Q = V/(S+1) may lead to the allocation of more seats than there are vacancies to fill. For example this occurs when nine seats are distributed among three parties with 70, 20 and 10 votes. If we use the formula Q = V/(S+1), the electoral quota is 10 (Q = 100/(9+1) = 10). In the first count, the parties get 7, 2, and 1 seats, respectively, which means that 10 seats are distributed in total. As will be shown, E. Hagenbach-Bischoff was aware of the possibility and formulated the calculation of this quota in such a way it is always the smallest integer greater than V/(S+1).

underestimated the mathematical aspects of the calculation of the electoral quota.

Hagenbach-Bischoff quota

E. Hagenbach-Bischoff formulated the calculation of his electoral quota in such a way as to ensure that under no circumstances will the number of distributed seats exceed the number of the actual number of seats available. In his study entitled *Die Frage der Einführung einer Proportionalvertretung statt des absoluten Mehres* (1888, s. 9), E. Hagenbach-Bischoff drew on the idea that a single-member election requires a minimum of more than one half of all the votes cast for the victory of a candidate to be granted; a double-member election requires a minimum of more than one third of all the votes cast; a triple-member election requires a minimum of more than one fourth of all the votes cast, etc. Based on his observation of this sequence, E. Hagenbach-Bischoff found, that it was possible to turn the sequence into a formula $Q > V/(S+1)$. Thus the calculation of the electoral quota is defined verbally as follows: "*Zu der Wahl eines Vertreters genügt eine bestimmte Zahl von Stimmen, die wir Wahlzahl nennen; dieselbe wird erhalten, indem man die Zahl der Wähler durch die um eins vermehrte Zahl der Vertreter dividirt und die auf den so erhaltenen Quotienten nächstfolgende ganze Zahl nimmt*" (1888, s. 9). This can be translated as follows: the electoral quota can be calculated by dividing the number of valid votes by the number of seats plus one. The result of this calculation must subsequently be rounded up to the nearest integer, which represents the actual electoral number (quota). E. Hagenbach-Bischoff also considered the possibility of the result calculated according to the formula $Q = V/(S+1)$ being an integer. In the circumstances, the quota have to be increased by one vote (Hagenbach-Bischoff, 1905, p. 7). This can be turned into a mathematical formula, namely $Q = [V/(S+1)]+1$, or $Q = [V/(S+1)+1]$ (brackets [ ] denoting the floor function).[1] Hagenbach-Bischoff's intention behind increasing the number of seats in the denominator by one was to ensure that the highest number of seats gets distributed among the individual parties concerned as soon as possible (in the first count).

In the mathematical explication of his method, E. Hagenbach-Bischoff (1905, p. 25 – 27) explained why it is important to increase the result of the formula $Q = V/(S+1)$ to the next integer. His explanation was based on the following procedure: $v_i(S+1)/V = s_i+\alpha_i$, which means that $S+1 = \Sigma s_i+\Sigma\alpha_i$ (v_i is votes of party *i*, s_i is seats allocated to party *i* and α_i is the remainders of the party *i*). In an extreme case (i.e. when $\Sigma\alpha_i = 0$), this leads to $\Sigma s_i = S+1$ (i.e. the total number of seats plus one gets distributed). For this reason, the electoral number (*Wahlzahl*) calculated according to the formula $Q = V/(S+1)$ has to be increased by one vote and rounded down (Hagenbach-Bischoff, 1905, p. 26).

Therefore, the academic discussion of the Hagenbach-Bischoff quota outlined earlier can be reduced to (mathematically irrelevant) dispute, which of the following electoral number calculation formulas is to be used for its calculation: $Q = [V/(S+1)+1]$, $Q = [V/(S+1)]+1$, $Q = \lfloor V/(S+1)\rfloor + 1$, $Q = \lfloor V/(S+1)+1\rfloor$, or $Q = V/(S+1)+i$, where *i* is the number necessary to reach the smallest integer greater than $V/(S+1)$[2]. The Hagenbach-Bischoff quota thus can take any of the following form, represented in a non-linear way:

$$Q = \left\lfloor \frac{V}{S+1} +1\right\rfloor, \left\lfloor \frac{V}{S+1}\right\rfloor + 1, \left[\frac{V}{S+1} +1\right], \text{ or } \left[\frac{V}{S+1}\right] + 1.$$

The formulas given above can be also associated with the Droop quota, which means that E. Hagenbach-Bischoff used the same formula for calculating the electoral quota as H. R. Droop. R. Taagepera and M. S. Shugart (1989, p. 30) point out that both the Droop and the Hagenbach-Bischoff quotas were intended to prevent situations when the number of seats actually distributed is higher than the number of seats available. The only difference they see between the two quotas is that the Droop quota is calculated according to the formula $Q = (V/(S+1))+1$ and the result is rounded down to the nearest integer, while the Hagenbach-Bischoff quota is calculated according to the formula $Q = V/(S+1)$ and the quota is the next larger integer, which means both quotas are always the same[3].

The difference between the Hagenbach-Bischoff quota and the Droop quota is that they are used in two different election systems, although both of them are developed in the same way. In an electoral system

1 Hagenbach-Bischoff's calculation of the electoral quota in the study entitled *Die Verteilungsrechnung biem Basler Gesetz nach dem Grundsatz der Verhältniswahl* (1905) is identical. The author stated that "the total number of valid votes has to be divided by the number of all the members of the Grand Council who are to be elected plus one" (Hagenbach-Bischoff, 1905, p. 7) and "the nearest integer that directly follows the quota arrived at in this way is the electoral number" (Hagenbach-Bischoff, 1905, p. 7).

2 For example: if V is 1000 and S is 9, the Hagenbach-Bischoff quota is 101; if V is 1000 and S is 10, the Hagenbach-Bischoff quota is 91 etc.

3 In 2007, R. Taagepera surprisingly defined the Hagenbach-Bischoff quota in the following way: $Q = (1+V)/(S+1)$ (2007, p. 30). He dif not mention any rounding off, so it can be assumed that there is no rounding off at all. From a mathematical perspective, the quota is correct as it does not allow the distribution of a greater number of seats than available. On the other hand, though, it has to be noted that this is not the way Hagenbach-Bischoff suggested it. What is surprising here is that R. Taagepera (with M. Shugart) defined the Hagenbach-Bischoff quota correctly in 1989 (Taagepera, Shugart, 1989, p. 30).

based on proportional representation (Hagenbach-Bischoff), the electoral quota is used for allocating seats to the political parties involved (electoral quota is divided into the vote that each party receives and the party wins one seat for each whole number produced). In the Single Transferable Vote system, the Droop quota is the minimum number of votes that are necessary for obtaining a seat. In the Single Transferable Vote system candidates do not benefit from the fact that they manage to exceed the Droop quota several times over.

In connection with Hagenbach-Bischoff seat allocation method it should be mentioned, that by the time he published his study of 1888, Hagenbach-Bischoff had not worked out his highest average method, which was later used to assign unallocated seats. In the event of a situation when all seats are not distributed in the first count, he suggested that the electoral quota be gradually reduced using the trial-and-error method so that all seats were distributed among the individual parties involved. In 1905, however, he pointed out that the law should be based on an exact (mathematical) method of distributing all seats (Hagenbach-Bischoff, 1905, p. 6), which is why he developed a procedure for allocating the remaining seats based on the method of the highest average according to the formula $a_i = v_i/(x_i+1)$, where v_i is the number of votes of the party *i*, and x_i is the number of seats already allocated to the party *i*. E. Hagenbach-Bischoff was strictly against using the largest remainder method for the distribution of remaining seats. E. Hagenbach-Bischoff (1905, p. 20) pointed out that using the largest remainder method might be a fast route to obtaining results, but it often led to mathematical anomalies (see Hagenbach-Bischoff, 1905 or 1908 for more information). For this reason, it is not acceptable to refer to any largest remainder method as the Hagenbach-Bischoff method.

Conclusions

The present study shows that Hagenbach-Bischoff quota is the same as Droop quota, so the correct formula that should be associated with name of E. Hagenbach-Bischoff is $Q = [V/(S+1)]+1$ or its mathematical equivalent. Interpretations and definitions of the Hagenbach-Bischoff quota can only be considered correct if the results are always identical to the results that can be arrived at using the quota formulated by E. Hagenbach-Bischoff; that is the definitions (interpretations) are compatible with the definition of electoral quota incorporated in the law for elections to the Grand Council in Basel in 1905 (§ 13)[1].

[1] Correct definitions can be found in the following works: A. D. Cridge (1904), J. H. Humphreys (1911), R. Taagepera, M. S. Shugart (1989, s. 30), D. M. Farrell (2001, p. 73 and p. 209), J. H. Humphreys & A. B. Poland (2004), or in the 1919 Constitutional Commission Report on proposed electoral regulations for elections for local boards (printing No. 302).

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