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Static and dynamic behavior of piezoelectric beams : Elastic and piezoelectric cases

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ABSTRACT

Piezoelectric materials are subject to a great deal of research in recent years, whether for mechanical purposes for their potential applications in the fields of mechanical engineering, as a tool of control and regulation, or for technological purposes in electronics such as crystal oscillators areas. They can also be used as actuators as well as vibration sensors.
The study of the behavior of piezoelectric beams is the subject of our work. We propose to solve the equations of motion obtained by a variational method, applying the NEWMARK approach and based on a finite element discretization. A MATLAB code is developed to simulate and compare the results of a system formed by a piezoelectric and an elastic beam.

1. Introduction

The advantage of the piezoelectric materials lies in their physical properties, mainly their ability to generate controllable deformations by application of an electric field and vice versa, converting a mechanical load (deformation) into an electrical signal. This reciprocity of the piezoelectric phenomenon is what allows their use as sensors or actuators.

Currently, the design and the control of piezoelectric structures is the subject of many theses and research works, we mention : Shen [1] proposed a finite element model based on the Timoshenko beam theory, modeling the beam and the piezoelectric element separately by using beam elements. Donthireddy Chandrashekhara and [2] develop a finite element model for analysis and control of piezoelectric beams form with integrated actuators stuck to the surface. Dettwiler and Al [3] conducted a finite element analysis of multilayer piezoelectric structures comprising actuators and sensors. Legrain and Petitjean [4] perform an experimental study on the behavior of piezoelectric materials. Sun and Huang [5] presented an analytical model of multilayer composite beams including piezoelectric sensors

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and actuators. Ishihara and N. Noda [6] studied a surface crack in a piezo-thermo-elastic body due to thermal loading. Liang Wang, Rui-xiang Bai and Chen Haoran [7] studied the static behavior of a crack situated at the interface between the piezoelectric actuator and the elastic beam. Ali Mahieddine Joël Pouget and Mohammed Ouali [8] presented a finite element model for piezoelectric beams presenting partially delaminated layers ; this model allows the modeling of delamination anywhere in the structure.

The aim of the present work is to develop a finite element model for the study of the static and dynamic behavior of piezoelectric actuator beams. This model is based on the first order Kirchoff's theory.

2. Mathematical formulation :

Piezoelectricity is an electromechanical interaction, as piezoelectric materials are dielectrics which are deformed under the effect of an electric field and polarized under the effect of deformation.

In our case, we consider a beam with length L, width b, thickness h, the density ρ and right section S.

Using Kirchhoff's first order theory, the displacement field is given by :

$$\begin{cases} U\left(x,y,z,t\right) = u\left(x,t\right) - z\theta\left(x,t\right) \\ V\left(x,y,z,t\right) = 0 \\ W\left(x,y,z,t\right) = W\left(x,t\right) \end{cases} \tag{1}$$

And the deformation field :

$$\begin{cases} \varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial \theta}{\partial x} \\ \varepsilon_{xz} = \frac{\partial W}{\partial x} - \theta \end{cases} \tag{2}$$

With : u, v and w are the displacements in the X, Y and Z directions.

The constitutive equations of piezoelectricity, if we neglect the thermal effect can be expressed in the following form :

$$\begin{aligned} \sigma &= \overline{C}\varepsilon - \overline{e}^t E \\ D &= \overline{e}\varepsilon + \overline{\epsilon} E \end{aligned} \tag{3}$$

σ, ε, $\overline{C}$, $\overline{e}$, D, $\overline{\epsilon}$, E are respectively the stress (6×1), the deformations (6×1), the stiffness matrix (6×6), the piezoelectric constant matrix (3×6), the electric displacement (3×1), the permittivity matrix (3×3) and the electric field (3×1).

By assuming that :

- The stresses : σ_y, σ_{yz}, σ_{xz} are negligible compared with σ_x.
- The upper surface of the beam is free to move in the Z direction, which results in $\sigma_z = 0$.
- The electric fields E_1 and E_2 are negligible in front of E_3.

Equations (3) become :

$$\left\{\begin{array}{c}\sigma_x\\ \sigma_{xz}\end{array}\right\}=\left[\begin{array}{cc}\tilde{C}_{11} & 0\\ \grave{a} & \tilde{C}_{11}\end{array}\right]\left\{\begin{array}{c}\varepsilon_x\\ \varepsilon_{XZ}\end{array}\right\}-\left\{\begin{array}{c}e_{11}\\ 0\end{array}\right\}E_Z \tag{4}$$

$$D_z=\tilde{e}_{31}\varepsilon_x+\tilde{\varepsilon}_{33}E_Z \tag{5}$$

With :
$\tilde{C}_{11}=C_{11}+C_{11}\left(\frac{C_{12}C_{26}-C_{16}C_{22}}{C_{22}C_{66}-C_{26}^2}\right)-C_{12}\left(\frac{C_{12}C_{66}-C_{16}C_{26}}{C_{22}C_{66}-C_{26}^2}\right)$
$\tilde{C}_{55}=C_{55}-\frac{C_{45}^2}{C_{44}}$
$\tilde{e}_{31}=e_{31}\left(1-\frac{C_{16}C_{26}-C_{12}C_{66}}{C_{26}^2-C_{22}C_{66}}\right)$
$\tilde{\varepsilon}_{33}=\varepsilon_{33}+\frac{C_{66}e_{31}^2}{C_{22}C_{66}-C_{26}^2}$
The expression of the kinetic energy is :

$$E_c=\frac{1}{2}\rho\int\left[\left(\frac{\partial u}{\partial t}-z\frac{\partial\theta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]d\Omega \tag{6}$$

The potential energy is given by :

$$E_p=\frac{1}{2}\int_\Omega(\sigma\varepsilon-ED)\,d\Omega \tag{7}$$

$$E_p=\frac{1}{2}\int_\Omega\left(\tilde{C}_{11}\varepsilon_x\varepsilon_x+\tilde{C}_{55}\varepsilon_{xz}\varepsilon_{xz}-\tilde{e}_{31}\varepsilon_xE_z-\tilde{e}_{31}E_z\varepsilon_x-\tilde{\varepsilon}_{33}E_zE_z\right)d\Omega \tag{8}$$

The vector of external forces :

$$W_{ext}=\int_A(wp_z+\theta_ym_y+\phi D)\,dA \tag{9}$$

Where D is the load applied on the surface, p_z is a distributed load and m_y is a pair of linear intensity.

2.1. Finite element formulation

For our study, the interpolation functions are chosen as follows :
$N_1=1-\frac{x}{L_e}$, $N_2=\frac{x}{L_e}$, $N_3=\frac{x^3}{L_e^2}-2\frac{x^2}{L_e}+x$, $N_4=2\frac{x^3}{L_e^3}-3\frac{x^2}{L_e^2}+1$, $N_5=\frac{x^3}{L_e^2}-\frac{x^2}{L_e}$, $N_6=3\frac{x^3}{L_e^2}-4\frac{x}{L_e}+1$, $N_8=6\frac{x^2}{L_e^3}-6\frac{x}{L_e^2}$, $N_9=3\frac{x^2}{L_e^2}-2\frac{x}{L_e}$, $N_{10}=-6\frac{x^2}{L_e^3}+6\frac{x}{L_e}$
L_e : elementary length.
These functions form the interpolation matrix N such that :

$$N=\left[\begin{array}{cccccc}N_1 & 0 & 0 & N_2 & 0 & 0\\ 0 & N_3 & N_4 & 0 & N_5 & N_6\\ 0 & N_7 & N_8 & 0 & N_9 & N_{10}\end{array}\right]$$

$N_\phi = \left[\begin{array}{cc} 1-\frac{z}{h} & \frac{z}{h} \end{array}\right]$ is the interpolation matrix for the electric field.
So :
$\left\{\begin{array}{c} u \\ \theta \\ w \end{array}\right\} = Nq_e$ and $\Phi = N_\Phi \Phi_e$

Where : $q_z \left\{\begin{array}{c} u_i \\ \theta_i \\ w_i \\ u_j \\ \theta_j \\ w_j \end{array}\right\}$ and Φ_e are respectively the nodal displacements and the nodal electric potential.

By replacing the displacements by their expressions we obtain :

$$E_c = \frac{1}{2}\dot{q}_e^T \left[M_e\right] \dot{q}_e \tag{10}$$

With :

$$\left[M_e\right] = \rho \int_\Omega \left(\left[N_u\right]^T \left[N_u\right] + z^2 \left[N_\theta\right]^T \left[N_\theta\right] + \left[N_w\right]^T \left[N_w\right] - 2z \left[N_{wu}\right]^T \left[N_\theta\right]\right) d\Omega$$

$$E_p = \frac{1}{2}\dot{q}_e^T \left[K_{uu}\right]_e \dot{q}_e \tag{11}$$

With :

$$\left[K_{uu}\right]_e = \int_\Omega \left[\begin{array}{c} \tilde{C}_{11}\left(\left[\frac{N_u}{dx}\right]^T \left[\frac{N_u}{dx}\right] + z^2 \left[\frac{N_\theta}{dx}\right]^T \left[\frac{N_\theta}{dx}\right] - 2z \left[\frac{N_u}{dx}\right]^T \left[\frac{N_\theta}{dx}\right]\right) + \\ +\tilde{C}_{55}\left(\left(\left[\frac{N_w}{dx}\right] - N_\theta\right)^T \left(\left[\frac{N_w}{dx}\right] - N_\theta\right)\right) \end{array}\right] d\Omega$$

$$E_{p\ piezo} = \frac{1}{2} q_e^T \left[K_{u\phi}\right] \phi + \frac{1}{2} \phi_e \left[K_{\phi u}\right] q_e^T \tag{12}$$

With :
$\left[K_{u\phi}\right] = -\int_\Omega \tilde{e}_{31} \left(\left[\frac{N_u}{dx}\right]^T \left[N_\theta\right] + z\left[\frac{N_\theta}{dx}\right]^T \left[N_\theta\right]\right) d\Omega$
$\left[K_{\phi u}\right] = -\int_\Omega \tilde{e}_{31} \left(\left[N_\theta\right]^T \left[\frac{N_u}{dx}\right] + \left[N_\theta\right]^T \left[\frac{N_\theta}{dx}\right]\right) d\Omega$

$$E_{p\ dielect} = \frac{1}{2} \phi_e^T \left[K_{\phi\phi}\right] \phi_e \tag{13}$$

With :
$\left[K_{\phi\phi}\right] = -\int_\Omega \tilde{\varepsilon}_{31} \left(\left[N_\theta\right]^T \left[N_\theta\right]\right) d\Omega$

$$\{F\}_e = \int_A \left([N_w]^T p_z + [N_\theta]^T m_y \right) dA \tag{14}$$

$$\{Q\}_e = \int_A \phi_e^T \left([N_\phi]^T D \right) dA \tag{15}$$

Where :

$$\{f\}_e = \left\{ \begin{array}{c} \{F\}_e \\ \{Q\}_e \end{array} \right\} \tag{16}$$

Once the elementary vectors and matrices are defined, we proceed to the assembly step, to express them in global coordinates, to form the global matrices and global vectors representing the entire structure.

$$\left[\begin{array}{cc} [M] & 0 \\ 0 & 0 \end{array} \right] \left\{ \begin{array}{c} \{\ddot{u}\} \\ \{\ddot{\Phi}\} \end{array} \right\} + \left[\begin{array}{cc} [K_{uu}] & [K_{u\phi}] \\ [K_{\phi u}] & [K_{\phi\phi}] \end{array} \right] \left\{ \begin{array}{c} \{u\} \\ \{\Phi\} \end{array} \right\} = \left\{ \begin{array}{c} F \\ Q \end{array} \right\} \tag{17}$$

To solve these equations, we opt for the Newmark approach, given its simplicity, accuracy and stability. Approximations for the Newmark method are provided by :

$$\begin{array}{l} \dot{u}_{n+1} \simeq \dot{u}_n + (1-\gamma)\, \Delta t\ \ddot{u}_n + \gamma \Delta t\ \ddot{u}_{n+1} \\ u_{n+1} \simeq u_n + \Delta t\ \dot{u}_n + \left(\frac{1}{2} - \beta\right) \Delta t^2 \ddot{u}_n + \beta \Delta t^2 \ddot{u}_{n+1} \end{array} \tag{18}$$

Introducing the numerical scheme (18) in the equations of motion (17) we obtain :

$$\tilde{M} \ddot{u}_{n+1} = F_{n+1} - K \left(u_n + \Delta t\ \dot{u}_n + \left(\frac{1}{2} - \beta \right) \Delta t^2 \ddot{u}_n \right) \tag{19}$$

With :
$\tilde{M} = M + \beta \Delta t^2 K$
$\gamma = \frac{1}{2}$
$\beta = \frac{1}{6}$

2.2. Results

For the validation of our model for the study of beams, a MATLAB code is developed. This code takes into account both elastic and piezoelectric case. In order to perform numerical simulations of an elastic beam bending, we consider a console beam-ie clamped-free with a length $L = 0.5m$, width $b = 0.03m$ and thickness $h = 0.01m$. This beam is subjected to the action of a uniformly distributed load over its entire length. The selected material (mild steel) has the following mechanical properties : Young's modulus : $E = 2 \cdot 10^{11} N/m^2$, Poisson coefficient : $\nu = 0.3$ and density : $\rho = 8000 kg/m^3$.

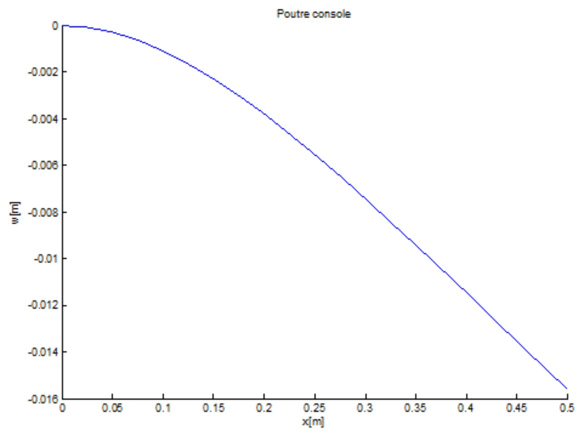

Fig. 1. The deflection of the elastic beam.

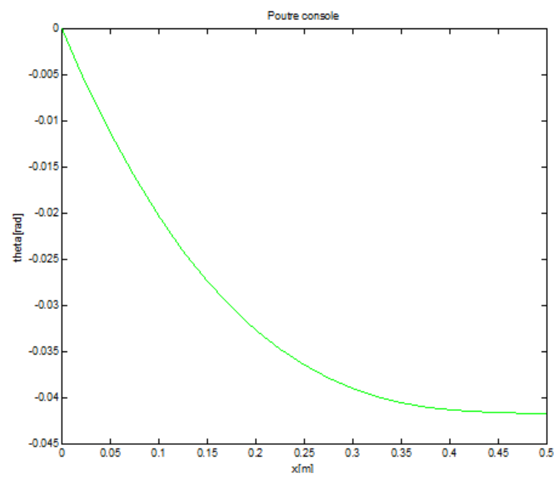

Fig. 2. Rotation of the elastic beam.

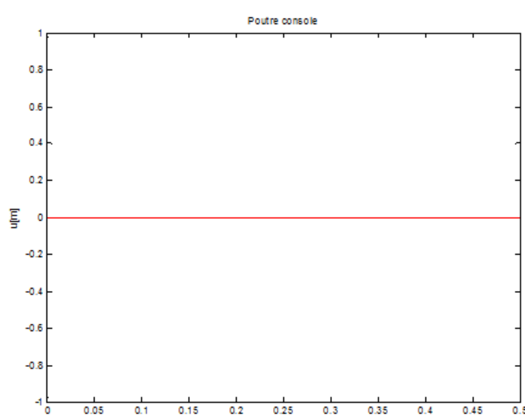

Fig. 3. The axial displacement of the elastic beam.

To validate the finite element model, we proceed to a comparison between analytical eigenvalues and those obtained by finite elements for the first 20 modes.

Tab. 1. Comparison between the analytical and numerical eigenvalues.

Modes	Frequencies (exact)	Frequencies (FE)	Relative error
1	202.974704	202.979418	0.002322
2	1272.112568	1271.384009	0.057272
3	3562.310339	3556.908165	0.151648
4	6980.317978	6961.565494	0.268648
5	11538.892984	11489.741877	0.425960
6	17237.111742	17130.495851	0.618525
7	24074.974252	23871.677245	0.844433
8	32052.480512	31699.167162	1.102296
9	41169.630525	40597.153041	1.390533
10	51426.424289	50548.361883	1.707415
11	62822.861804	61534.309574	2.051088
12	75358.943071	73535.556627	2.419602
13	89034.668090	86531.964641	2.810931
14	103850.036860	100502.948007	3.223002
15	119805.049382	115427.715894	3.653714
16	136899.705655	131285.500183	4.100962
17	155134.005680	148055.765653	4.562662
18	174507.949457	165718.399369	5.036762
19	195021.536984	184253.876833	5.521267
20	216674.768264	203643.403014	6.014251

This comparison shows that the finite element model is well-chosen as the relative error does not exceed 6% for the 20th mode, and the results are almost identical.

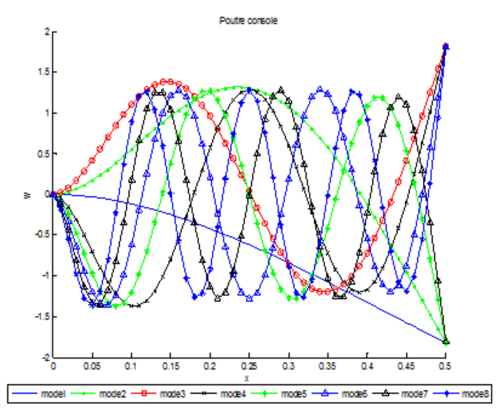

Fig. 4. Overlapping 8 eigenmodes of vibration of the elastic beam.

To talk over the dynamic problem, we consider the same beam (forced undamped vibration mode) subjected to the action of a harmonic force $F = F.cos(100.t)$.

The resolution of this problem by Newmark approach gives the graph (Figure 5), which represents the vertical vibrations of the beam.

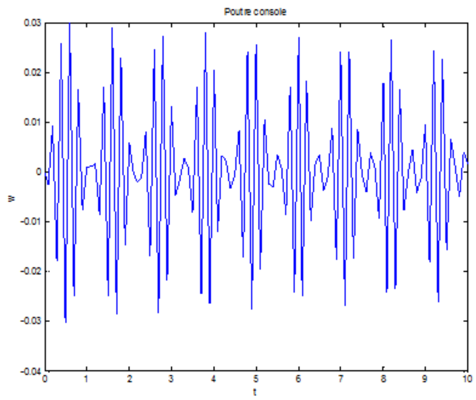

Fig. 5. The vertical vibration end of the elastic beam.

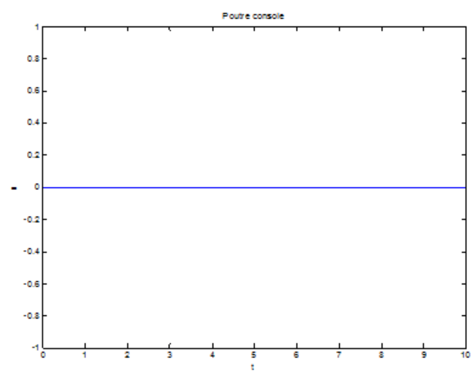

Fig. 6. The axial vibration end of the elastic beam.

To investigate the piezoelectric case, we consider a beam having the same dimensions as those of the elastic case. This beam is subjected this time to the action of a mechanical load uniformly distributed over the entire length and a difference in electric potential V in the thickness direction. The material of the beam is piezoelectric, it is a piezoelectric ceramic, PZT-4, which has the following mechanical and electrical properties :

Density $\rho = 7500kg/m^3$

The stiffness matrix is :

$$C = \begin{bmatrix} 13.90 & 7.78 & 7.43 & 0 & 0 & 0 \\ 7.78 & 13.90 & 7.43 & 0 & 0 & 0 \\ 7.43 & 7.43 & 11.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.56 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.56 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.07 \end{bmatrix} \cdot 10^{10} \left[N/m^{\acute{e}}\right]$$

The matrix of piezoelectric constants is :

$$e = \begin{bmatrix} 0 & 0 & 0 & 0 & 12.7 & 0 \\ 0 & 0 & 0 & 12.7 & 0 & 0 \\ -5.2 & -5.2 & 15.1 & 0 & 0 & 0 \end{bmatrix} \left[C/m^2\right]$$

The matrix of dielectric constants is :

$$\varepsilon = \begin{bmatrix} 13.06 & 0 & 0 \\ 0 & 13.06 & 0 \\ 0 & 0 & 11.51 \end{bmatrix}$$

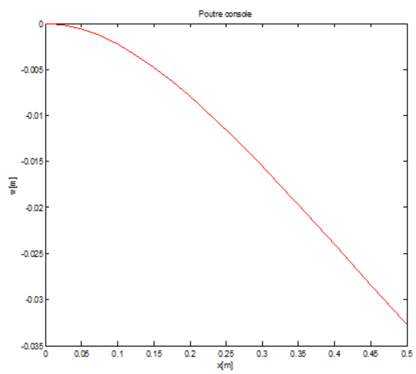

Fig. 7. The deflection w of the console beam in actuator mode.

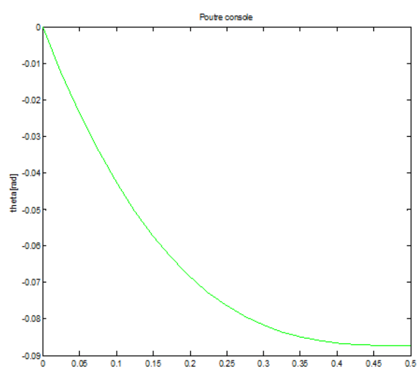

Fig. 8. The rotation ? of a console beam in actuator mode.

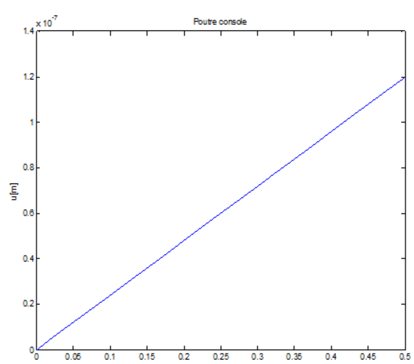

Fig. 9. The axial displacement u of a console beam in actuator mode.

The first important finding, we can notice by comparing the results of the elastic and piezoelectric case is that in the later case, the axial displacement u is non-zero, but in linear variation, which results of the application of an electric potential difference between the upper and lower surfaces of the beam.

Turning now to the results of the dynamic problem (ie the forced vibration) without taking into account the structural damping of the beam. The external excitation is harmonic $F = F.cos(100.t)$.

The graph (Figure 10) and (Figure 11) give the vertical and the axial vibrations of the piezoelectric beam.

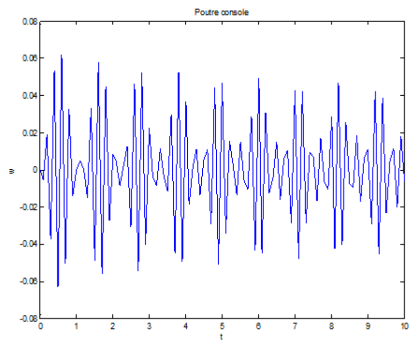

Fig. 10. The vertical vibrations of the piezoelectric beam.

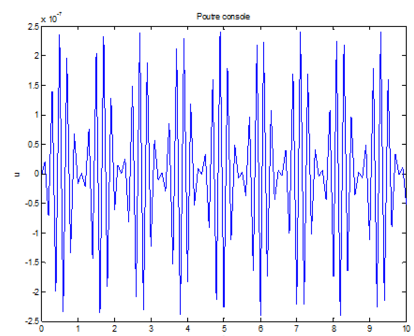

Fig. 11. The axial vibrations of the piezoelectric beam.

In order to show the importance of choosing the piezoelectric material on the behavior of the beam, we propose to study the same beam considered in actuator mode, but for different materials : PZT2, PZT-4, PZT-5A, PZT-5H, PZT-7A, PZT8, ZnO and ZnS.

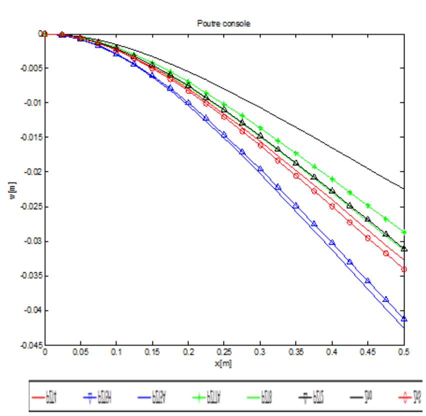

Fig. 12. Comparison of the beam deflection in actuator mode for different materials.

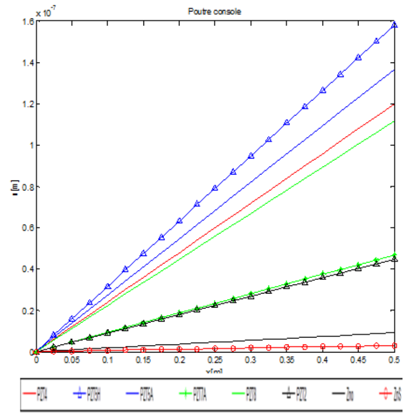

Fig. 13. Comparison of the axial displacements of the beam in actuator for different materials.

The graphs (12) and (13) show that the shape of the deflection w and the slope of the axial displacements u differ from piezoelectric material to another, mainly because the mechanical and electrical characteristics are different. Hence we can conclude that each piezoelectric material is intended for a certain class of applications. For example, if the application requires a large deflection and axial movement PZT-5A and PZT-5H are used instead of using a PZT-2, and so on ...

In this section we try to see the influence of the beam's thickness on the static and dynamic behavior of the piezoelectric beam, that's why we consider the same beam, but for different thicknesses.

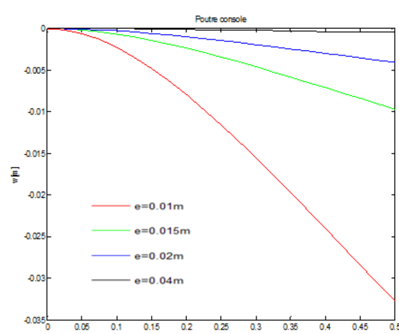

Fig. 14. Influence of the thickness on the deflection of the piezoelectric beam.

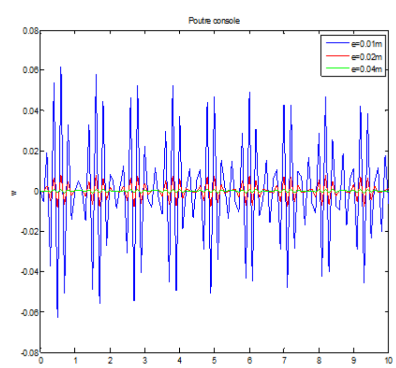

Fig. 15. Influence of the thickness on the vertical vibrations of the beam.

It is clear that the choice of the dimensions of the beam is also important because of their impact on the static and dynamic behavior of the beam. For example, if we consider the influence of the thickness, we can simply find that increasing the thickness leads to reduce the deflection in the static case and to reduce the amplitude of the vertical vibrations of the beam in the dynamic case.

3. Conclusions

Throughout this work, we study the impact of the application of piezoelectric effect on the behavior of beams and modeling of these beams on the one hand. And on the other hand, we have numerically simulated the behavior of these beams in both elastic and piezoelectric case. A finite element formulation is presented for piezoelectric beams in actuator mode.

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