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Study and Analysis of Flow of an In-compressible Fluid past an Obstruction

[1]Deepak Kumar, Assistant Professor

[1]Department of Mechanical and Automation Engineering, Amity University, Haryana, India

E-mail: deepak209476@gmail.com

Contact No: 09416627599

Abstract— In present work, the time independent laminar flow of a viscous, incompressible fluid in two dimensions flow has been studied. The fluid had been allowed to flow in a channel with an obstruction which was a rectangular plate of definite dimensions. The flow of water had been considered to be steady with uniform incident velocity with various boundary conditions. The resulting Navier- Stokes equations had been solved with the help of software “FLEX PDE”. Reynolds number had been varied as 10, 50, 100 and variation of streamlines, velocity, and pressure along with flow pattern in the form of velocity vector had been investigated.

Keywords— Rectangular Obstruction, Incompressible Fluid Flow, FLEX PDE, Computational Fluid Dynamics, Navier- Stokes Equations, Streamlines, Reynolds’s Number..

INTRODUCTION

Understanding the complexities of laminar and turbulent flow is a problem that has been studied for many years. Researchers in this field have been creative and innovative by introducing several new techniques and definitions. Here, the well-known problem of laminar flow in a channel with a rectangular obstruction had been studied and investigated. Alvaro Valencia [1] studied laminar flow past square bars arranged side by side in a plane channel. His aim was to provide information on the unsteady flow processes. He captured the effects of vortex shedding by solving the continuity and the Navier-Stokes equations in two dimensions. He made computations for 11 transverse bar separation distances for constant Reynolds number. The numerical results reveal the complex structure of the flow. B.N. Rajani [2] focused on the analysis of two- and three-dimensional flow past a circular cylinder in different laminar flow regimes. Here an implicit pressure-based finite volume method is used for time-accurate computation of compressible flow. Results are studied for pressure, skin friction coefficients, and also for the Strouhal frequency of vortex shedding. The complex three dimensional flow structure of the cylinder wake is also reasonably captured. M. Boubekri and M. Afrid [3] considered the numerical simulation of the two-dimensional viscous flow over a solid ellipse with an aspect ratio equal 3.5. Sufficiently far from the ellipse, the flow is assured potential. The flow is modeled by the two dimensional partial differential equations of conservation of mass and momentum. The numerical solutions revealed that the flow over the ellipse is steady with zero vortex up to Re = 40. For Reynolds numbers between 50 and 190, the flow is steady with two vortices in the wake. For Re = 210 the flow becomes unstable. B. H. Lakshmana Gowda and Myong-Gun Ju [4] analyzed the reverse flow in a square duct with an obstruction at the front (which is a square plate). The gap g between the obstruction and the entry to the duct was systematically varied, and it was found that maximum reverse flow occurs around a g/w value of 0.75. Wisam K. Hussam, et al. [5] for shallow flow past an obstacle in a channel, the channel depth and blockage ratio play a significant role .In this study, the flow past a confined circular cylinder is investigated numerically using a spectral element algorithm. The incompressible Navier-Stokes equations are solved over a two dimensional domain. A parametric study is performed for the two-dimensional flow by varying the Reynolds number (Re) and blockage ratio (β), over the ranges $20 \leq Re \leq 2000$ and $0.2 \leq \beta \leq 0.6$.

Shivani T. Gajusingh [6] did experimental study to investigate the impact of a rectangular baffle inside a square channel. The measurements were conducted for two Reynolds numbers in the fully turbulent regime. The changes to the flow structure due to the insertion of a baffle were quantified by a direct comparison with the flow structure in the absence of a baffle, under similar conditions. Significant enhancement of turbulence was observed in a region up to two times the baffle height immediately downstream of the baffle and the thickness of this layer increased to three times the baffle height further downstream of the baffle. Zou Lin [7] carried out a three-dimensional numerical investigation of cross-flow past four circular cylinders in a diamond arrangement at Reynolds number of 200. With the spacing ratios (L/D) ranging from 1.2 to 5.0, the flow patterns can be classified into three basic regimes. The relationship between the three-dimensional flow patterns and force characteristics around the four cylinders shows that the variation of forces and Strouhal numbers against L/D are generally governed by these three kinds of flow patterns. It is concluded that the spacing ratio has important effects on the force and pressure characteristics of the four cylinders. S. B. Doma et al. [8] described the motion of steady flow of a viscous incompressible fluid passing a rectangular plate. The cross section of the plate is considered to be in the form of a rectangle. The fluid is assumed to be steady flow of water. The boundary conditions are discussed in details. The resulting equations are solved numerically. The Reynolds number is varied as 0.5, 1, 10, 20, 100, 200 and 300 and the variation of streamlines is studied. Also the values of the pressure force, the velocity magnitude, vorticity magnitude are analyzed at each position point. Gera.Bet al. [9] carried out a numerical simulation for a two dimensional unsteady flow past a square cylinder for the Reynolds number (Re) considered in the range 50250 so that flow is laminar. The main objectives of this study were to capture the features of flows past a square cylinder in a domain with the use of CFD. The variation of Strouhal number with Reynolds number was found from the analysis. It was found that up to Reynolds number 50, the flow is steady. Between Reynolds numbers 50 to 55, instability occurs and vortex shedding appears and flow becomes unsteady. Vikram C. K. [10] analyzed numerical investigation of two dimensional unsteady flow past two square cylinders with in-line arrangements in a free stream. The main aim of the study is to systematically investigate the influences on size of the eddy, velocity, frequency of vortex shedding, pressure coefficient and lift coefficient by varying pitch to perimeter Ratio of two square cylinders. It has been found that the size of the eddy and the monitored velocity in between the square cylinders increases with increase in PPR. Frequency of vortex shedding is found to be same in between the cylinders and in the downstream of the cylinder. The pressure distribution near to the surface of the cylinder is quite low due to viscous effects. The upstream cylinder is found to experience higher lift compared to the downstream cylinder.

PROBLEM SPECIFICATION

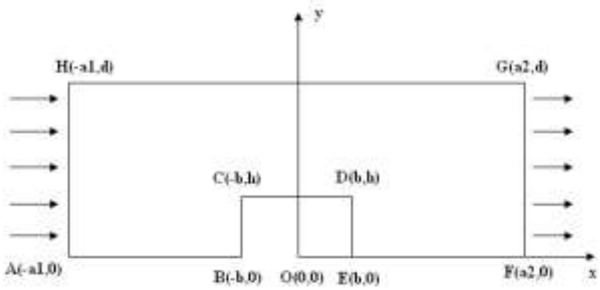

Figure A

In order to solve the flow of a viscous, incompressible fluid in a channel with a rectangular obstruction some assumptions are necessary, since mathematical expressions become simpler and the solutions are still close to real cases

Assumptions

- Flow is 2-Dimensional and laminar
- The fluid is water which is considered incompressible and Newtonian
- Flow is not temperature dependent
- Flow is not affected by the gravity field

GOVERNING EQUATIONS

The flow in a channel with an obstruction was computed by solving the Navier-Stokes equations for incompressible fluid in a two-dimensional geometry. The governing equations are:

A. *Continuity Equation*

This equation states that mass of a fluid is conserved.

Rate of increase of mass in fluid element = Net rate of flow of mass into fluid element

For three dimensional and unsteady flow

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

For 2-D, incompressible, steady flow

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

B. *X- Momentum Equation*

Momentum equations are based on Newton's second law which states that, the rate of change of momentum equals the sum of forces on fluid particle.

For three dimensional and unsteady flow

$$\frac{\partial(\rho u)}{\partial t}+u\frac{\partial(\rho u)}{\partial x}+v\frac{\partial(\rho u)}{\partial y}+w\frac{\partial(\rho u)}{\partial z}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left[\lambda\nabla.V+2\mu\frac{\partial u}{\partial x}\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]+\rho f_x$$

Where $V=u_i+v_j+w_k$ is velocity vector field f = Body force per unit mass and f_x is its x – component, $\lambda = -\frac{2}{3}\mu$

For 2-D, incompressible, unsteady and with no body forces

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

C. Y- Momentum Equation

$$\frac{\partial(\rho v)}{\partial t}+u\frac{\partial(\rho v)}{\partial x}+v\frac{\partial(\rho v)}{\partial y}+w\frac{\partial(\rho v)}{\partial z}=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial y}\left[\lambda\nabla.V+2\mu\frac{\partial v}{\partial y}\right]+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right]+\rho f_y$$

Where V=$u_i + v_j + w_k$ is velocity vector field

f = Body force per unit mass and $f_{y\,=}$ y – Component of body force, $\lambda = -\frac{2}{3}\mu$

For 2-D, incompressible, unsteady and with no body forces

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

Now we will convert Continuity equation, X- momentum equation, and Y- momentum equation into non dimensional form. Now let us consider

$$x^*=\frac{x}{L},\quad y^*=\frac{y}{L},\quad t^*=\frac{t}{T},\quad u^*=\frac{u}{U},\quad v^*=\frac{v}{U},\quad p^*=\frac{p}{P}$$

To make these equations dimensionless, we must have to derive the non- dimensional form of various time and space derivate. The time derivatives with respect to the dimensional variable can be written as:

$$\frac{\partial(\,)}{\partial t}=\frac{\partial(\quad)}{\partial t^*}\frac{\partial t^*}{\partial t}=\frac{1}{T}\frac{\partial(\,)}{\partial t^*}$$

Similarly, the spatial derivate are given by

$$\frac{\partial(\,)}{\partial x}=\frac{\partial(\quad)}{\partial x^*}\frac{\partial x^*}{\partial x}=\frac{1}{L}\frac{\partial(\,)}{\partial x^*}$$

$$\frac{\partial(\,)}{\partial y}=\frac{\partial(\quad)}{\partial y^*}\frac{\partial y^*}{\partial y}=\frac{1}{L}\frac{\partial(\,)}{\partial y^*}$$

and

$$\frac{\partial^2(\,)}{\partial x^2}=\frac{1}{L^2}\frac{\partial^2(\,)}{\partial x^{*2}}$$

$$\frac{\partial^2(\,)}{\partial y^2}=\frac{1}{L^2}\frac{\partial^2(\,)}{\partial y^{*2}}$$

Thus continuity equation becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\frac{1}{L}\frac{\partial u^*U}{\partial x^*}+\frac{1}{L}\frac{\partial v^*U}{\partial y^*}=0$$

$$\frac{U}{L}\left(\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}\right)=0$$

The non- dimensional form of continuity equation is given by,

$$\left(\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}\right)=0$$

Using similar process, non- dimensional Navier-Stokes equations can be given by,

$$\left(\frac{L}{UT}\right)\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\left(\frac{P}{\rho U^2}\right)\frac{\partial p^*}{\partial x^*}+\left(\frac{\mu}{\rho UL}\right)\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

$$\left(\frac{L}{UT}\right)\frac{\partial v^*}{\partial t^*}+u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\left(\frac{P}{\rho U^2}\right)\frac{\partial p^*}{\partial y^*}+\left(\frac{\mu}{\rho UL}\right)\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

There are three dimensionless groups in the non-dimensional Navier-Stokes equations, these are

$$\frac{L}{UT},\quad \frac{P}{\rho U^2}\,and\,\frac{\mu}{\rho UL}$$

Therefore, the continuity and Navier-Stokes equations in dimensionless form for two dimensional channel flow where width (w) of channel is the characteristics length (L) and U is the free stream uniform velocity at the entry of the test channel. Here μ is the dynamic viscosity; ρ is the density of the fluid, and Re is the Reynolds number. The continuity and momentum equations for steady flow in dimensionless form can be written as,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

D. Equation of Stream Function

We know that vorticity (ω) is given by

$$\omega=\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$$

Also $\frac{\partial \psi}{\partial x}=u$ and $\frac{\partial \psi}{\partial y}=-v$

Putting these values in the equation of vorticity we will get

$$\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}=-\omega$$

Hence the governing equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

RESULTS AND DISCUSSION

A computer program was developed here for quantitative and qualitative analysis of laminar incompressible flow through a channel with a rectangular obstruction. The equations used in the program are Navier- stokes equations.

We have taken different Reynolds's number as 10, 50, 100 and at these Reynolds's number the variation of following is seen by taking height of obstruction constant = 0.5.

(1) Streamlines
(2) Velocity in x- direction (u)
(3) Flow pattern
(4) Pressure

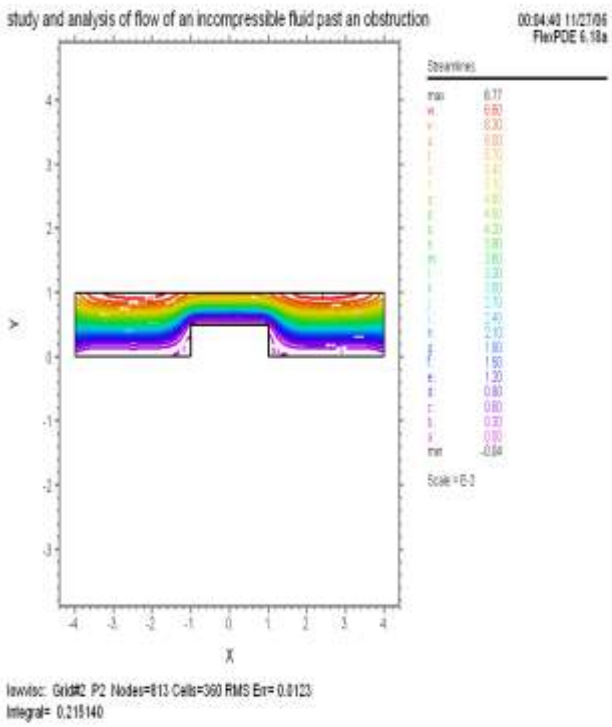

Figure 1: Streamlines at Re=10

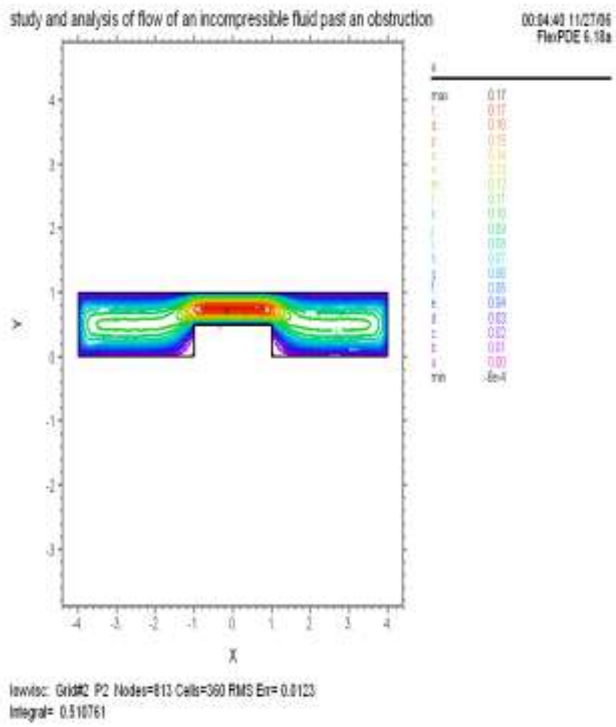

Figure 2: Velocity in x-direction (u) at Re=10

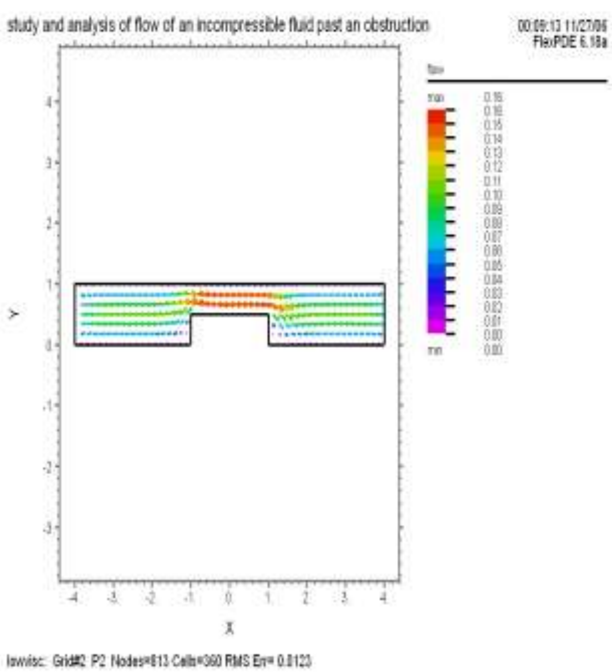

Figure 3: Flow at Re=10

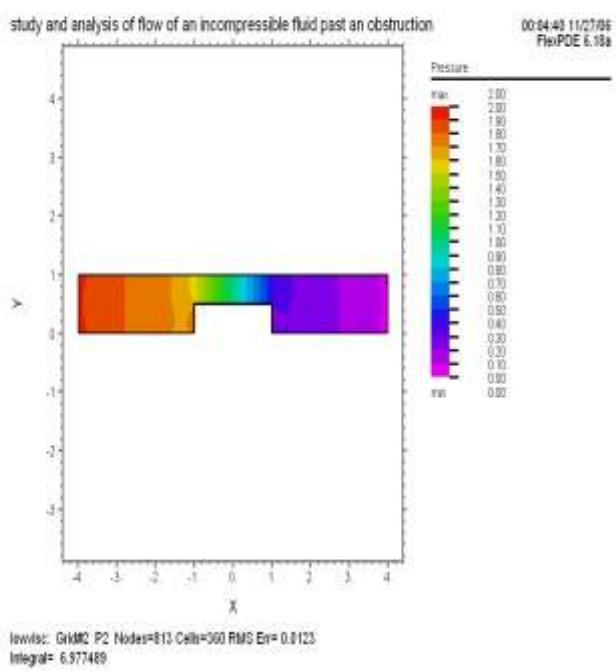

Figure 4: Pressure at Re=10

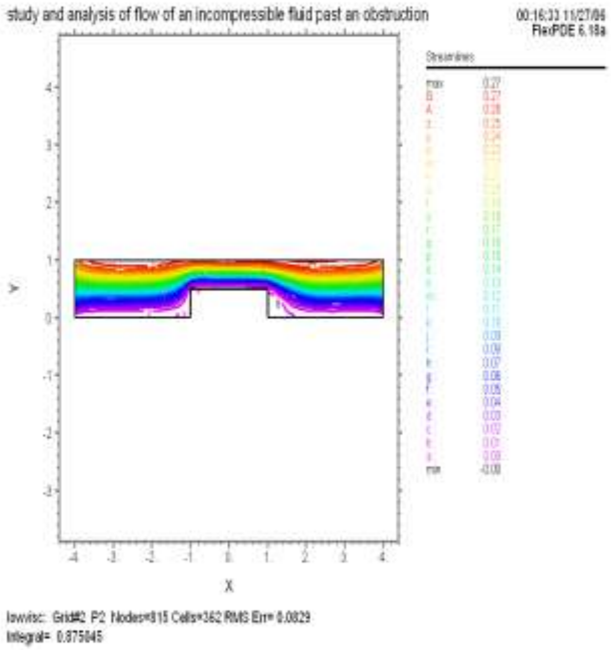

Figure 5: Streamlines at Re=50

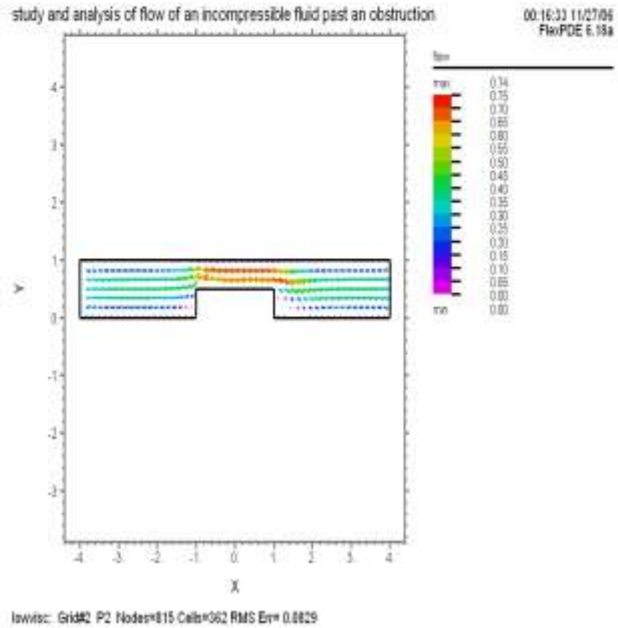

Figure 6: Flow at Re=50

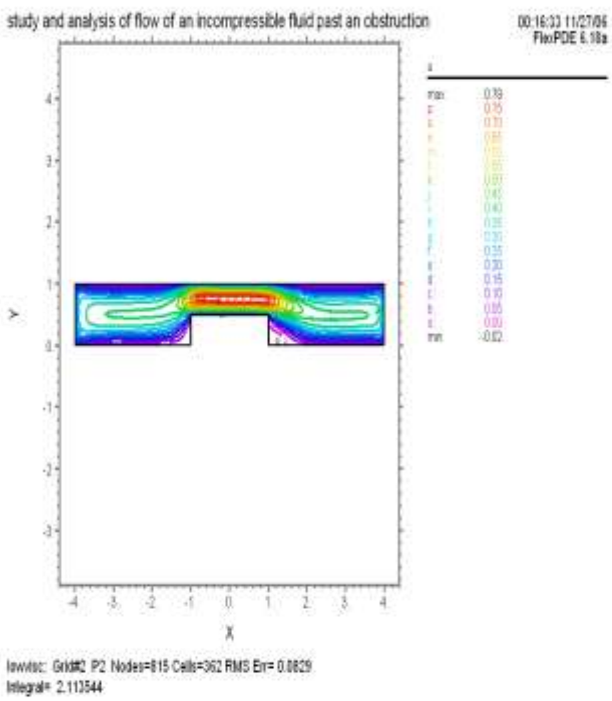

Figure 7: Velocity in x-direction (u) at Re=50

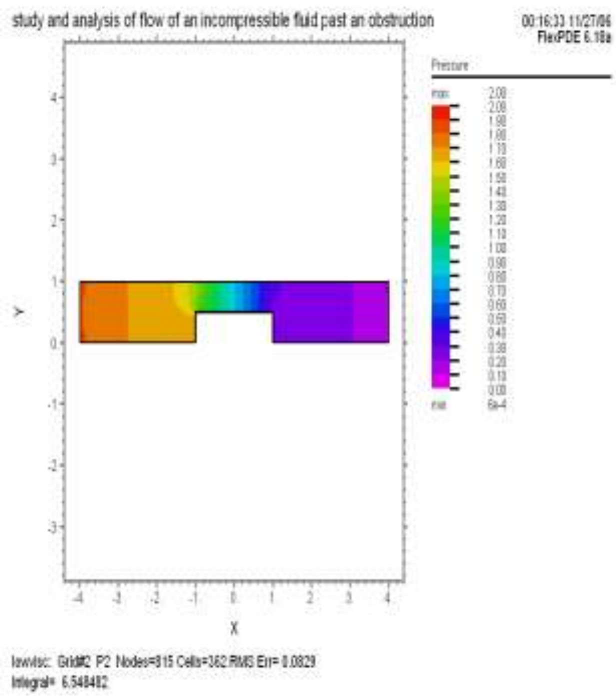

Figure 8: Pressure at Re=50

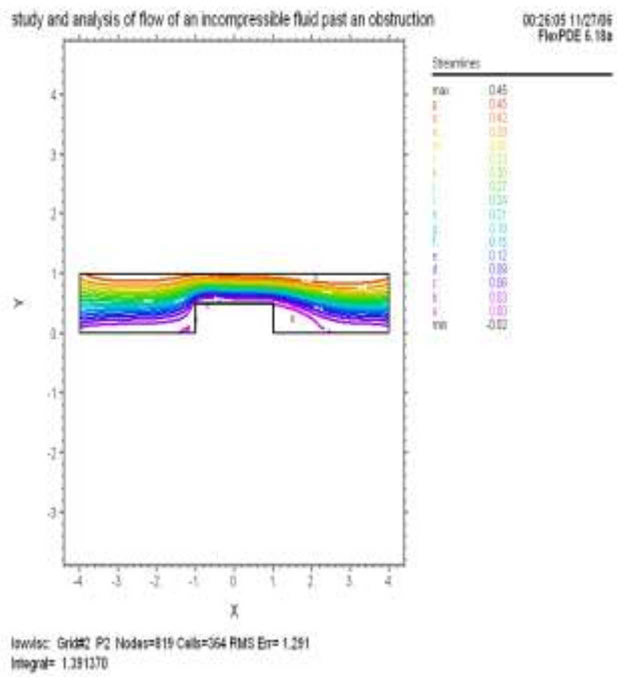

Figure 9: Streamlines at Re=100

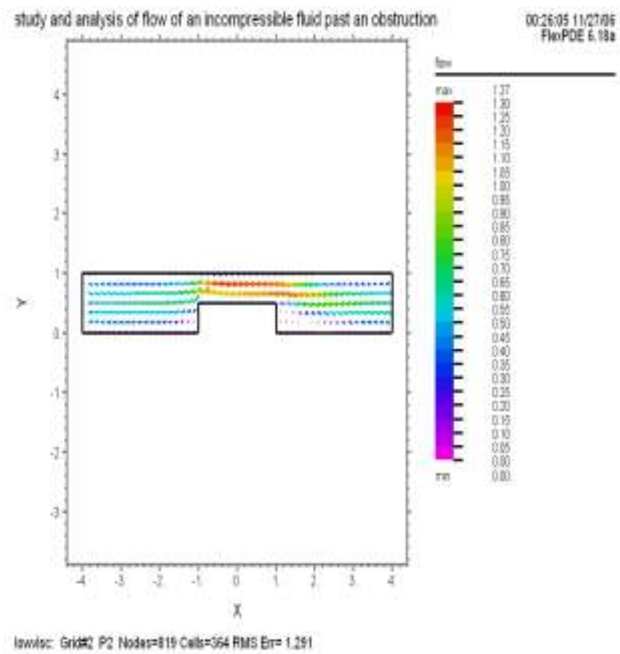

Figure 10: Flow at Re=100

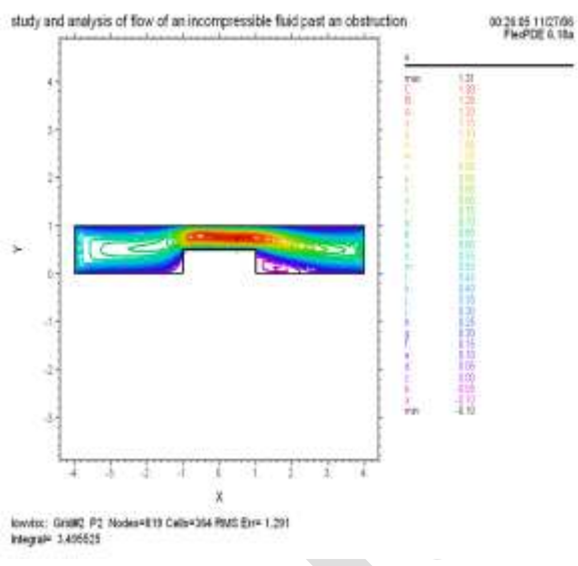

Figure 11: Velocity in x-direction (u) at Re=100

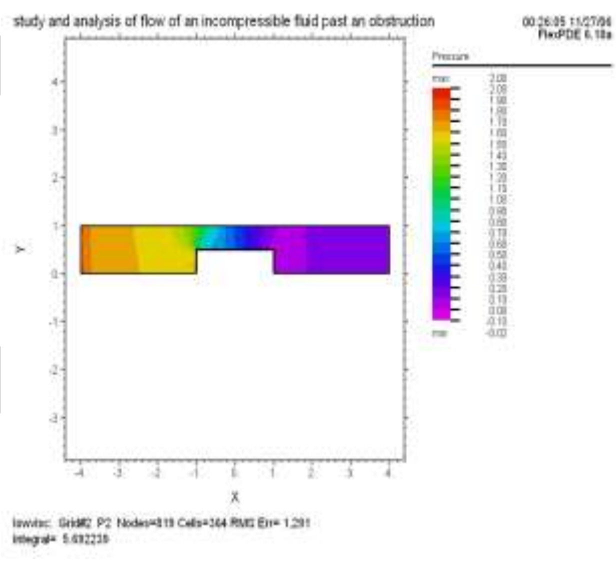

Figure 12: Pressure at Re=100

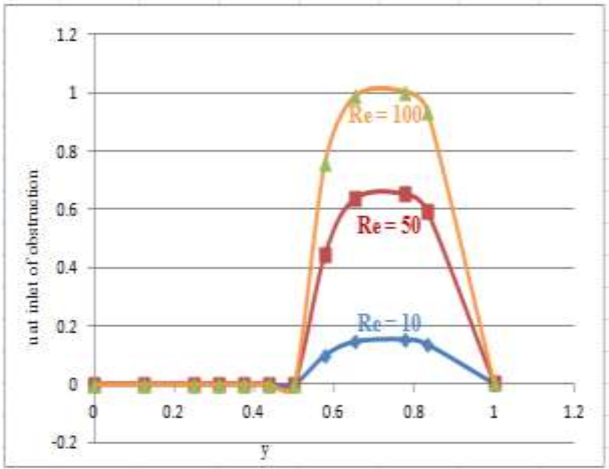

Figure 13: Velocity in x - direction at inlet of obstruction

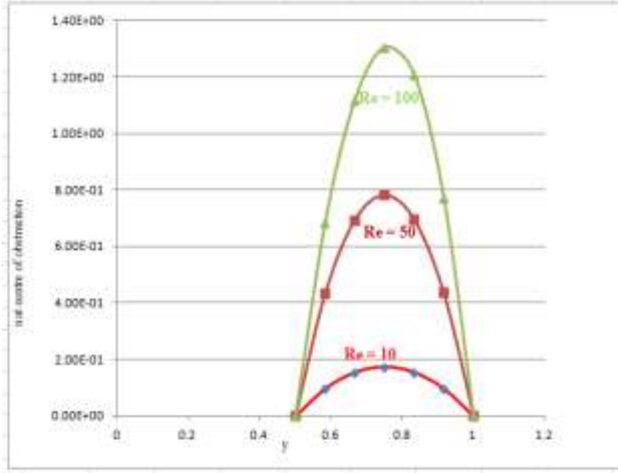

Figure 14: Velocity in x - direction at centre of obstruction

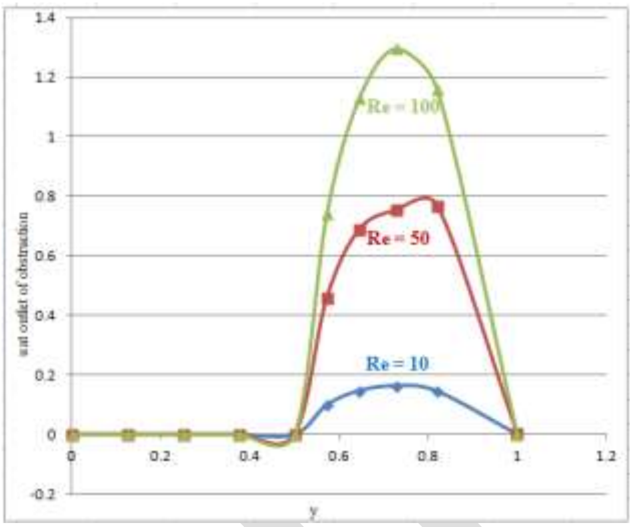

Figure 15: Velocity in x - direction at outlet of obstruction

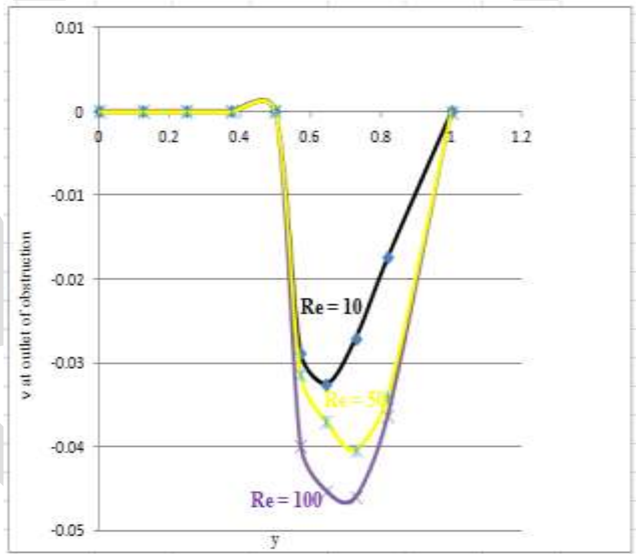

Figure 16: Velocity in y - direction at outlet of obstruction

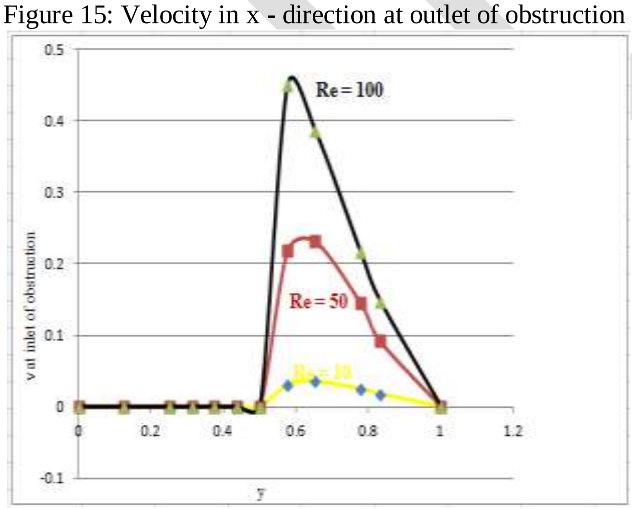

Figure 17: Velocity in y - direction at inlet of obstruction

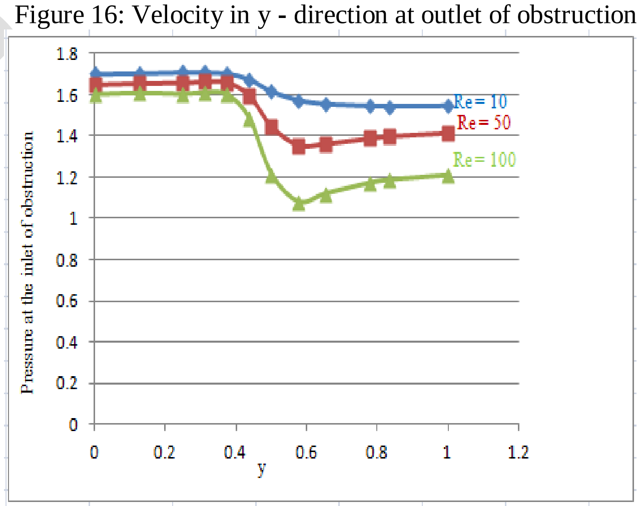

Figure 18: Pressure at inlet of obstruction

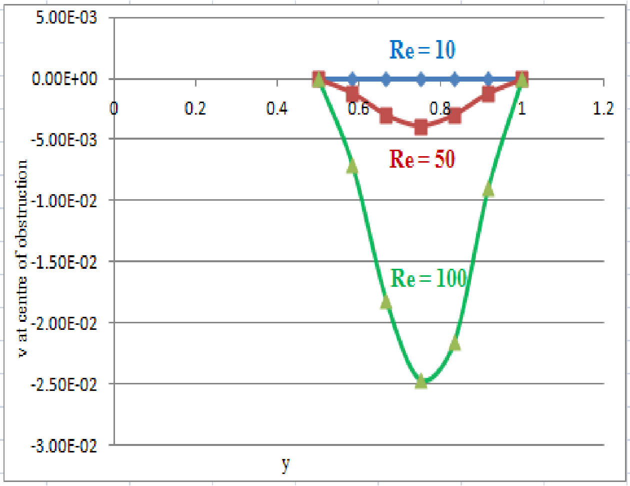

Figure 19: Velocity in y - direction at centre of obstruction

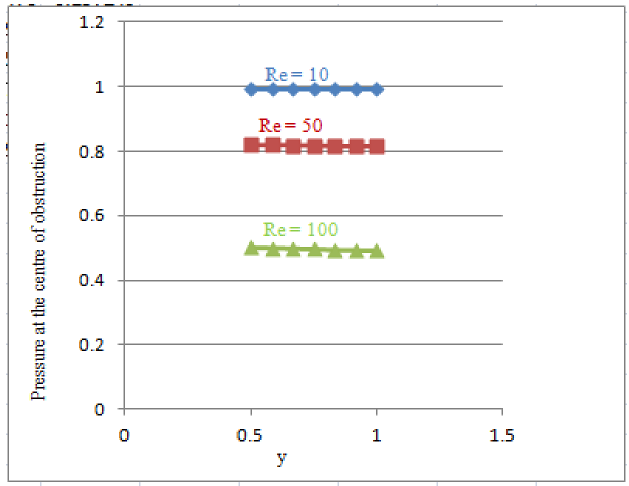

Figure 20: Pressure at centre of obstruction

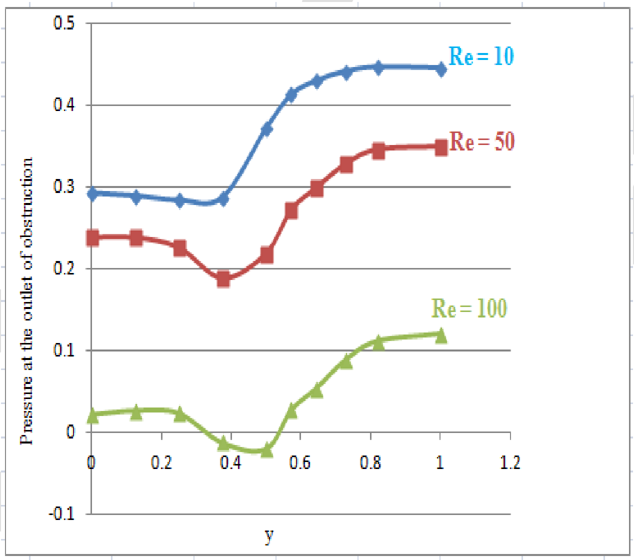

Figure 21: Pressure at outlet of obstruction

CONCLUSION

The FLEX PDE software enables rapid evaluation of the flow characteristics past an obstruction (rectangular plate). It is found to be very effective to solve the partial differential equations. It makes the solution of coupled PDE's very easy going and less calculating efforts are required. The main points of conclusion are

- It is analyzed that there is a variation in the magnitude of velocity in x –direction at different Reynolds numbers but it is found that variation in the magnitude of velocity in y – direction is negligible. It nearly remains constant.
- With the increase in Reynolds number, the magnitude of stream function gets also increased.

As we increase the Reynolds number the size of right vortex also increases.

REFERENCES:

[1] B. H. lakshmanaGowda' and E. G. Tulapurkaraz "Reverse flow in a channel with an obstruction at the entry" J. Fluid Mech. (1989), vol. 204, pp. 229-244.
[2] M.M. Zdravkovich "conceptual overview of laminar and turbulent flows past smooth and rough circular cylinders" Journal of Wind Engineering and Industrial Aerodynamics, 33 (1990) 53-62.
[3] J. LI,* A. Chambarel, M. Donneaud and R. Martin "numerical study of laminar flow past one and two circular cylinders" Computers & Fluids Vol. 19, No. 2, pp. 155--170, 1991.
[4] Yinglong Zhang and Songping Zhu "Open channel flow past a bottom obstruction" Journal of Engineering Mathematics 30: 487--499, 1996.
[5] Alvaro Valencia "laminar flow past square bars arranged side by side in a plane channel".
[6] K.M. Lam∗, M.Y.H. Leung "Asymmetric vortex shedding flow past an inclined flat plate at high incidence" European Journal of Mechanics B/Fluids 24 (2005) 33–48.
[7] A. K. Singha1, A. Sarkar2 and P. K. De "wall effect on heat transfer past a circular cylinder" - a numerical study International Conference on Mechanical Engineering 2005 (ICME2005) 28- 30 December 2005, Dhaka, Bangladesh.
[8] A. Mueller1, J. Anthoine1 and P. Rambaud1 "Vortex shedding in a confined laminar flow past a square cylinder" 5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008) June 30 –July 5, 2008 Venice, Italy.
[9] B.N. Rajani A. KandasamySekharMajumdar "Numerical simulation of laminar flow past a circular cylinder" Applied Mathematical Modelling 33 (2009) 1228–1247.
[10] Chang-Hyun Sohn*, B. H. LakshmanaGowda and Myong-Gun Ju "Reverse flow in a square duct with an obstruction at the entry" Journal of Mechanical Science and Technology 23 (2009) 2376~2389.
[11] Shivani T. Gajusingh, NasiruddinShaikh, Kamran Siddiqui "Influence of a rectangular baffle on the downstream flow structure" Experimental Thermal and Fluid Science 34 (2010) 590–602.
[12] ZouLin Lin Yu-feng Lu Hong "flow patterns and force characteristics of laminar flow past four cylinders in diamond arrangement" 2011,23(1):55-64 DOI: 10.1016/S1001-6058(10)60088-1.
[13] S. B. Doma , I. H. El-Sirafy and A. H. El-Sharif "Two-Dimensional Fluid Flow Past a Rectangular Plate with Variable Initial Velocity".
[14] Vikram C. K., Dr. Y. T. KrishneGowda, Dr.H.V. Ravindra "Numerical simulation of two dimensional unsteady flow past two square cylinders" ©gopalax -International Journal of Technology and Engineering System (IJTES): Jan –March 2011-Vol.2.No.3.
[15] M.A. Kabir, M.M. K. Khan and M. G. Rasul "Numerical Modelling of Reverse Flow Phenomena in a Channel with Obstruction Geometry at the Entry" wseas transactions on fluid mechanics.
[16] D. Greenspan "Numerical Studies of Steady, Viscous, Incompressible Flow in a Channel with a Step" Journal of Engineering Mathematics, Vol. 3, No. l, January 1969.
[17] A. K. Dhiman, R. P. Chabra and V. Eswaran, "Steady Flow Across a Confined Square Cylinder: Effects of Power-Law Index and Blockage Ratio" J. Non-Newtonian Fluid Mech., (2008).
[18] S. Bhattacharyya, Kharagpur, and D. K. Maiti, "Vortex shedding suppression for laminar flow past a square cylinder near a plane wall: a two-dimensional analysis" ActaMechanica, 184: 15–31 (2006).
[19] Jonhn D. Anderson, Jr. "Computational Fluid Dynamics" McGraw-Hill International Editions.
[20] Wisam K. Hussam, Mark C. Thompson and Gregory J. Sheard "A Quasi- two-dimensional investigation of unsteady transition in shallow flow past a circular cylinder in a channel" Seventh International Conference on CFD in the Minerals and process Industries CSIRO, Melbourne, Australia 9-11 December 2009.
[21] S. Kumar, C. Cantu and B. Gonzalez **"**Flow past a rotating cylinder at Low and high rotation rates "Journal of Fluids Engineering Copyright 2011 by Asme April 2011, vol. 133 / 041201-1