

# Comparative Study of type-1 and Type-2 Fuzzy Systems

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**Abstract** TYPE-2 fuzzy sets (T2 FLSs), originally introduced by Zadeh [3], provide additional design degrees of freedom in Mamdani and TSK fuzzy logic systems (FLSs), which can be very useful when such systems are used in situations where lots of uncertainties are present [4]. The implementation of this type-2 FLS involves the operations of fuzzification, inference, and output processing. We focus on “output processing,” which consists of type reduction defuzzification. Type-reduction methods are extended versions of type-1 defuzzification methods. In this paper we represent a comparison of different techniques of fuzzy logic systems.

**Key Words**—fuzzy logic systems, interval sets, uncertainties, membership function, defuzzification.

## I. INTRODUCTION

IN this paper, we introduce a new class of fuzzy logic systems—*type-2 fuzzy logic systems*—in which the antecedent or consequent membership functions are type-2 fuzzy sets. The concept of a *type-2 fuzzy set* was introduced by Zadeh [1] as an extension of the concept of an ordinary fuzzy set (henceforth called a *type-1 fuzzy set*). Such sets are fuzzy sets whose membership grades themselves are type-1 fuzzy sets; they are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set; hence, they are useful for incorporating uncertainties. Quite often, the knowledge used to construct rules in a fuzzy logic system (FLS) is uncertain. This uncertainty leads to rules having uncertain antecedents and/or consequents, which in turn translates into uncertain antecedent and/or consequent membership functions. For example:

- 1) A fuzzy logic modulation classifier described in [2] centers type-1 Gaussian membership functions at constellation points on the in-phase/quadrature plane. In practice, the constellation points drift. This is analogous to the situation of a Gaussian membership function (MF) with an uncertain mean. A type-2 formulation can capture this drift.
- 2) Previous applications of FL to forecasting do not account for noise in training data. In forecasting, since antecedents and consequents are the same variable, the uncertainty during training exists on both the antecedents and consequents. If we have information about the level of uncertainty, it can be used when we model antecedents and consequents as type-2 sets.
- 3) When rules are collected by surveying experts, if we first query the experts about the locations and spreads of the fuzzy sets associated with antecedent and consequent terms, it is very likely that we will get different answers from each expert [4]. This leads to statistical uncertainties about locations and spreads of antecedent and consequent fuzzy sets. Such uncertainties can be incorporated into the descriptions of these sets using type-2 membership functions. In addition, experts often give different answers to the same rule-question; this results in rules that have the same antecedents but different consequents. In such a case, it is also possible to represent the output of the FLS built from these rules as a fuzzy set rather than a crisp number. This can also be achieved within the type-2 framework [6].

## 2 TYPE – 1 FLS

In a type-1 FLS, the inference engine combines rules and gives a mapping from input type-1 fuzzy sets to output type-1 fuzzy sets. Multiple antecedents in rules are connected by the  $\wedge$ -norm (corresponding to intersection of sets). The membership grades in the input sets are combined with those in the output sets using the *sup-star* composition. Multiple rules may be combined using the  $\vee$ -norm operation (corresponding to union of sets) or during defuzzification by weighted summation. In the type-2 case, the inference process is very similar. The inference engine combines rules and gives a mapping from input type-2 fuzzy sets to output type-2 fuzzy sets. To do this one needs to find unions and intersections of type-2 sets, as well as compositions of type-2 relations.

In a type-1 FLS, the defuzzifier produces a crisp output from the fuzzy set that is the output of the inference engine, i.e., a type-0 (crisp) output is obtained from a type-1 set. In the type-2 case, the output of the inference engine is a type-2 set; so we use “extended versions” (using Zadeh’s extension principle [5], [7], of type-1 defuzzification methods. This extended defuzzification gives a type-1 fuzzy set. Since this operation takes us from the type-2 output sets of the FLS to a type-1 set, we call this operation “type reduction” and the type-reduced set so obtained a “type-reduced set.”

To obtain a crisp output from a type-2 FLS, we can defuzzify the type-reduced set. The most natural way of doing this seems to be by finding the centroid of the type-reduced set, however, there exist other possibilities like choosing the highest membership point in the type-reduced set.

□

From our discussions so far, we see that in order to develop a type-2 FLS, one needs to be able to: 1) perform the set theoretic operations of *union*, *intersection*, and *complement* on type-2 sets [8]; 2) know properties (e.g., *commutativity*, *associativity*, *identity laws*) of membership grades of type-2 sets [8]; 3) deal with type-2 fuzzy relations and their compositions [8]; and 4) perform type reduction and defuzzification to obtain a set-valued or crisp output from the FLS [8], [7].

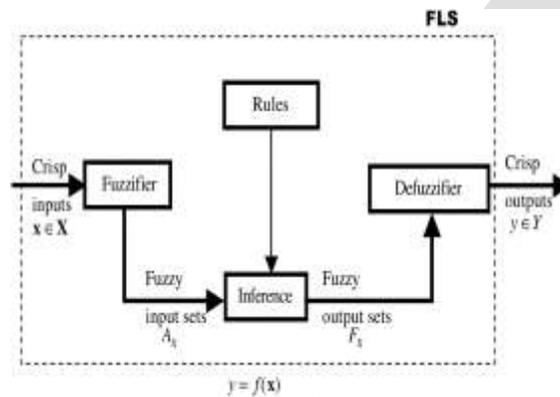


Fig. 1. Type-1 FLS.

### 3 TYPE – 2 FLS

Type-2 fuzzy sets allow us to handle linguistic uncertainties, as typified by the adage “words can mean different things to different people [20].” A fuzzy relation of higher type (e.g., type-2) has been regarded as one way to increase the fuzziness of a relation and, according to Hisdal [11], “increased fuzziness in a description means increased ability to handle inexact information in a logically correct manner.” According to John [12], “Type-2 fuzzy sets allow for linguistic grades of membership, assisting in knowledge representation and they also offer improvement on inferencing with type-1 sets.”

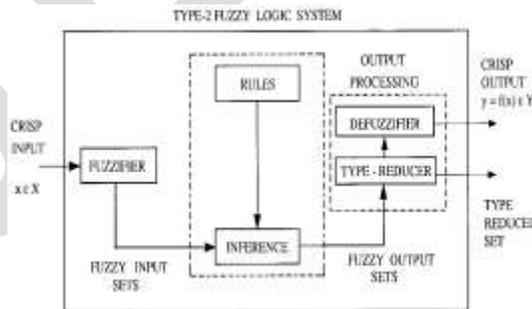
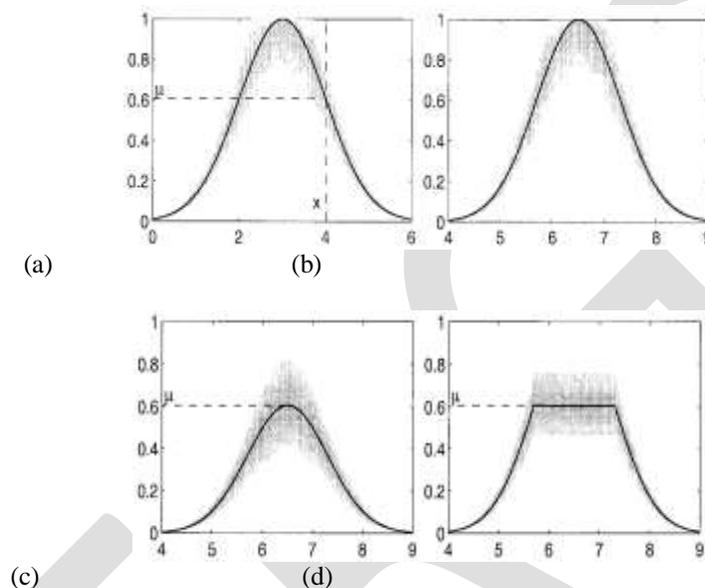


Fig. 2. The structure of a type-2 FLS.

Fig. 2 shows the structure of a type-2 FLS. It is very similar to the structure of a type-1 FLS [26]. For a type-1 FLS, the *output processing* block only contains the defuzzifier. We assume that the reader is familiar with type-1 FLS's, so that here we focus only on the similarities and differences between the two FLS's.

The fuzzifier maps the crisp input into a fuzzy set. This fuzzy set can, in general, be a type-2 set, however, in this paper, we consider only *singleton* fuzzification, for which the input fuzzy set has only a single point of nonzero membership

Fig.3 shows an example of product and minimum inference for an arbitrary single-input single-output type-2 FLS using Gaussian type-2 sets. Uncertainty in the primary membership grades of a type-2 MF consists of a dark region that we call the *footprint of uncertainty* of a type-2 MF. The footprint of uncertainty represents the union of all primary memberships. Darker areas indicate higher secondary memberships. The *principal* membership function, i.e., the set of primary memberships having secondary membership equal to one, is indicated with a thick line



**Fig. 3. Illustrations of product and minimum inference in the type-2 case. (a) Gaussian type-2 antecedent set for a single input system. The membership of a certain input  $x = 4$  in the principal membership function is also shown, equal to 0.6. (b) Consequent set corresponding to the antecedent set shown in (a). (c) Scaled consequent set for  $x = 4$  using product inference. Observe that the secondary membership functions of the consequent set also change depending upon the standard deviation of the membership grade of  $x$ . (d) Clipped consequent set for  $x = 4$  using minimum inference.**

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