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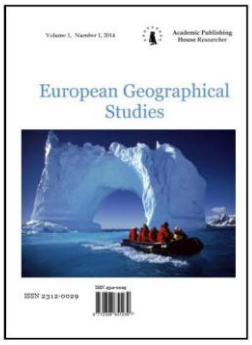

UDC 911

Approximation and Modeling of "Shark Tooth" Stalactites Using Griewank Function & Particle Swarm Optimization Approach (Short Note for Geophysical Structure Modeling)

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Abstract

In this short note we consider the way in which a useful approximation of "shark tooth" stalactites morphology can be obtained with a very simple mathematical function. The approximation is not applicable for other stalactite morphologies, because this possibility can be used only in very special applications, where the solution (or approximation) process is complicated by the presence of some more patterning effects. We shall not consider this complicated question in this article. We only pay attention to the fact that some stalactite patterning mechanisms admit a simple geometrical interpretation in the frame of particle swarm optimization theory.

Keywords: shark tooth stalactites; caves; colloid; emulsion; suspension; particles; Griewank function; PSO; particle swarm optimization; PSO.

Introduction

The shark tooth stalactite is broad and tapering in appearance. It may begin as a small driblet of lava from a semi-solid ceiling, but then grows by accreting layers as successive flows of lava rise and fall in the lava tube, coating and recoating the stalactite with more material; and they can vary from a few millimeters to over a meter in length [1-6]. Any material which is soluble, can be deposited as a colloid, or is in suspension, or is capable of being melted, may form a stalactite [7-9].

Let us begin with defining more carefully what we mean by physical nature in this case. For the purpose of analysis we shall assume that stalactite precursor (colloid or suspension) is a particle manifold [10]. Incidentally, it is to be noted that colloidal particle manifold under external field is an ordered manifold [11]. Moreover, it is easily possible to demonstrate that gravitational behavior of particle swarm in stalactite formation is coherence behavior [12] or synergism [13, 14]. Practitioners (like ourselves) rarely worry about mathematical rigor, but if necessary this can be proved without difficulties. In this short note we shall not pay any attention to mathematical aspects of this problem.

It is important to understand how to apply the concept of physical similarity [15-21] for appropriate approximation selection in stalactite formation hydrodynamics. This method is applicable for a large class of physical systems, but we shall not consider this very extensive question. For example, an application of collective particle dynamics laws gives very simple explanation for particle swarm optimization [22-24] applicability in approximation of cooperative dynamics in a liquid carrier. This is also in accordance with experimental physical observations [25, 26].

Thus we should confine ourselves to finding correct (morphologically similar) approximation for shark tooth stalactite surfaces [27] visualized as regular peak and valley patterns using the function choice within particle swarm optimization. Henceforth, we shall not attempt to distinguish between stalactite forms and forms of particle swarm optimization models of shark tooth stalactites, because it is possible to establish one-to-one mapping between them. A problem that we should inevitably face while using this concept is one-to-one mapping between forms of shark tooth stalactites and force fields of stalactite formation. Of course this approach applies only if we know all of them, but for the present approximation, however, we neglect most of the second order effects. Difficulties arise as soon as we try to approximate stalactite surface forms using *ab initio* approach, but in the first-order approximation we may ignore some technical complications. Nevertheless, attention needs to be paid to morphological similarity between approximation visualization form and elementary shark tooth stalactite surface deformation.

Methods and results

In this section we illustrate the application of the above approach. For simplest example, a useful approximation is obtained by a function from particle swarm optimization area, known as Griewank function [28, 29]:

$$f(x) = 1 + \sum_{i=1}^{n} \frac{x_i^2}{4000} - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}}\right) \quad (1)$$

or

$$f(x) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1 \quad (2)$$

We have taken a number of algorithms and MATLAB codes as a starting point for our findings. Early investigators have used various approaches for Griewank function visualization, such as given below for C++ (by M. Clerck [30]):

```
    E=exp(1); two_pi=2*acos(-1);
sum1=0;sum2=0;
for (d=0;d<D;d++) {xd=x.x[d]; sum1=sum1+xd*xd;
sum2=sum2+cos(two_pi*xd);}
f=(-20*exp(-0.2*sqrt(sum1/(double)D))-
exp(sum2/(double)D)+20+E);
```

or (by Zabinsky, Khompatraporn and Ali [31]):

```
float fvalue;

   fvalue = 0.0;

   float gvalue=0.0;
   float hvalue =0.0;
   for(int index = 0 ; index<dimension ; index ++)
   {
      gvalue = gvalue + pow( *(position+index),2.0 );
      hvalue = hvalue + cos(*(position +
index)*2*3.14159265359);
   }
   fvalue = -20 exp(-0.0.2 * pow((gvalue/dimension),0.5)
```

```
) - exp(hvalue/dimension)+20+exp(1);

   return fvalue;
};
```

and their analogues for MATLAB:

```
function z = ft_ackley(x,y)
a =   20;
b =  0.2;
c = 2*pi;
d =  5.7;
f =  0.8;
n =    2;
z = (1/f)*( -a*exp(-b*sqrt((1/n)*(x.^2+y.^2))) - ...
    exp((1/n)*(cos(c*x) + cos(c*y))) + ...
    a + exp(1) + d);
```

or for "Axes 3D" (http://deap.gel.ulaval.ca/doc/0.8/api/benchmarks.html)

```
from mpl_toolkits.mplot3d import Axes3D
from matplotlib import cm
import matplotlib.pyplot as plt

try:
    import numpy as np
except:
    exit()

from deap import benchmarks

def griewank_arg0(sol):
    return benchmarks.griewank(sol)[0]

fig = plt.figure()
ax = Axes3D(fig, azim = -29, elev = 40)
# ax = Axes3D(fig)
X = np.arange(-50, 50, 0.5)
Y = np.arange(-50, 50, 0.5)
X, Y = np.meshgrid(X, Y)
Z = np.zeros(X.shape)

for i in xrange(X.shape[0]):
    for j in xrange(X.shape[1]):
        Z[i,j] = griewank_arg0((X[i,j],Y[i,j]))

ax.plot_surface(X, Y, Z, rstride=1, cstride=1, cmap=cm.jet,
linewidth=0.2)

plt.xlabel("x")
plt.ylabel("y")

plt.show()
```

We have widely used these or similar algorithms and codes in our computational practice, but have not recently appealed to C++. In recent years several authors (except us) unfortunately no longer use this procedure [32].

Now we should consider an important question: how good this approximation is?

This approximation is valid whenever an obvious visual isomorphism exists between this approximation and surface forms of the shark tooth stalactite. The nature of the approximation is illustrated in Fig. 1 and the photo of the shark tooth stalactite surface is given in Fig. 2. A similar relationship exists between some physical objects only when one of them approximates the other. A better approximation can be obtained by numerical parameter choice, but in our calculations we used particle swarm optimization approach, so such numerical approaches are of little significance in our case [33]. From the arguments completely analogous to those presented in the previous chapter we conclude that morphological similarity in this case corresponds to the similarity of physical principles, because optimization of particle trajectories (for energetically profitable) in cooperative particle dynamics as a physical basis of stalactite formation under well-known physical field provides applicability of a similar approach (known as particle swarm optimization) to computer algorithms for mathematical calculations in this area. Another example of a forecited approach is illustrated in Fig. 3 + Fig. 4 and Fig. 5 + Fig 6 in morphological comparison. The nature of the approximation is illustrated in Fig. 3, Fig 5 and the photo of stalactite surface is given in Fig. 4 and Fig. 6.

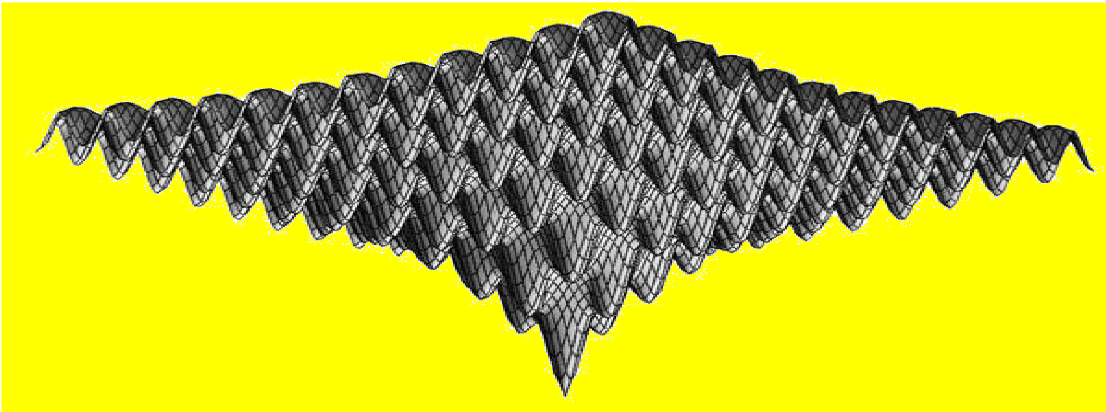

Fig. 1. Griewank function (inverted visualization by Mathcad)

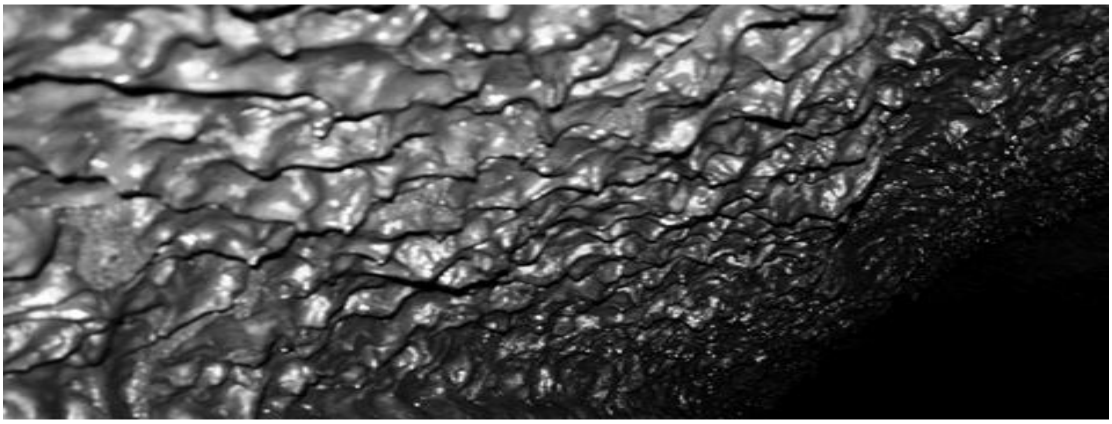

Fig. 2. “Shark tooth” stalactites

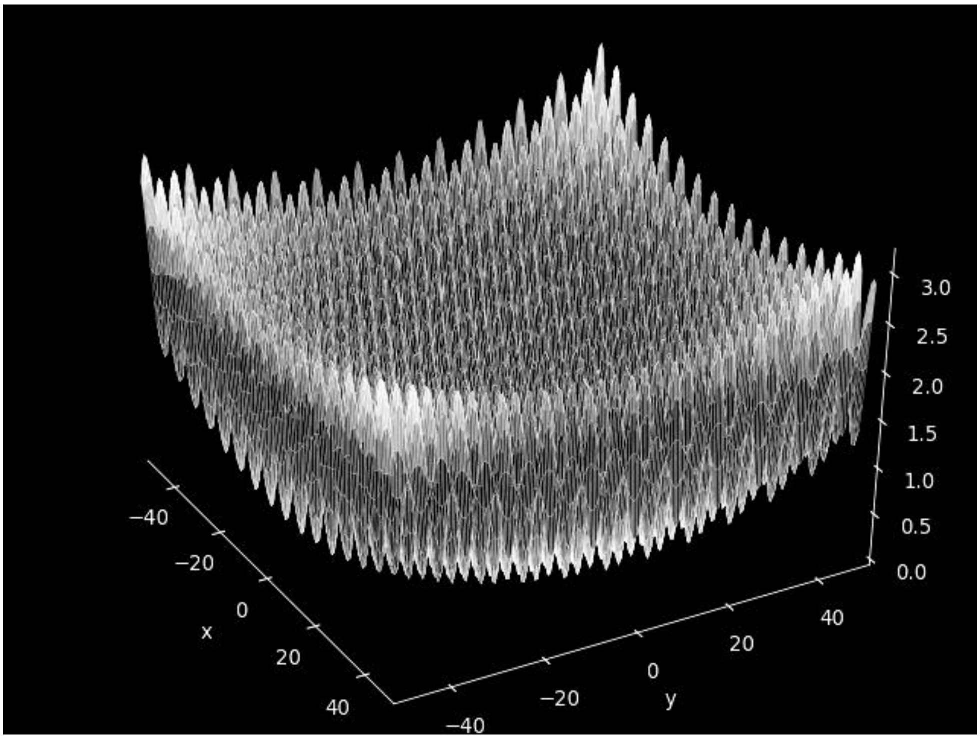

Fig. 3a. “Concave” visualization of Griewank function (created using “Axes 3D”).

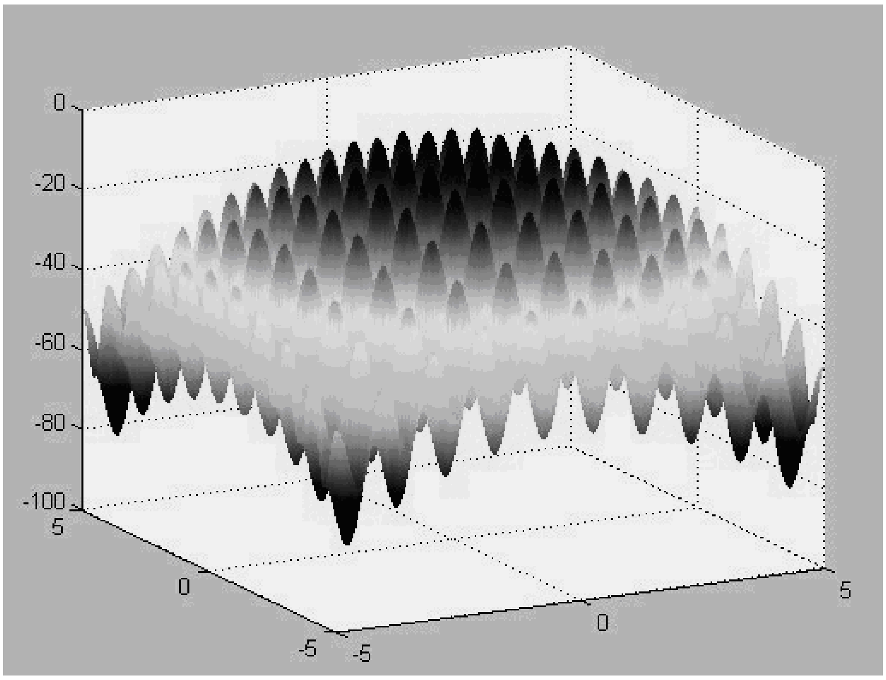

Fig. 3. “Sloping” \ “convex” or “arched” visualization Griewank function (created using “Axes 3D”).

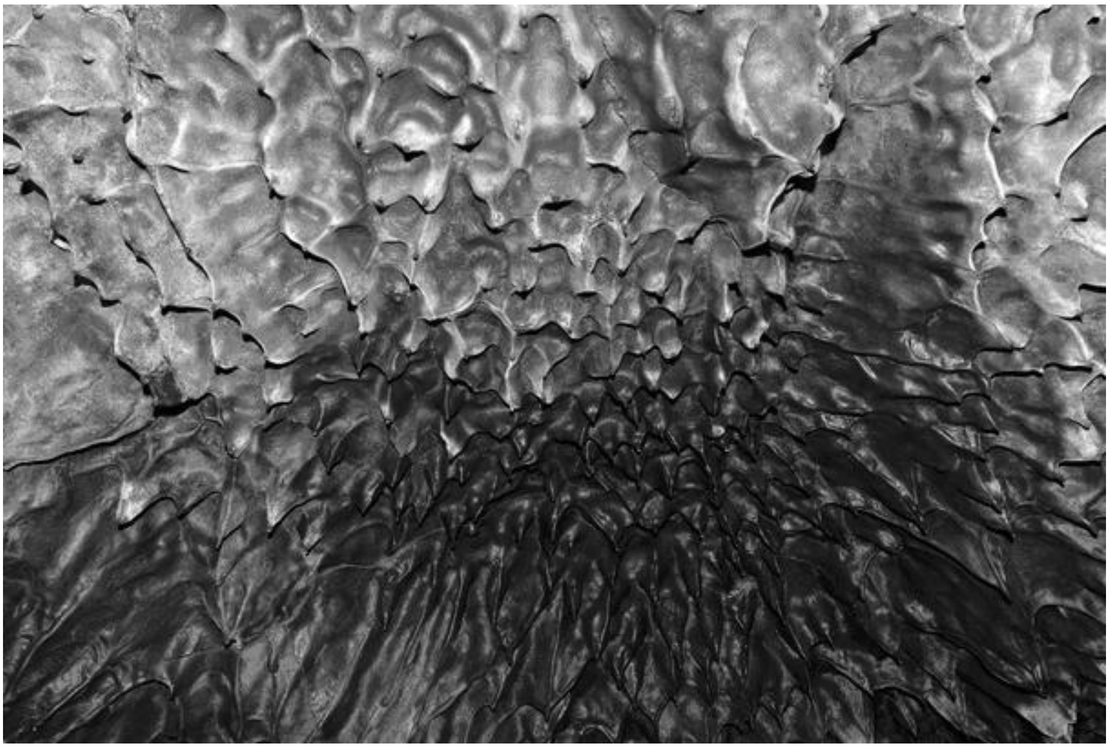

Fig. 4. “Shark tooth” stalactites

Conclusion

Thus we have focused our attention at physical mechanisms of shark tooth-like stalactite self-organization based on force field induced [34] collective (multi-agent [35, 36]) behavior of colloidal / suspended particles in shark tooth stalactite formation . Strictly speaking, such a definition does not make sense because it is very obvious. We therefore limit ourselves to the most simple case, which is morphologically observable. We have not paid much attention to some more complicated situations, but it's clear that the viewpoint adopted in this article possesses a more wide application than we have already mentioned here. Our next step was to apply this idea for modelling of biomimetic pattern formation and a corresponding article has just been submitted to another biological journal. It was not our purpose to give a comprehensive development of the idea proposed, so in this article we have only laid a theoretical foundation for its further application. Actually, we are also little concerned about our priority, because this complex problem is still fairly difficult to be solved without collective efforts. These results can easily be described in terms of particle swarm optimization theory. Such elementary cases can be covered by the general Griwenk equation. The foregoing results are a very brief and simplified implementation of this basic idea, because in general case the described procedure itself is known in PSO, but its application to ferrofluidics is our contribution. The described approach is expected to possess a wide range of potential applicability in ferrofluid science.

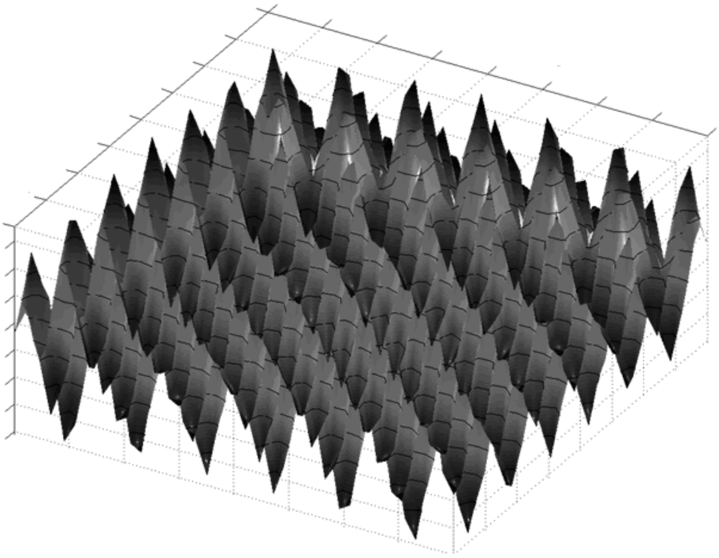

Fig. 1. Griewank function.

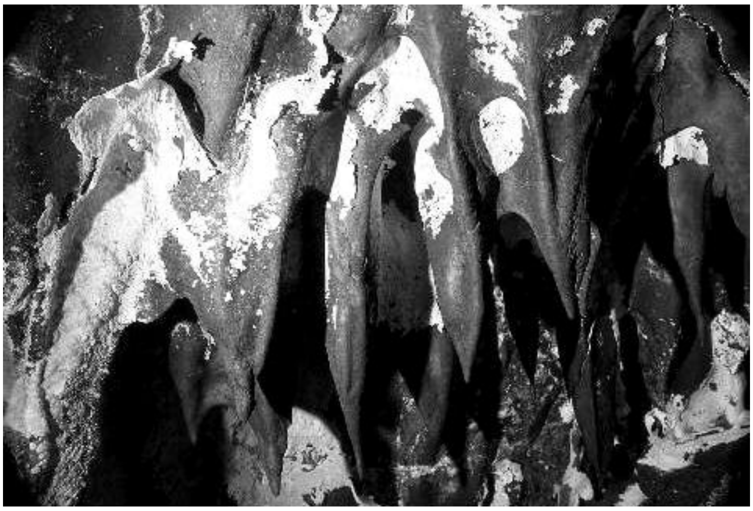

Fig. 6. "Shark tooth" stalactites

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