

Teaching Materials “Surface Area of Geometric Figures,” Created Using the Software Package *GeoGebra*

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Abstract. Social development, the progress of technology, and changing economic forces certainly affect the development of the current educational system. One of the main problems of today’s school system is how to maintain focus, concentration, and interest among students with regards to the learning that takes place during classes. An important feature of modern teaching is multimedia. However, the use of multimedia requires a certain transformation in the teaching process. Given the fact that the focus of the teaching process has been shifting away from the curriculum and the teacher, and towards the student, multimedia will undoubtedly contribute in a significant way to the modernization of traditional teaching. It is not unrealistic to think that technology will become a regular part of the daily routines of teaching. In this work, an innovative approach to teaching mathematics in elementary and high schools through the use of the software known as GeoGebra is introduced. This approach is demonstrated using the example of ‘surface area’ as a mathematical concept, wherein the goal was to increase interactivity between teachers and students, and to improve the quality of teaching.

Keywords: teaching mathematics; GeoGebra; modern education; interactivity

1. Introduction

The future of the school cannot be understood without understanding its development and history (Cross, 2004). One of the key needs of the human race is education, which is reflected in the maintenance of the species. In order to fulfil that need, adult community members have cared about youth. As the knowledge and skills that were to be transferred to the next generation kept accumulating, it was clear that the care of young people had to stand apart from everyday life, and needed to be organized differently. Over time, education and teaching the young have become a special social activity, one that has gradually been formalized and institutionalized. One characteristic of the formal organization of youth education was the genuine concern about society’s survival and the maintenance of the human species.

The school is one of the oldest institutions of known societies, dating back approximately 7000 years to Sumeria and ancient China. Education continued to formally develop through the Greek and Roman cultures, was influenced

greatly by various world religions and international conflicts, and has continued to evolve during the modern era. The school is able to survive so long because it is constantly changing, adapting, and harmonizing its work with the needs and demands of the communities within which it has existed. The school has changed and continued to take on new forms, always transforming into a new school, one that meets the needs of the community.

For all those concerned with education in any way, it is of great interest to understand how schooling will play a social role in the future of modern society. Social development, the progress of technology, and changing economic forces certainly affect the development of the current educational system. The problems that we will encounter in the future, the direction of technical advances, the development of economic systems can no longer be predicted using past experience. Currently, we cannot even be sure of what the world will look like in ten or twenty years—the world in which these students that we are teaching today, will work, live, and raise families.

Our goal as teachers is to prepare students properly to face problems that await them in the future. Unlike the previous school system, where it was required to be submissive and obedient to reproduce the stored or known information, we now ask students to understand, to think, and to choose the correct answers. We must value creativity and freedom of thought, which has often been neglected in formal schooling.

Before we start talking about the development of the current education system, we must look at its shortcomings. Perhaps the greatest example is the lack of attention and concentration demonstrated by students during the average lesson. The goal of every teacher is to make lessons fun and interesting enough so that more students will be motivated to be attentive during classes, to think, and to follow the material that is being presented (Ruthven, Hennessy, & Brindley, 2004). However, when it comes to mathematics classes, especially with abstract mathematical topics, this is not an easy task. Teachers can use the most modern technology and educational software to present abstract mathematical concepts (Polya, 1962) in a virtual environment (e.g., using computer software) in which students feel very comfortable. In this way, the space where students are accustomed to playing and having fun is turned into a space where in students can be free to explore and to learn (Pećanac, Lambić, & Marić, 2011).

Classes were formerly designed to fit the needs of the *average student*, and this was another drawback of the traditional education system. The use and implementation of multimedia and educational software classes may significantly contribute to the modernization and individualization of learning (Prawat, 1992). Educational software that is designed with good didactic and methodical preparation of materials is able to better correspond with the existing knowledge, abilities, and skills of each student. Instruction is adjusted to each student in light of their individual needs, since the progress of students

does not have to be tied to the overall progress of the class, but rather to their own work and learning opportunities. The focus of the teaching process has indeed shifted away from teachers and teaching materials, towards the student and learning materials. One might say that the student is finally in the centre of our teaching.

GeoGebra is a mathematical software developed by Markus Hohenwarter for teaching mathematics in schools. It was originally created as part of his graduate studies (Masters), and has since become a very popular software package that is used around the world. Hohenwarter continued to improve the program through the completion of his doctoral dissertation. He has won a number of European and international awards in the field of educational software. GeoGebra is a software package that connects algebra, geometry, and analysis—this combination forms the basis of its name (i.e., geometry + algebra). It is a freely-available and open-source software, and it is currently available in over 50 languages. User guides, a user forum, and many useful related resources are also available through the main website (<http://www.geogebra.org/>). GeoGebra is written in Java, and as such can be run in any web browser. GeoGebra combines two different approaches of visualizing mathematical objects. More precisely, geometry and algebra are equally and dynamically represented, i.e., it is possible to assign objects to equations, and then to change the graphic objects and observe how the equations change, or vice versa.

One can use GeoGebra to create interactive web pages, so-called interactive worksheets, and the easy publication of these drawings to the web is yet another benefit of the software. Without knowledge of HTML programming or how to create web pages, one can easily use the GeoGebra “Wizard” that requires only a few clicks and the entering of basic information related to the activity. This method allows students to better interact with the material, as well as providing a robust and reliable method of sharing files with their peers in the classroom, school, and beyond. As a result, students are often more engaged and interested in their mathematical work (Hohenwarter & Preiner, 2007). Interactivity of applets in web pages is increased if Javascript buttons are present on the site, allowing the interaction of text and applets. By preparing these kinds of interactive worksheets, teachers can capitalize on a powerful tool that can bring abstract mathematical concepts to life for students of all ages (Chen-Wo, Jiann-Min, Quo-Ping, & Maiga, 2009).

2. Interactive Educational Activity

In this section, an interactive educational activity for the elementary and high school level is presented, related to the term “surface area” of a figure.* Dynamic interactivity is achieved by using JavaScript and PHP functions, and a

* The activity is available for downloading in the form of GeoGebra applets at the website: <http://alas.matf.bg.ac.rs/~ml06125>

MatJax function enables the writing of mathematical text and formulas using standard LaTeX commands. All educational materials are written primarily for students (students can learn independently, expanding their knowledge and making gains in individual progress) but also for teachers (including ideas for multi-media classes). All of the available activities are divided into categories based on various mathematical topics relating to the surface area of figure, and according to the educational standards issued by the Ministry of Education of the Republic Serbia. Each category contains four different sections of interactive materials that are mutually complementary.

The first section related to the aspect of learning. By using interactive GeoGebra applets (see Figures 1-3) and dynamic web pages, geometric figures/objects and their basic properties are represented to students in interesting ways, as they learn to calculate the surface areas of a figure.

☐ The concept of surface

Elementary School

☐ 4. grade

Comparison and measurement of surface

Calculation of the rectangle area

Calculation of the surface of the cuboid and the cube

Interesting tasks

Test of knowledge

Homework

☐ 6. grade

☐ 7. grade

☐ 8. grade

High School

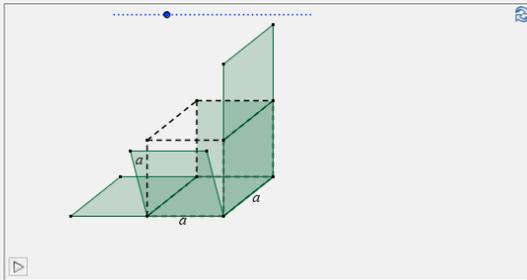
☐ 3. grade

☐ 4. grade

☐ Literature

Calculation surface of the rectangular and the cube

Objectives and tasks of these classes are introducing students to the figures of cubes and squares, edges, vertices, sides, and to develop in the surface plane of the figure. The following applet shows that the network consists of 6 blocks of matching square with sides equal to the edge of a cube. If P_1 surface of a square, then the surfaces of the cube

$$P = 6P_1 = 6a^2.$$


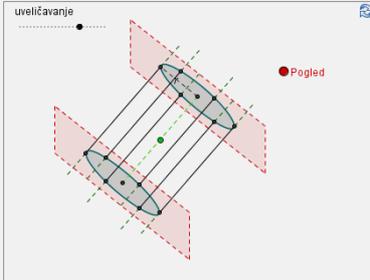
The network rectangular consists of a six rectangles, of which two nonadjacent congruent. The surfaces of these rectangles are

$$P_1 = a \cdot c, \quad P_2 = a \cdot b, \quad P_3 = b \cdot c,$$

and rectangular area:

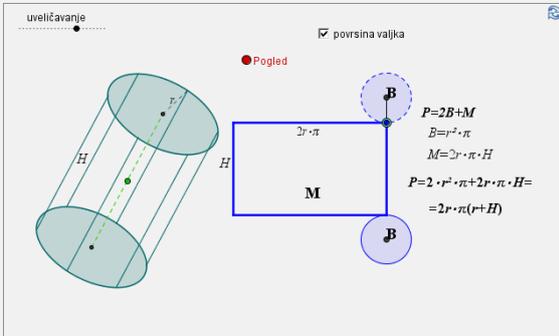
Figure 1. Grade 4 activity: Surfaces of the cube

uveličavanje



uveličavanje

površina valjka



$$P = 2B + M$$

$$B = r^2 \cdot \pi$$

$$M = 2r \cdot \pi \cdot H$$

$$P = 2 \cdot r^2 \cdot \pi + 2r \cdot \pi \cdot H = 2r \cdot \pi (r + H)$$

Figure 2. Grade 8 activity: Area of the cylinder

☐ The concept of surface

elementary School

☐ 4. grade

☐ 6. grade

☐ 7. grade

☐ 8. grade

☐ 8. grade

High School

☐ 3. grade

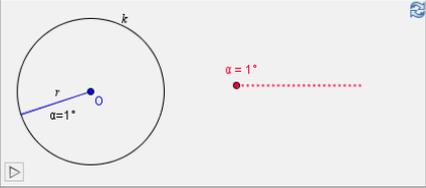
☐ 4. grade

☐ Literature

Surface of the sector, slice, and ring

The circular slice is part of the circle limited by radius and arc. Each circular slice has a corresponding central angle α and an arc length l .

Let draw circular slice of which is the central angle 1° . Its area is equal to 360-part of the circle area $P_t = \frac{r^2 \cdot \pi}{360}$.



If a central angle of the sector 2° then its surface is $P_t = \frac{r^2 \cdot \pi}{360} \cdot 2$, if a central angle is 3° than is $P_t = \frac{r^2 \cdot \pi}{360} \cdot 3$, and so on...

If α central angle of the sector, then its surface is:

$$P_t = \frac{r^2 \cdot \pi}{360} \cdot \alpha.$$

Figure 3. Grade 7 activity: Surface area of the sector, slice, and ring

The main feature of this presentation of educational material is that it allows students to individually explore and discover relationships between observed objects.

By moving objects, points, and the slider on the applets, students notice changes that occur, and in this way draw their own conclusions. In every Web page with applets included, there is mathematical text that introduces students to the event that takes place on the applet, and students can later check to see if they have discovered the main conclusion of that particular lesson.

The second part refers to the tasks for exercises. When students learn the material that concerns the properties of geometric objects and the ways in which one computes the surface area of figures, the next step is to explore the learning further with problem assignments. By opening the link “Interesting tasks” (Figure 4) within each class, we get a web page with the problems that need to be solved. The problems are divided into several groups, depending on the level of difficulty or the type of task. If students have some problems while solving the tasks, they may simply click on the “solution” button that appears below each task. This button opens a field in which the steps of the solution are explained in detail and the exact results are given. This type of initiation of the problem allows students to solve tasks independently, and (if needed) simultaneously check if are they working well or if there is another, perhaps simpler, way of solving the problem.

② The concept of surface

Elementary School

- ② 4. grade
- ② 6. grade
- ② 7. grade
- ② 8. grade
- Surface of prism
- Surface of pyramid
- Surface of the cilinder
- Surface of buy
- Surface of the ball
- Interesting tasks

High School

- ② 3. grade
- ② 4. grade
- Literature

Interesting task

Prism Pyramid Cylinder Cup Ball

1. What is the height of equal edges a three-sided pyramid edges 9 cm? Solution

We can see the right-angled triangle on it and apply the Pythagorean Theorem:

$$H^2 = a^2 - \left(\frac{2}{3}h_a\right)^2$$

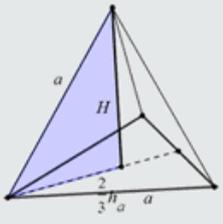
$$H^2 = a^2 - \left(\frac{2}{3} \cdot \frac{a\sqrt{3}}{2}\right)^2$$

$$H^2 = 9^2 \text{ cm}^2 - \left(\frac{9\sqrt{3} \text{ cm}}{3}\right)^2$$

$$H^2 = 81 \text{ cm}^2 + 27 \text{ cm}^2 = 108 \text{ cm}^2$$

than $H = 6\sqrt{3} \text{ cm}$.

Close solution.



2. Calculate the height of regular hexagonal pyramid if the side edge is length 6 cm formed with a the plane base angle of 45°. Solution

As in the previous task, draw a picture and then look at what it looks like a diagonal cross-section.

The angle at the top of the pyramid is straight, if you put the the height from the top of the pyramid it divides the right-angled triangle into two congruent isosceles triangles. Hence we can conclude that the height of the pyramid is equal to the basic edge a and using the Pythagorean theorem, we obtain:

$$s^2 = a^2 + H^2$$

$$6^2 \text{ cm}^2 = 2H^2$$

$$H^2 = 18 \text{ cm}^2, \text{ from here is } H = 3\sqrt{2}.$$

Close solution.

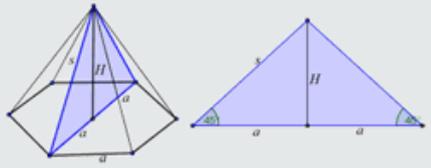


Figure 4. Interesting tasks

The third section is related to the self-test regarding knowledge (Figure 5). Students who have successfully completed tasks in the part of the site dedicated to the activity can further test their knowledge by solving a related test. By clicking on the “Test” button within each class, one can open the web page that features the instructions for students and the selection of the level assignment difficulty. Tasks are grouped in three tests, arranged by difficulty, as instructed by “educational standards for the end of compulsory education for mathematics”, according to the Ministry of Education and the Institute for the Evaluation of Education of the Republic of Serbia. When the test is opened, the measuring of time begins.

At the end of each question, there is a place to enter the correct answer, and the number of points awarded for each part of the question is also indicated. When a student is done with the task, by clicking on the “Check” button, his answers are evaluated and he accumulates a certain number of points. The student can win a total of up to 100 points. The “Check” button opens a new page where all of the tasks are reviewed and where the student can read information about the test, such as which tasks are correct, the number of points that were won, the time spent solving the test, and the exact solutions for the tasks (in order to realize their mistakes).

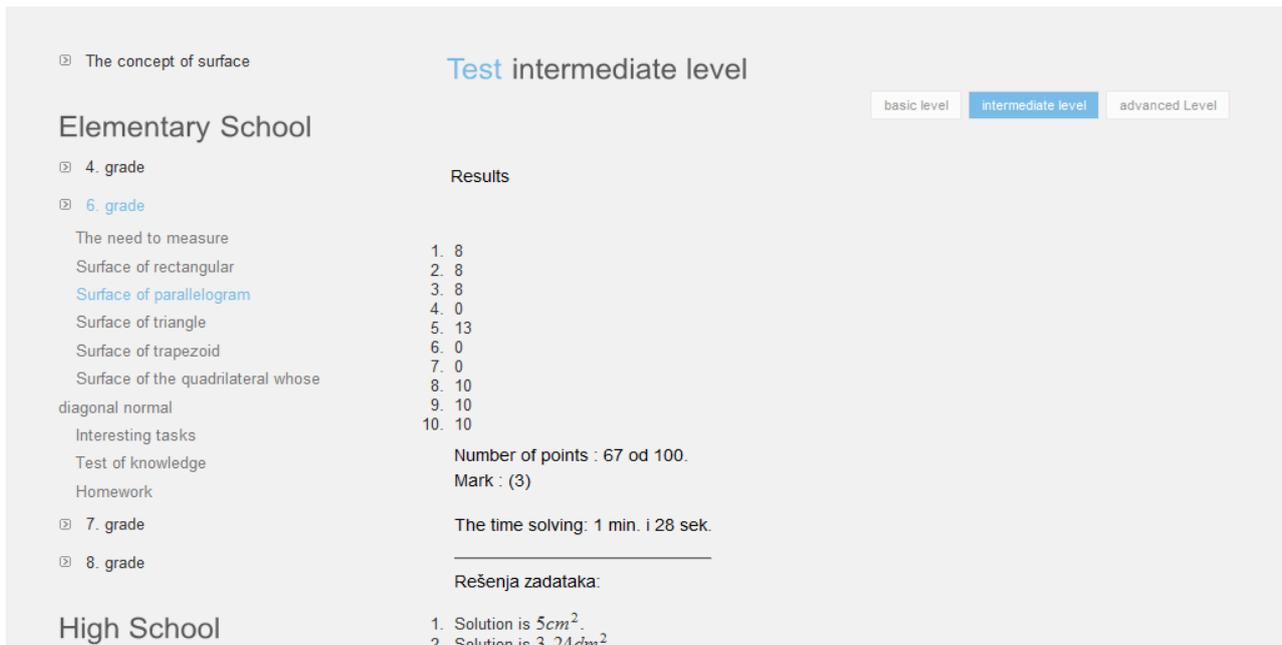


Figure 5. Test of knowledge

The fourth part refers to homework (Figure 6). This part of the site is dedicated to students who work at home and whose knowledge teachers may want to check by using e-mail. When a student opens a homework app, before he/she starts to work, it is necessary to enter his/her name, email, and the teacher's email in the corresponding fields. When all of the tasks are completed, the student fills in the last field which refers to the comments on homework, then clicks on "send homework" in order to send the answer to the teacher.

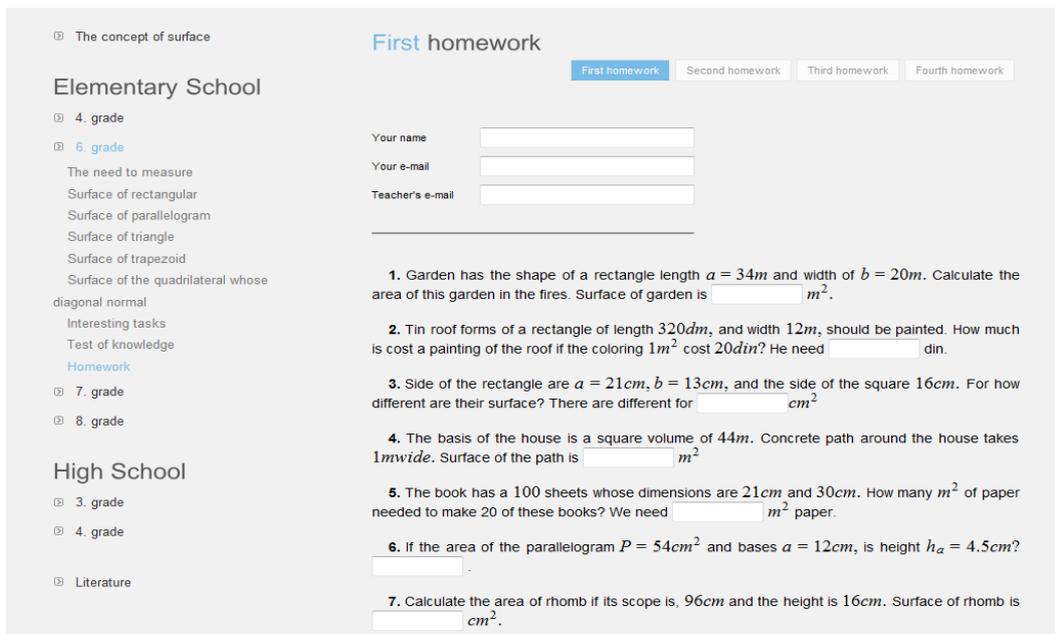


Figure 6. Homework

When a student clicks on "send" button, the student receives a confirmation email (see Figure 7) indicating that the homework has been sent to the teacher.

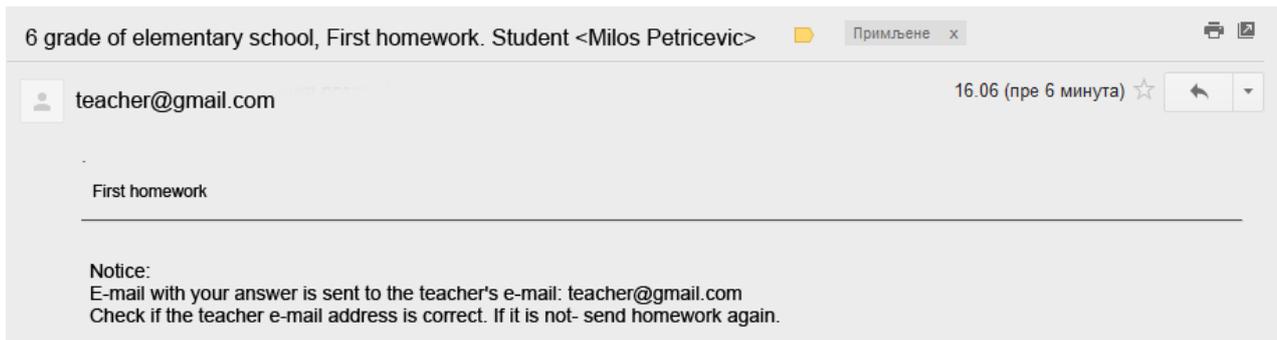


Figure 7. Automatic e-mail received by the student

The teacher also gets an email with the homework title, the student's name, an email address, text relating to the tasks, the correct answer(s), and the student's answer(s) in table format (see Figure 8). The teacher needs to review mail and to inform students on the reception of the email, as well as the grade received for the homework tasks.

First homework, 6. grade of elementary school		
Student name:	Milos Petricevic	
Student e-mail:	milos.petricevic@gmail.com	
Text of problem	Corect answer	Student answer
1. Vrt ima oblik pravougaonika dužine $a=34m$ i širine $b=20m$. Izračunati površinu toga vrta u arima.	680m ²	6,8
2. Limeni krov oblika pravougaonika dužine 320dm, a širine 12m, treba obojiti. Koliko košta bojenje tog krova ako se za bojenje 1m ² plaća 20din ?	7680	7680
3. Stranice pravougaonika su $a=21cm$, $b=13cm$, a stranica kvadrata je 16cm . Za koliko se razlikuju njihove površine?	17cm ²	17
4. Temelj kuće je kvadrat obima 44m . Oko kuće vodi betonska staza širine 1m . Površina staze je ?	48m ²	48
5. Sveska ima 100 listova čije su dimenzije 21cm i 30cm. Koliko m ² papira je potrebno da bi se napravilo 20 takvih svezaka?	1260m ²	126
6. Ako je površina paralelograma $P=54cm^2$ i osnovica $a=12cm$, da li je visina $h_a=4,5cm$?	Da.	da
7. Izračunati površinu romba ako mu je obim 96cm, a visina 16cm.	384cm ² .	65
8. Obim paralelograma je 24cm. Kraća stranica je dva puta manja od duže stranice. Ako je visina koja odgovara dužoj stranici $h_a=2cm$, odrediti dužinu visine koja odgovara kraćoj stranici.	4cm.	3
Comments on homework		

Figure 8. E-mail received by the teacher

3. Conclusion

The main task of teachers is to present a problem to a student, to make it more understandable, and to prepare interactive GeoGebra worksheets that enable students to individually research the concept and to verify new

properties of known objects. In this sense, the software package GeoGebra as a tool for modelling and dynamic interactivity, can greatly enhance learning through student discovery activities. In this paper, we have only been able to present a sampling of the many features of GeoGebra and the possibilities that it offers to teachers of mathematics. We have demonstrated a series of interactive educational activities used in elementary and highschools, relating to the term “surface area of figure.” Four aspects of a created GeoGebra environment were presented: the learning aspect, explanations, self-testing, and homework extensions. This approach can be applied to any mathematical topic. Teachers can use GeoGebra educational software and present abstract mathematical concepts in a virtual environment wherein students feel very comfortable and confident. In this way, the GeoGebra environment can be viewed as a place where students can learn and play with mathematics at the same time.

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