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DYNAMIC STABILITY OF A CRACKED PIPE CONVEYING FLUID AND RESTING ON A PASTERNAK ELASTIC FOUNDATION

Summary. Pipeline transport is used worldwide in many sectors of the economy. Its main advantages are continuity of transport, large transportation volumes, small energy consumption, safety, reliability and high environmental benefits. However, the safety problems of pipes attract much interest in science and industry. This paper deals with a cracked pipe with a static scheme of a simply supported beam. It rests along its entire length on a Pasternak elastic foundation. The flowing fluid is considered non-compressible and heavy. The Galerkin method is employed to approach the problem numerically. Conclusions are drawn based on the influence of the crack and the parameters of the Pasternak elastic foundation on the critical flow velocity of the fluid.

Keywords: pipe, fluid, dynamic stability, crack, Pasternak elastic foundation, critical velocity

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1. INTRODUCTION

Pipelines conveying fluid have many engineering applications. They are widely used in the petroleum industry for the transportation of oil and gas. Another established use of them is in the transport of water. They also find applications in nuclear engineering, aviation and aerospace.

Nanoscale tubes find applications in nanophysics, nanobiology and nanomechanics as nanofluidic devices in nanocontainers for gas storage and nanopipes conveying fluid. The experiments at the nanoscale are difficult and expensive. That is why the continuum elastic models have been used to study the fluid-structure interaction. The carbon nanotubes are considered with Euler- and Timoshenko-beam models.

Tubes conveying fluid may also be found in pulmonary and urinary systems and haemodynamics.

The interaction of a tube and the fluid flowing in it has been the subject of much research. The flow of the fluid in the tube causes oscillations in it. The dynamic characteristics of the pipe's oscillations depend on the velocity and the mass of the conveyed fluid. The system is stable for flow velocities that are less than a certain value called critical flow velocity. The research on the dynamic stability of pipes conveying fluid is branched into two directions: a) dynamic stability of pipes with a rectilinear axis, and b) dynamic stability of curved pipes.

The most common methods used for the dynamic analysis of pipes conveying fluid are the transfer matrix method (TMM) and the generalized differential quadrature method (GDQM). Both methods have a significant advantage over the finite element method (FEM). The conventional FEM can be very time-consuming when it comes to the investigation of a pipeline with a high number of spans. The order of the overall property matrices for the whole multi-span pipeline increases with the number of spans. This is unlike the TMM, in which the order of the overall transfer matrix is independent of the number of spans and is kept unchanged.

The GDQM approximates a derivative of a function in the partial differential equation of the lateral vibration of the pipe at any discrete point as a weighted sum of the function values at all discrete values at the domain. The main advantage of this method is its high convergence with a few grid points.

Pipelines often rest with their entire length or with part of it on an elastic medium. The first suggested model of that medium is the Winkler elastic foundation. Although it has some shortcomings, it is still being widely used in civil engineering since its introduction in 1867. The Winkler model of the elastic medium consists of mutually independent vertical linear springs.

In 1954, a refined model of the elastic medium was introduced by Pasternak. He introduced shear interaction between adjacent linear springs in the Winkler model. The Pasternak foundation is a two-parameter model. The values of the parameters for practical application are the subject of much research in the field of geotechnical mechanics.

Cracks are the most encountered damage in the structures. They reduce the stiffness of the structural element, causing a decrease in its natural frequencies and a change in the mode shapes. In pipes conveying fluid, cracks lead to a decrease in the critical velocity of the fluid. The cracks could be hazardous for the system. They might lead to loss of stability if the reduced critical velocity of the transported fluid, due to the crack, is exceeded. This is why crack detection is a topic of great interest in scientific research. Some of the studies for crack detection deal with the changes in the natural frequencies and eigenforms, and others with dynamic responses to harmonic loads.

The book [16] deals with the dynamics of slender cylindrical bodies in contact with axial flow. It not only covers the fundamentals of the problem but also solves some examples that have direct applications in engineering and physiological systems.

M. Paidoussis and N. Issid in [15] investigated the dynamic stability of pipes with internal flow. They considered clamped-clamped and pinned-pinned pipes.

M. Siba et al. [19] reviewed studies of the oscillations of a tube conveying fluid. The need for more experiments in this area is justified.

L. Shiwen et al. [18] studied the flow-induced vibration characteristics of a pipeline system. Fluid-structure interaction numerical simulation is conducted for a typical fluid-conveying pipe network with the help of software.

Bing Chen et al. [2] used Galerkin's method and the complex mode method to find the natural frequencies of a pinned-pinned pipe conveying fluid and lying on a Pasternak foundation.

Eslami Ghiyam et al. [8] investigated the vibrations of a pipe with a crack and embedded in a visco-elastic medium.

Son In-Soo et al. [21] investigated vibrations of a cracked pipe conveying fluid with concentrated mass and supported on elastic supports.

In this paper, a fluid-conducting tube resting on a Pasternak elastic foundation is investigated. The results obtained reflect the dependence of the critical fluid velocity on the parameters of the Pasternak elastic foundation. They also show the effect of an open crack on the critical velocity of the fluid.

This paper is structured as follows. First, the model of the pipe and the governing differential equation of its transverse vibration is presented. The Galerkin method is employed to approach the solution to the problem. It is shown how to obtain the characteristic equation of the problem. Based on its roots, conclusions could be drawn about the stability of the system. Second, it is shown how to model the crack with the help of Castigliano's theorem. Finally, the obtained results from the numerical solution are presented, and several important conclusions are summarized.

2. PROBLEM FORMULATION

This paper uses the Euler-Bernoulli beam theory to investigate the dynamic stability of a pipe of length l, conveying fluid and resting on a Pasternak elastic foundation. The pipe, shown in Figure 1, is hinged at both ends and is supposed to have an open edge crack, which is modelled as a rotational spring with a lumped stiffness k_{rs} [14]. The position of the crack is shown in Figure 1 through the axial coordinate x_c.

The pipe is divided into two segments. The first segment is the left-hand side of the crack, and the second – the right-hand side of the crack.

The transverse vibration of a straight pipe conveying inviscid fluid and lying on a Pasternak elastic foundation is governed by the following differential equation:

$$EI\frac{\partial^4 w}{\partial x^4}+\left(m_f V^2-k_r\right)\frac{\partial^2 w}{\partial x^2}+2m_f V\frac{\partial^2 w}{\partial x\partial t}+\left(m_f+m_p\right)\frac{\partial^2 w}{\partial t^2}+kw=0, \tag{1}$$

where t is the time, $w(x,t)$ is the lateral displacement of the pipe axis, x is the coordinate along the axis, EI is the rigidity of the pipe. The mass of the pipe per unit length is denoted by m_p and the mass of the fluid per unit length of the pipe by m_f. V is the flow velocity of the fluid in the pipe. While k and k_r, are the parameters of the Pasternak elastic foundation.

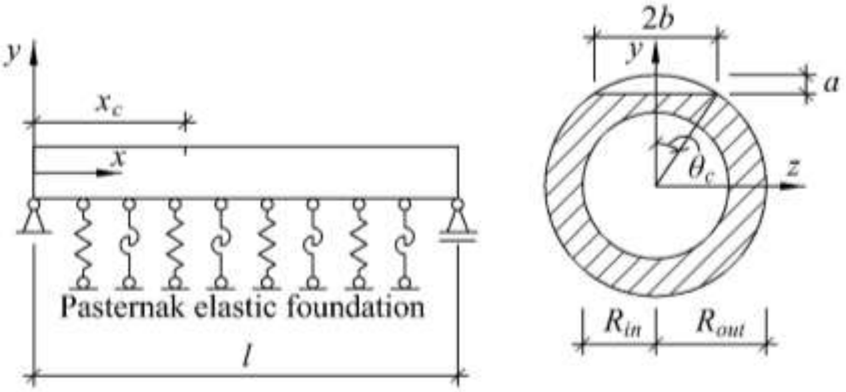

Fig. 1. Static scheme and cross-section of the investigated pipe

For simplicity, the following dimensionless parameters are introduced:

$$\xi = \frac{x}{l};\ \xi_c = \frac{x_c}{l};\ u = lV\sqrt{\frac{m_f}{EI}};\ \beta = \frac{m_f}{m_f + m_p};\ \tau = \frac{t}{l^2}\sqrt{\frac{EI}{m_f + m_p}};$$

$$\bar{k}_r = \frac{k_r l^2}{EI};\ \bar{k} = \frac{k l^4}{EI};\ \bar{k}_{rs} = \frac{k_{rs} l}{EI} \tag{2}$$

The dimensionless equations for transverse vibration in the two segments of the pipe are:

$$\eta_n^{IV} + \left(u^2 - \bar{k}_r\right)\eta_n^{II} + 2u\sqrt{\beta}\dot{\eta}_n^{I} + \ddot{\eta}_n + \bar{k}\eta_n = 0,\ n = 1,2 \tag{3}$$

In (3) and in the sequel, primes denote derivatives with respect to ξ and dots with respect to the dimensionless time τ.

The spectral Galerkin method is applied to approximate the solution of the differential equation (3). The solution for each segment of the pipe is sought in the following form:

$$\eta_n(\xi,\tau) = \sum_{i=1}^{m} W_{ni}(\xi) q_i(\tau),\ n = 1,2, \tag{4}$$

where:
$q_i(\tau)$ - are unknown functions;
$W_{ni}(\xi)$ - are basic functions that satisfy the boundary conditions of the pipe. Such functions describe the i-th mode of vibration of a beam with the same static scheme as the pipe.

The boundary conditions of the cracked simply supported beam, shown in Figure 1 are:
For the left end of the beam:

$$W_{1i}(0)=0 \text{ and } W_{1i}^{II}(0)=0 \;. \tag{5}$$

For the right end of the beam:

$$W_{2i}(1)=0 \text{ and } W_{2i}^{II}(1)=0 \tag{6}$$

For the cracked section of the pipe [21]:

$$W_{1i}(\xi_c)=W_{2i}(\xi_c)\,,\; W_{1i}^{II}(\xi_c)=W_{2i}^{II}(\xi_c)\,,\; W_{1i}^{III}(\xi_c)=W_{2i}^{III}(\xi_c)\,,$$

$$\left[W_{1i}^{I}(\xi_c)-W_{2i}^{I}(\xi_c)\right]\overline{k}_{rs}=W_{2i}^{II}(\xi_c) \tag{7}$$

Inserting equation (4) in equation (3), yields:

$$|M|\ddot{q}+|C|\dot{q}+|K|q=0\,, \tag{8}$$

where the elements of the matrices in equation (8) are:

$$M_{ij}=\sum_{n=1}^{2}\int_{0}^{\xi_n}W_{ni}(\xi)W_{nj}(\xi)d\xi \tag{9}$$

$$C_{ij}=2u\sqrt{\beta}\sum_{n=1}^{2}\int_{0}^{\xi_n}W_{ni}(\xi)W_{nj}^{I}(\xi)d\xi \tag{10}$$

$$K_{ij}=\sum_{n=1}^{2}\int_{0}^{\xi_n}W_{ni}(\xi)W_{nj}^{IV}(\xi)d\xi+\sum_{n=1}^{2}\int_{0}^{\xi_n}W_{ni}(\xi)W_{nj}^{II}(\xi)d\xi \tag{11}$$

The general solution of the system (8) is expressed through the roots of the equation:

$$\det X=0 \tag{12}$$

The elements of the matrix X are given by:

$$X_{ij}=\lambda^2 M_{ij}+\lambda C_{ij}+K_{ij} \tag{13}$$

Based on the obtained roots could be concluded the stability of the system. The system is stable if the real part of all the roots of the characteristic equation (13) is negative.

The roots depend on all the parameters of the system. If all of them are fixed except the velocity of the conveyed fluid V, one could obtain the corresponding critical velocity.

3. CRACK MODELLING

It is considered that the bending vibrations of the Euler-Bernoulli beam is in the plane $x-y$ (Figure 1), which is also a plane of symmetry for the cross-section. The crack is assumed to be open. Castigliano's theorem is used to obtain the local flexibility in the presence of the crack [8]:

$$c=\frac{\partial^2 U}{\partial M^2}=\frac{1-\nu^2}{E}\int_{-b}^{b}\int_{0}^{a}\frac{\partial^2\left(K_I^2\right)}{\partial M^2}dx\,dy, \quad (14)$$

where E and ν are Young's module and Poisson's ratio, respectively. K_I is the stress intensity factor of bending. a and b are the crack dimensions as shown in Figure 1. M is the bending moment.

$$K_I=\frac{M}{\pi R^2 t_p}\sqrt{\pi R\theta_c}\,F(\theta_c), \quad (15)$$

where $R=0{,}5(R_{in}+R_{out})$, t_p and θ_c are the thickness of the pipe and the half central angle of the crack, respectively (Figure 1). $F(\theta_c)$ is calculated from the following formula [21]:

$$F(\theta_c)=1+A_t\left[4{,}5967\left(\frac{\theta_c}{\pi}\right)^{1{,}5}+2{,}6422\left(\frac{\theta_c}{\pi}\right)^{4{,}24}\right] \quad (16)$$

$$A_t=\sqrt[4]{\frac{1}{8}\frac{R}{t_p}-\frac{1}{4}} \text{ for } 5\le\frac{R}{t_p}\le 10 \quad (17)$$

$$A_t=\sqrt[4]{\frac{2}{5}\frac{R}{t_p}-3} \text{ for } 10\le\frac{R}{t_p}\le 20. \quad (18)$$

The equivalent rotational spring stiffness:

$$k_{rs}=\frac{1}{c}. \quad (19)$$

4. RESULTS AND DISCUSSION

Numerical studies have been carried out for the system in Figure 1.

The geometric and material characteristics of the pipe are: the inner and the outer radii of the cross-section of the pipes - $R_{in}=0{,}012\,m$ and $R_{out}=0{,}014\,m$, Young's modulus

$E = 210GPa$, the density of the material of the pipe $\rho = 7800kg/m^3$. The density of the flowing fluid is $\rho = 1000kg/m^3$. The dimensions of the crack are $a = 1mm,\ b = 5mm$.

The finite element method was used to obtain the basic functions $W_{ni}(\xi)$. The eigenfunctions for the pipe with stationary fluid ($V = 0$) are used as the basic functions in this paper. The first 10 modes were used in the calculations.

At first, the position of the crack is fixed with the coordinate $x_c / l = 0{,}33$ at the top edge of the beam. The aim is to investigate the influence of the parameters of the Pasternak elastic foundation on the stability of the system. When the parameter $k_r = 0$ and the parameter $k \neq 0$, the foundation is known as Winkler elastic foundation. When $k = 0$ and $k_r \neq 0$, the foundation is known as the rotational elastic foundation. The results obtained in this work allow to investigate the influence of the Winkler elastic foundation and rotational elastic foundation on the stability of the system.

Based on the obtained roots of the characteristic equation (12) could be drawn conclusions about the stability of the system. The system is stable if the real part of all the roots is negative. If one or more roots have positive real parts, then the system is unstable. When one or more roots of the characteristic equation have real parts equal to zero, the system is at the edge of loss of stability, the corresponding fluid velocity is the critical fluid velocity. The roots depend on all the parameters of the system. If all of them are fixed, except the velocity of the conveyed fluid V, one could obtain the corresponding critical velocity.

The obtained results for a cracked pipe are compared with the results of an undamaged pipe.

For the pipe in Figure 1 are obtained the critical velocities for different values of the parameters of the Pasternak elastic foundation. The results shown in Figure 2 are calculated for a crack fixed with a coordinate $x_c / l = 0{,}33$.

The obtained results show that the Pasternak foundation has a stabilizing effect on the pipe - by increasing the parameters of the foundation, the critical velocity increases. The crack has a destabilizing effect on the system, leading to decreasing in the critical velocity.

For the Winkler elastic foundation ($k_r = 0$), the results show the stabilizing effect of the foundation on the system. For both cracked and undamaged pipes, increasing the rigidity of the foundation leads to an increase in the critical velocity.

The same dependence between the rigidity of the rotational foundation and the critical velocity of the fluid is observed.

The second part of the survey investigated the influence of the position of the crack on the stability of the system. It considered not only the position of the crack along the length of the beam but also if the crack is on the top or bottom edge of the pipe. The results are shown in Figure 3.

When the coordinate of the crack x_c / l increases, the critical flow velocity decreases. The system is less stable when the crack is in the middle cross-section of the pipe $x_c / l = 0{,}5$.

There is a slight difference in the critical velocities when the position of the crack is on the top and bottom edge of the pipe. For all investigated coordinates of the crack x_c / l , the position of the crack at the top has a destabilizing effect on the system.

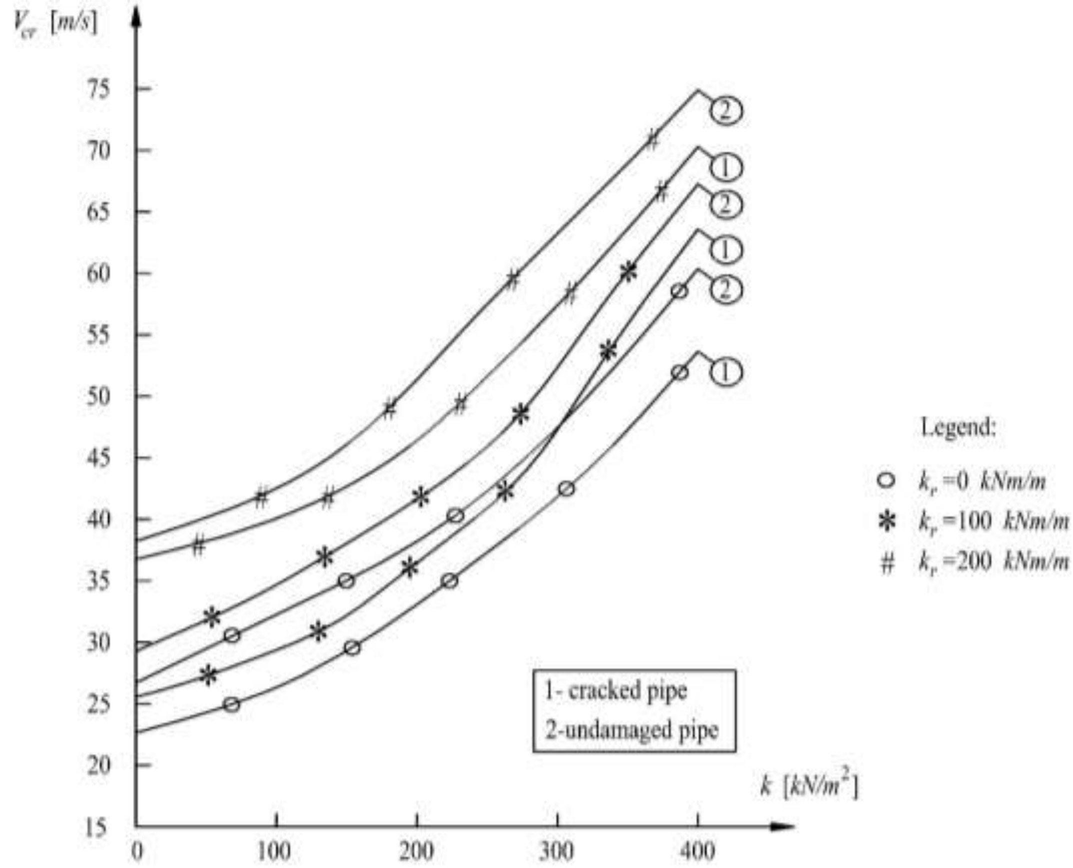

Fig. 2. Critical velocity versus the rigidity of the Pasternak elastic foundation

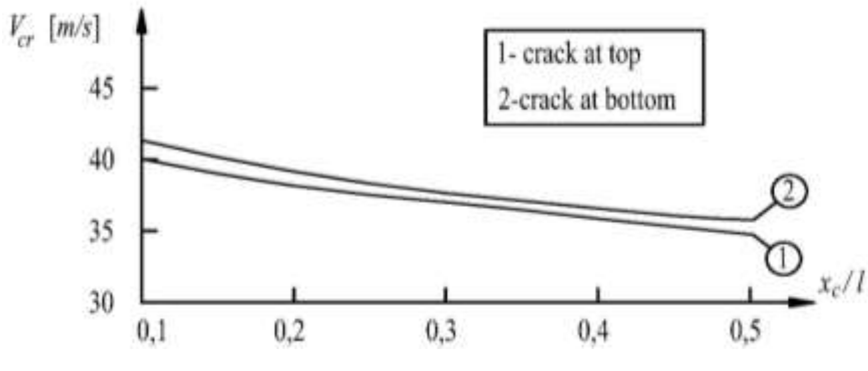

Fig. 3. Critical velocity versus the position of the crack ($k = 200\,kN/m^2$; $k_r = 100\,kNm/m$)

5. CONCLUSION

The classical Winkler foundation is often used as a model in geotechnical analyses. It states that the deflection at any point at the surface of an elastic medium is proportional to the load applied at the point and does not depend on the applied loads at other points of the surface, and that is its major shortcoming. To overcome this, the Pasternak model is introduced. It is an improved two-parameter model of the elastic medium.

Cracks are the most encountered damage in the structures. When a structure is cracked, its stiffness is reduced, with a consequent reduction in the natural frequencies and a change in the eigenforms.

This paper investigated the influence of the parameters of the Pasternak elastic foundation on the stability of a cracked pipe conveying fluid.

The pipe is modelled as two segments connected by a rotational elastic spring at the cracked cross-section. Castigliano's theorem is employed to calculate the stiffness of the spring. The spring stiffness depends on the geometry of the cross-section of the pipe and the severity of the crack.

The results obtained in this study could be summarized as follows:

1. The Pasternak foundation has a stabilizing effect on the system. This means that the fluid could flow through the pipe at a higher velocity without causing a loss of stability in the system.
2. When the parameter of the Pasternak elastic foundation $k_r = 0 kNm/m$, the foundation is known as a Winkler elastic foundation. The obtained results show that the Winkler elastic foundation also has a stabilizing effect on the system.
3. In the case when the pipe rests on the rotational elastic foundation (Pasternak elastic foundation with $k = 0$) – the increasing of the rigidity of the foundation leads to an increase in the critical velocity.
4. A pipe resting on Pasternak elastic foundation has higher critical velocity compared with the same pipe but resting on Winkler elastic foundation when the coefficient k of both foundations is equal.
5. The position of the crack affects the stability of the system. If the crack severity remains unchanged, the critical velocity of the fluid is higher when the crack is located at the bottom edge of the pipe than when the crack is at the upper edge of the pipe. Also, the position of the crack along the length of the pipe affects the stability of the system. The closer the crack is to the middle of the span, the more unstable the system becomes.

It is worth mentioning that the damping of the Pasternak elastic foundation also affects the stability of the system; however, this effect was not considered in this paper.

The results obtained contribute to the safety of pipes conveying fluid. To avoid damages, the operator of the pipe should not allow higher transportation velocities than the critical velocity of the system. As the critical velocity depends on many parameters of the system, among which is the severity and position of the crack, the operator of the pipe should perform strict crack detection tests, then based on the results, correct the velocity of the fluid to the damaged system, not to lose stability.

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