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DEVELOPMENT OF A LINEAR REGRESSION MODEL BASED ON VEGETATION INDICES OF AGRICULTURAL CROPS

Abstract: The article is devoted to the study of vegetation indices for assessing the productivity of agricultural crops of the North Kazakhstan Agricultural Experimental Station (NKAES) LLP. The research was carried out using a modern software package for processing satellite images, EOS Land Viewer. The work used images from the Landsat 8 (USA) and Sentinel 2 (European Space Agency) spacecraft. Digitized Earth remote sensing data for the last 3 years are presented, showing changes in the amount of moisture reserves on the territory of NKAES LLP. Time series of distribution of the studied coefficients were constructed according to different phases of active vegetation biomass in the study area. The resulting time series made it possible to identify annually repeating patterns, a linear trend of increasing and decreasing NDWI and NDVI on the territory of the NKAES LLP.

Review of studies over the past 5 years, published in highly rated foreign journals, on various vegetation indices, including indices designed to assess moisture content in vegetation and soil. It is noted that the first normalized water index, NDWI, using the SWIR infrared channel, unlike the widely used NDVI vegetation index, actually penetrates 80% of the atmosphere.

Analysis of the obtained NDWI allowed us to identify dry, moderately dry and fairly humid periods on the territory of the NKAES LPP from 2020 to 2023. Based on the research carried out, the feasibility of using normalized difference water indicators and normalized vegetation indices for further use in forecasting yields in the conditions of the North Kazakhstan region is substantiated. Next, using vegetation indices and additional agrometeorological factors, a linear model for predicting crop yields was developed. The coefficient of determination of the resulting model is 0.90 which indicates that the selected trend line reliably approximates the process under study.

Keywords: Normalized difference water index; NWVI coefficient; soil moisture; remote sensing data; vegetation indices; time series analysis.

Introduction

Crop condition assessment using remotely sensed satellite data is a scientifically advanced field that is enabling qualitatively new results in precision agriculture in a number of Asian countries, including Kazakhstan. Satellite data are currently used in agriculture, geology and ecology. Satellite information is used to study various agro-climatic measures, as well as in analysing vegetation quality and forecasting crop yields [1]. The remote sensing data obtained from satellite images, as a result of operations with different spectral bands, are called vegetation indices. There are about 160 variants of spectral vegetation indices to date that are selected on the basis of known features of spectral reflectance curves of vegetation and soils. The efficiency of vegetation indices is determined by reflectance features. Vegetation indices are usually calculated in the most stable parts of the spectral reflectance curve of plants by combining the reflectance intensities in two or three different bands [2]. The authors of [3] distinguished several types of vegetation indices: greenness indices calculated from data in wide spectral channels; greenness indices calculated from data in narrow spectral channels; light utilization efficiency indices; indices of nitrogen content of vegetation; indices of carbon content in the form of lignin and cellulose; indices of pigment content of leaves; indices for estimating vegetation moisture content.

Literature review

In many studies, vegetation indices calculated from infrared and red spectrum bands are used to estimate the yield of different crops [4]. The authors of the studies showed that a single index cannot provide a reliable estimation in all cases, the efficiency of the index depends on factors such as climate, soil, crop type, growing season or limitations of the device used. Therefore, researchers have proposed more than 150 types of vegetation indices [5]. Vegetation condition is represented by various indices: historical data from local hydrological monitoring data with higher resolution and distributed controller data with a high degree of separation [6].

In a study [7], water indices namely water range index (WBI), normalised difference water index (NDWI), normalised difference infrared index (NDII) and land surface water index (LSWI) were examined in detail. According to the results of this study, WBI and NDWI indices helped to recognise saturated soil and also NDII and LSWI indices showed better dynamic range of recognition due to stronger absorption of infrared rays to display different degrees of soil moisture. Other studies by these authors have shown that hydrological regimes such as flood frequency and inundation area determine habitat quality affect vegetation cover and biomass [8].

Further, the authors of [9], in order to improve the knowledge of autumn phenology at the landscape scale, proposed to investigate the autumn state dynamics based on several vegetation indices from remotely sensed data. Using time series of the MODIS Normalised Difference Water Index (NDWI), the onset of drying and the dormancy period of semi-arid grasslands were determined.

Compared to NDVI dynamics, NDA II variations may partly reflect a more rapid decline in photosynthetic activity than any other form of physiological activity during initial drought disturbance [10]. The study [11] presents evaluations of the use of red edge parameters to measure spectral characteristics and growth condition of winter wheat under water stress conditions. The study was conducted using correlation data between soil moisture, spectral indices and optimised soil moisture equation models.

In [12], the authors considered the interchangeability of a common and a new remote sensing parameter to assess biochemical, functional and structural responses of plants to the development of soil water limitation. In [13], in addition to modelling the soil moisture field and associated indicators (indices) on the basis of satellite data, the task of assessing the contribution of these indicators to the formation of soil moisture using statistical methods was solved.

In general, most studies are conducted to investigate the relationships between SM and soil spectral indices using different models, as this direction in the field of remote sensing, still attracts the attention of researchers. This work considers the NDWI vegetation index, which is designed to determine the quality and boundaries of water bodies, to calculate the soil moisture content and the degree of moisture content of vegetation cover, and to estimate the moisture content of vegetation cover.

Purpose and Objectives of Research

The purpose of this paper is to analyse the amount of moisture in the North-Kazakhstan region for 2020-2023. In accordance with the objective, the following tasks were set:

- to create a database of satellite images for the studied period in EOS Landviewer system;
- to calculate vegetation water difference indices;
- on the basis of obtained indices to analyse the degree of wetting of vegetation cover of North-Kazakhstan region.

- create a yield forecasting model based on the linear regression method.

Multispectral images of Sentinel 2 and Landsat 8 satellite for LLP NKAES for 2020-2023 were selected as the object of the study.

Materials and methods

Normalized Difference Water Index (NDWI). Normalized Difference Water Index (NDWI) is used for the water bodies analysis. The index uses Green and Near infra-red bands of remote sensing images. The NDWI can enhance water information efficiently in most cases. It is sensitive to build-up land and result in over-estimated water bodies. The NDWI products can be used in conjunction with NDVI change products to assess context of apparent change areas [14]. Water bodies having low reflectance. It only reflects within visible portion of the electromagnetic spectrum. Water bodies in their liquid state are generally high reflectance on Blue (0.4–0.5 μm) spectrum than Green (0.5–0.6 μm) and Red (0.6–0.7 μm) spectrum. Clear water having greatest reflectance in the blue portion of the visible spectrum. So, water appear blue. Turbid water has higher reflectance in visible spectrum. There is no reflection in Near Infrared (NIR) and beyond. NDWI is developed by Gao (1996) to enhance the water related features of the landscapes. This index uses the near infrared (NIR) and the Short-Wave infrared (SWIR) bands. NDWI can be calculated by following formula:

$$NDWI = \frac{Green - Nir}{Green + Nir} \quad (1)$$

where *Green* is a green band such as TM band 2 and *NIR* is a near infrared band such as TM band 4. This index is designed to

- (1) maximize reflectance of water by using green wavelengths;
- (2) minimize the low reflectance of NIR by water features;
- (3) take advantage of the high reflectance of NIR by vegetation and soil features [15, 16].

As a result, water features have positive values and thus are enhanced, while vegetation and soil usually have zero or negative values and therefore are suppressed. Sentinel-2 NDWI for agricultural monitoring of drought and irrigation management can be constructed using either combinations:

- band 8A (864nm) and band 11 (1610nm)
- band 8A (864nm) and band 12 (2200nm).

The interpretation of NDWI coefficient values are shown at the next table 1:

Table 1. Interpretation of Normal Water Vegetation Indexes (NDWI)

NDWI coefficient values		
From	To	Interpretation
0,2	1	water surface
0,0	0,2	humidity or flooding
-0,3	-0,0	moderate drought and non-water surfaces
-1	-0,3	drought and non-water surfaces

Experimental analysis and discussion

For experimental analysis of our study, high resolution Sentinel 2 and Landsat 8 images for the last three years of NKAES LLP were downloaded (Fig.1).

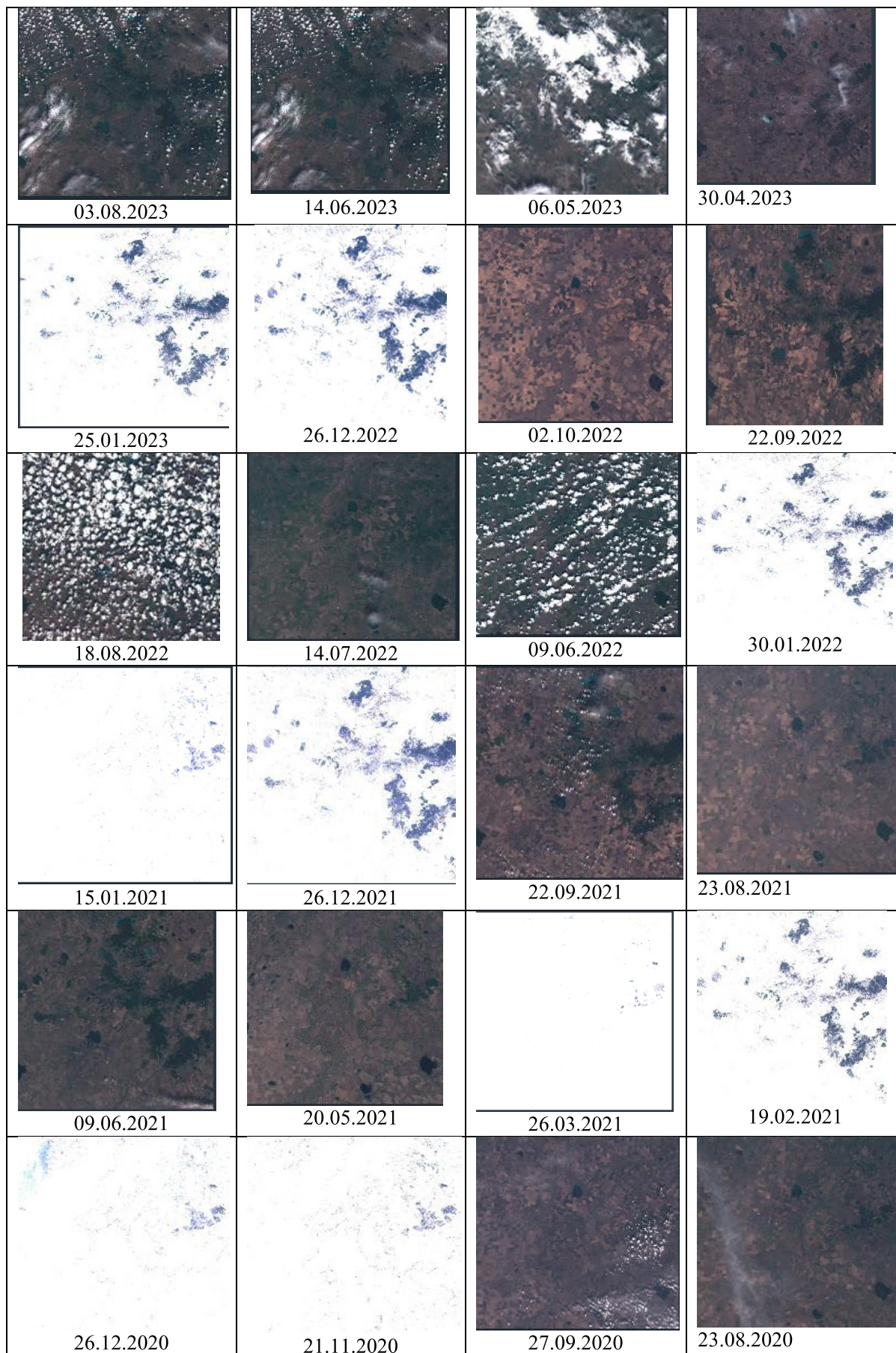


Figure 1. Satellite images in EOS Land Viewer for the last 3 years

The images obtained were digitised into NDWI coefficients and descriptive statistics were then obtained (Table 2).

Table 2. Digitised values of normalised water indices and their descriptive statistics

scene_id	date	min	max	std	variance	median
S2A_tile_20200823	23.08.2020	-0.516	-0.105	0.072	0.0052	-0.303
S2B_tile_20200927	27.09.2020	-0.193	0.055	0.05	0.0025	-0.061
S2A_tile_20201121	21.11.2020	-0.171	0.055	0.024	0.0006	-0.080
S2B_tile_20201206	06.12.2020	-0.139	0.097	0.021	0.0004	-0.067
S2B_tile_20201226	26.12.2020	-0.143	0.119	0.019	0.0004	-0.064
S2A_tile_20210219	19.02.2021	-0.067	0.008	0.005	0	-0.033
S2B_tile_20210326	26.03.2021	-0.049	0.015	0.004	0	-0.004
S2A_tile_20210520	20.05.2021	-0.523	-0.120	0.076	0.0058	-0.359
S2A_tile_20210609	09.06.2021	-0.539	-0.162	0.064	0.0041	-0.329
S2B_tile_20210823	23.08.2021	-0.371	-0.107	0.043	0.0019	-0.186
S2B_tile_20210922	22.09.2021	-0.349	-0.133	0.037	0.0014	-0.220
S2A_tile_20211226	26.12.2021	-0.113	0.054	0.011	0.0001	-0.053
S2A_tile_20220115	15.01.2022	-0.053	-0.032	0.003	0	-0.043
S2B_tile_20220130	30.01.2022	-0.087	0.024	0.007	0.0001	-0.038
S2B_tile_20220609	09.06.2022	-0.439	-0.141	0.057	0.0033	-0.281
S2A_tile_20220714	14.07.2022	-0.519	-0.164	0.060	0.0036	-0.267
S2B_tile_20220818	18.08.2022	-0.472	-0.150	0.056	0.0032	-0.275
S2A_tile_20220922	22.09.2022	-0.342	-0.114	0.039	0.0016	-0.217
S2A_tile_20221002	02.10.2022	-0.330	-0.131	0.032	0.001	-0.240
S2A_tile_20221121	21.11.2022	-0.100	0.056	0.010	0.0001	-0.045
S2B_tile_20221226	26.12.2022	-0.130	0.103	0.012	0.0002	-0.062
S2A_tile_20230130	30.01.2023	-0.126	0.048	0.009	0.0001	-0.045
S2A_tile_20230430	30.04.2023	-0.355	-0.018	0.044	0.002	-0.232
S2B_tile_20230803	03.08.2023	-0.531	-0.145	0.057	0.0033	-0.265

Figure 2 illustrates the time series of water vegetation indices from NKAES LLP.

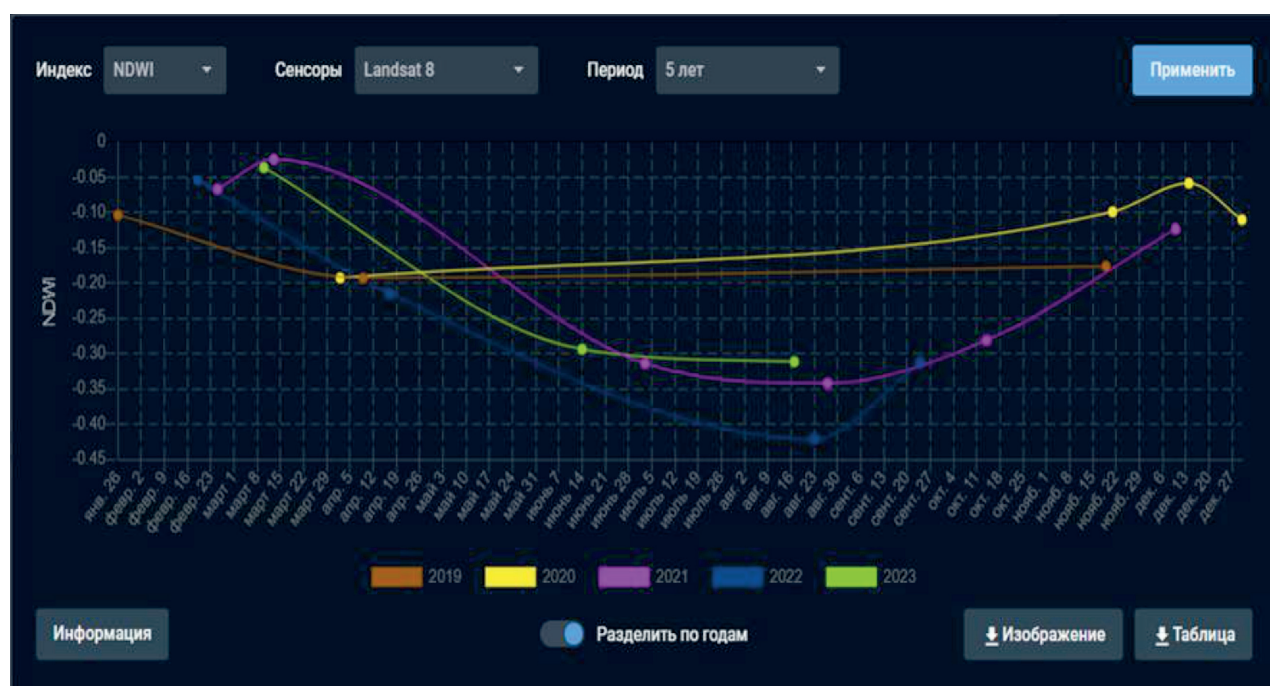


Figure 2. Time series of water vegetation indices

Based on a temporary analysis of water indices, a table has been compiled that can be used to analyze the soil conditions of the North Kazakhstan region by vegetation phases.

Table 2. Values of NDWI coefficients for 2021-2023 in the North Kazakhstan region

Years	January, February	March, April, May	June, July, August
2021	$-0,06 > C_{NDWI} > -0,78$	$-0,78 > C_{NDWI} > -0,33$	$-0,33 > C_{NDWI} > -0,04$
2022	$-0,58 > C_{NDWI} > -0,28$	$-0,28 > C_{NDWI} > 0,12$	$0,12 > C_{NDWI} > -0,28$
2023	$-0,04 > C_{NDWI} > -0,06$	$-0,09 > C_{NDWI} > -0,07$	$-0,07 > C_{NDWI} > -0,05$

Let's analyze the values of these coefficients using Table 2. In the winter months of 2021 and 2023, the soil of the studied area was arid, in 2022 the soil was moderately arid. In the spring months of 2021, the soil was arid, in 2022 the coefficient shows moisture and flooding, in 2023 in the spring the soil was moderately dry. In the summer months of 2021, the coefficient shows moderate and average aridity, in 2022 the soil was well moistened, which indicates the presence of abundant precipitation and good soil moisture supply before going into winter, in 2023 the soil turned out to be arid. Figure 3 illustrates time series of normalized difference vegetation indices in the North Kazakhstan region for the last years.

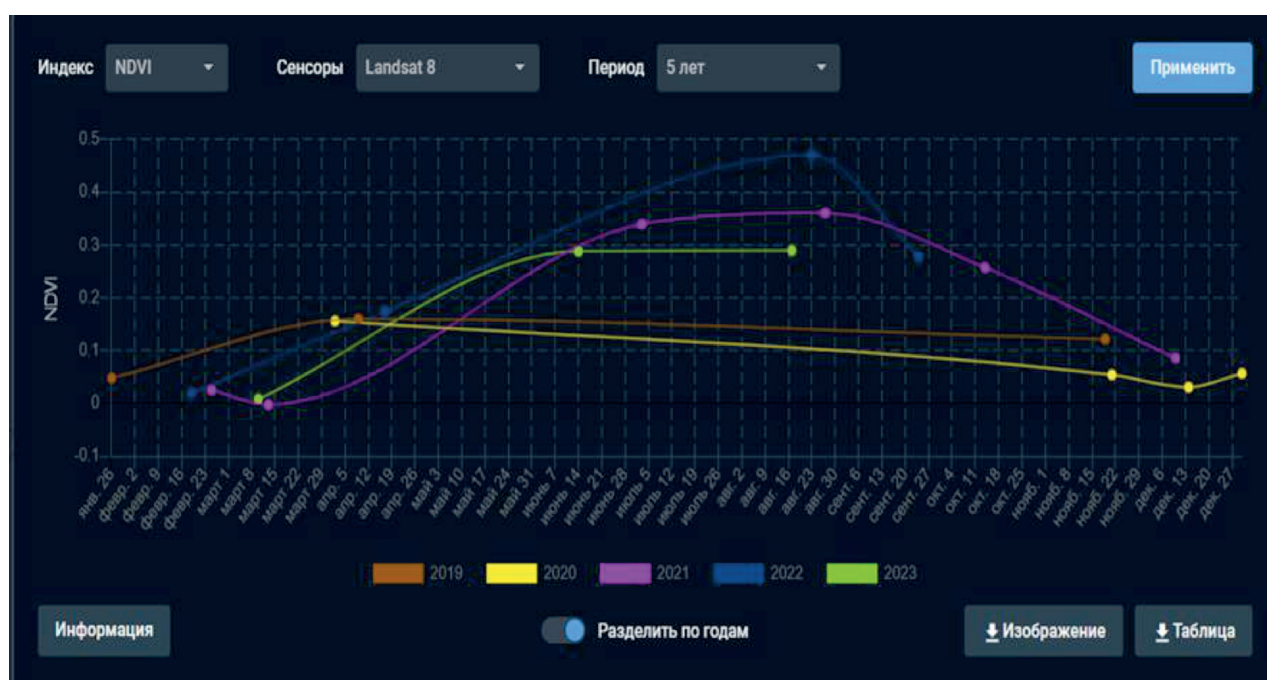


Figure 3. Time series of Normalized Difference Vegetation Indices coefficients

A table 3 shows the time series of Normalized Difference Vegetation Indices of agricultural crops for the last 5 years by NKAES LLP.

Based on data on vegetation coefficients and agrometological data, a basis was collected for constructing a crop forecasting model.

Table 3. Initial information for building a crop yield forecasting model NKAES LLP

Years	T_{vp} , °C	H_{vp} , %	S_{sm} , mm	$NDVI_{max}$	$NDWI_{max}$
2017-2018	18,20	49,00	218,00	0.2365	0.2575
2018-2019	18,10	66,00	43,00	0.6338	0.5555
2019-2020	19,10	58,00	160,00	0.8405	0.6608
2020-2021	19,50	90,00	68,00	0.8006	0.539
2021-2022	19,30	66,00	68,00	0.8191	0.5972

T_{vp} - average temperature during the growing season of the North Kazakhstan region, H_{vp} - atmospheric humidity during the growing season of the North Kazakhstan region, S_{sm} – soil moisture growing season North Kazakhstan region, $NDVI_{max}$ – maximum normalized vegetation index, $NDWI_{max}$ – maximum water difference index. To make a model, a multiple linear regression method was used in Python program. (Figure 4).

```

import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error, r2_score, mean_absolute_error

ndvi_data = pd.read_csv("NDVI.csv")[["max"]]
ndwi_data = pd.read_csv("NDWI.csv")[["max"]]
crop_yield_data = pd.read_csv("Crop_yields.csv")[['Actual grain yield, c/ha']]
additional_data = pd.read_csv("Additional_data.csv")[['Tvp, °C', 'Hvp, %', 'Ssm, mm']]

merged_data = pd.concat([ndvi_data, ndwi_data, additional_data, crop_yield_data], axis=1)
merged_data.columns = ['NDVI_max', 'NDWI_max', 'Tvp, °C', 'Hvp, %', 'Ssm, mm', 'Crop_yield']

X = merged_data[['NDVI_max', 'NDWI_max', 'Tvp, °C', 'Hvp, %', 'Ssm, mm']]
y = merged_data['Crop_yield']
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, random_state=42)

model = LinearRegression()
model.fit(X_train, y_train)

y_pred = model.predict(X_test)

mae = mean_absolute_error(y_test, y_pred)
mape = np.mean(np.abs((y_test - y_pred) / y_test)) * 100
mse = mean_squared_error(y_test, y_pred)
rmse = np.sqrt(mse)
r2 = r2_score(y_test, y_pred)

print("Mean Absolute Error:", mae)
print("Mean Absolute Percentage Error (MAPE):", mape)
print("Mean Squared Error:", mse)
print("Root Mean Squared Error:", rmse)
print("R-squared (R2) Score:", r2)

# Visualize the Linear Regression model
plt.scatter(X_test['NDVI_max'], y_test, color='blue', label='Actual')
plt.scatter(X_test['NDVI_max'], y_pred, color='red', label='Predicted')
plt.xlabel('NDVI_max')
plt.ylabel('Crop_yield')
plt.legend()
plt.title('Linear Regression Model')
plt.show()

plt.scatter(X_test['NDWI_max'], y_test, color='blue', label='Actual')
plt.scatter(X_test['NDWI_max'], y_pred, color='red', label='Predicted')
plt.xlabel('NDWI_max')
plt.ylabel('Crop_yield')
plt.legend()
plt.title('Linear Regression Model')
plt.show()
print(model.intercept_)

#print(y_pred)

Mean Absolute Error: 0.30512332535469433
Mean Absolute Percentage Error (MAPE): 1.570295142859697
Mean Squared Error: 0.10873036817668763
Root Mean Squared Error: 0.3297428819196673
R-squared (R2) Score: 0.9013783508601474
13.403948921241415

```

Figure 4. Code fragment using the linear regression method to predict crop yields North-Kazakhstan Agricultural Experimental Station

As a result, a model was obtained to predict agricultural yields over the last 9 years.

$$C_{North\ kaz} = 1.9681NDVI_{max} - 2.1488NDWI_{max} - 3.0405T_{vp} + 7.0877H_{vp} + 9.1758S_{sm} + 13.403 \quad (2)$$

The coefficient of determination of this model is 0.90, which indicates that the selected trend line reliably approximates the process under study. The obtained values of the root mean square error $MSE=0.108 < E$ and the root of the root mean square error $RMSE=0.329 < E$, which demonstrates a small difference between the predicted and observed values in the model.

Conclusion

The main advantage of vegetation indices is their reliability and ease of obtaining, as well as a wide range of tasks that can be solved with their help. NDWI vegetation difference index is the main tool for analyzing moisture availability in agricultural plots.

According to the objectives of this study, the following work has been done:

1. A database of Landsat 8 and Sentinel 2 high spatial resolution space images was formed in EOS Land Viewer geo information system of the North-Kazakhstan Agricultural Experimental Station. In total 24 images were uploaded for the period 23.08.2020- 03.08.2023.

2. The time series of normalized difference indices for the indicated period is illustrated and analyzed.

3. Vegetation water difference indices and their descriptive statistics standard deviation, median, maximum and minimum values for each month were calculated on the basis of received images. According to the obtained values it can be seen that the highest normalised water difference index on the territory of the North Kazakhstan experimental station is equal to 0.119, obtained in December 2020 and the minimum difference index for all considered decades is equal to -0.539, obtained in 09.06.2021. The paper also analysed and identified the dry and moderately dry periods in the last 3 years.

4. Using a machine learning algorithm, a linear regression model was created to predict yield. The statistical measure of the model – the coefficient of determination is equal to $R^2=0.90$, which shows the adequacy of this model. This model can be used in predicting yields, having previously checked them for adequacy.

In the future, to increase the accuracy of forecasting, the quality of the model can be improved by adding additional parameters, such as biological and man-made factors.

The above analysis will be useful for doctoral students and students conducting research in the field of remote sensing, as well as in planning the fertility of agricultural crops on the territory of the North-Kazakhstan experimental station.

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