A TOOL OF TECHNOLOGY-BASED LABORATORY ENABLED STUDENTS TO PRECISELY DESCRIBE SCIENTIFIC PHENOMENA

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Abstract. *Proposing scientific descriptions is critical for individuals to cope with daily problems and acquire essential information. Nonetheless, few classes have enhanced students' ability to describe facts of scientific phenomena. Thus, using a tool of technology-based laboratory, this research examined whether students' scientific descriptions and mathematical modelling behaviours could be improved. The participants included 52 undergraduate students randomly assigned to the experimental and control group. Two prompts were developed to remind the experimental group that it is common to place 'time' along the x-axis and that mathematical modelling is important in physics. Results showed that as expected, all participants generated more propositions in scientific descriptions, especially the experimental group. However, contrary to the hypothesis, the participants did not propose more correct propositions and the effect of group was limited. Moreover, the hypotheses were partially supported that the participants used more image-based and mathematics-based representations to describe phenomena, and the proportion of participants whose propositional type was quantitatively increased, though no main effects of group were observed. Most participants adjusted their mathematical models by keeping slightly changing the coefficients/constants to fit the data, rather than applying relevant physics knowledge to revise models, illustrating their difficulties in connecting mathematical representations with actual phenomena.*
Keywords: *mathematical modelling, model-building behaviours, modelling activities, physics teaching/learning strategies, technology-based laboratory*

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Introduction

In daily life, scientific descriptions are central to individuals coping with everyday problems and receiving the requisite information, such as illustrating the past week's health and fitness data on your mobile phone for your doctors or sharing your cycling route over time in the Global Positioning System with friends. Darwin (2018) proposed that grouping facts constitutes science, revealing that describing facts of scientific phenomena, that is, generating scientific descriptions, should be an ability of great value. In the classroom or laboratory, describing scientific phenomena often involves making observations, collecting data, analysing the obtained data, and drawing conclusions about the results, and thus serves as a solid foundation for further scientific investigation.

Since many scientific phenomena cannot be directly introduced to the classroom, it is critical to help students describe and represent the phenomena by constructing their models, which allows the abstract phenomena to be visualised, simplified, manipulated, scale-transformed, and mathematized (National Research Council, 2012; Zwickl et al., 2015). The process of modelling involves essential skills in scientific practices, including model constructing, data collecting, phenomenon observing, and model improvement (National Research Council, 2012; Nicolaou & Constantinou, 2014). Starting from constructing models, individuals should be able to develop external representations of a physical phenomenon (Namdar & Shen, 2015) after directly observing the phenomenon or indirectly using secondary sources. However, it has long been challenging for students to coordinate various representations and connect physics representations with real-world phenomena (Ibrahim & Rebello, 2013; Kozma & Russell, 2005; Stull et al., 2016). To actively scaffold and support students' learning of physical phenomena, technology-based laboratories hold

great promise. Technology-based laboratories have been developed to help students gain a better understanding of scientific phenomena by simultaneously presenting the obtained data in terms of selected variables and displaying diagrams depicting the connections between/among the variables (Chien et al., 2015; Hsu & Wu, 2016). In essence, these technology-based laboratories provide multiple representations among which students can make connections and comparisons and construct their mental models.

In the previous research, a technology-based laboratory, InduLab, was developed to assist students in mathematical modelling of physics experiments (e.g., pendulum motion), and its benefits for laboratory physics in high school were confirmed (Liu et al. 2017). Taking a step further, could individuals describe and represent a real-world phenomenon differently with the help of InduLab? The pilot study found that the effect of InduLab on undergraduate students' scientific descriptions was less profound. More details of the pilot study are reported in the *Research Focus* section.

Hence, the current research provided two prompts as treatment and then examined the effect of InduLab on undergraduate students' construction of scientific descriptions, that is, describing facts of scientific phenomena, as well as their mathematical modelling behaviours. The first research question centres on learners' scientific descriptions: Do the use of InduLab and receipt of the prompts affect undergraduate students' scientific descriptions? The second research question concerns learners' modelling behaviours: Which modelling behaviours are performed by undergraduate students with and without the prompts when engaging in InduLab? Before continuing, a clarification about terminology and limitations should be provided. It is easy but utterly incorrect to equate modelling activities with simply finding appropriate formulas. In this research, there was something about fitting of a function to empirical data, which could be called also as modelling, but there is more: The rich process of modelling with mathematical functions in an authentic context was cautiously examined. This kind of model-building activity is merely a small piece of the big picture since various kinds of complex models (e.g., stochastic models; Erickson, 2006) and modelling approaches exist (Sevinc & Lesh, 2018). However, it would be difficult to handle all of the issues regarding models in a single research study. In Haverty et al. (2000)'s research, they specifically asked the participants to perform modelling with mathematical functions, that is, discover a mathematical function that represented the connections between two variables, to reveal the cognitive processes in mathematics. Namdar and Shen (2015)'s synthesized literature review of modelling in the last three decades also found that most research emphasised only a part of the modelling process (i.e., model interpretation) since it could be time-consuming and challenging to consider the whole process. Thus, this research limited itself to examine the performances and behaviours of modelling with elementary mathematical functions in a technology-based context. It is noteworthy that 'model' and 'mathematical model' were used in this limited sense.

Literature Review

Scientific Descriptions and Models

In this research, 'scientific descriptions' refer to facts of scientific phenomena. They are constructed from collected data that illustrate scientific phenomena based on an individual's mental model and prior knowledge. Mental models are analogical representations of external reality within an individual's mind (Johnson-Laird, 1983). Mental model plays a critical role in generating scientific descriptions since individuals must understand and explain the phenomena by referencing to the model (Johnson-Laird, 1983). Mental models are the syntheses of various representations (i.e., propositions, mental images, and mental models). Ainsworth (2006) suggested that learners' mental models for scientific concepts would be more complete and coherent with multiple representations.

In addition to mental models, other modelling perspectives also exist. Though these perspectives might differ in whether they involve mental representations, situation models, or real models, most of them include the goal of constructing a mathematical model to explain real physical phenomenon (Sevinc & Lesh, 2018). For instance, Hestenes (1987, 441) particularly emphasised the mathematical formulation in physics: 'models are physical properties represented by quantitative variables.' Namdar and Shen (2015) also concluded that mathematical modelling of the physical world must be prioritized to obtain scientific knowledge successfully. Mathematical modelling has long been emphasised in science education (Greca & Moreira, 2002), because it reflects the nature of a physical system, allowing the quantitative estimation of the behaviour of the system and because it is beneficial for learning physics concepts and processes. Moreover, mathematical modelling activities have many plausible answers, making it unnecessary for students to seek only one correct answer. For example, Angell et al. (2008) proposed a mathematical modelling approach in a Norwegian secondary physics classroom, finding that students became more aware of the modelling nature of physics. In addition, by engaging in modelling activities, learners also gain opportunities to utilise representations to develop a deeper understanding of natural phenomena (Baumfalk et al., 2018).

Nonetheless, mastering different representations has always been a challenge for learners (Ibrahim & Rebello, 2013; Kohnle & Passante, 2017; Kozma, 2003; Kozma & Russell, 2005; Stull et al., 2016). Ibrahim and Rebello (2013) explored how engineering students utilised mental representations for open-ended tasks on kinematics and work. They found that most students used propositions as their main mental representations in both tasks. Moreover, most students tended to use a single representation, rather than meaningfully integrating multiple representations. In the studies of Kozma (2003) and Kozma and Russell (2005), they indicated that chemistry students were unable to transform one representation into another compared to expert chemists. In addition, the novice students generated descriptions based merely on the superficial features of one representation, without precise interpretation. To assist learners in successfully developing scientific descriptions, Kozma suggests that instructional technologies can be a potential route to present multiple representations for further coordination and integration.

Technology-Based Laboratory

Researchers have suggested that technology-based laboratories have significant advantages (Bernhard, 2018; Chien et al., 2015; Hochberg et al., 2020; Hsu & Wu, 2016; Wang et al., 2017). First, the experimental data can be shown in multiple representations, enabling students to make sense of the data by transforming and communicating them between different representations. Second, the selected variables can be visualised almost instantaneously, which provides students with instant feedback to construct and revise models. Based on the two advantages, students can devote greater effort to reading and interpreting the representations automatically generated by computers.

Although technology-based laboratories support individuals' learning, assistances such as prompts or instructions to scaffold students' interactions with educational technologies are needed (Kohnle & Passante, 2017; Wang et al., 2017). Kohnle and Passante (2017) examined the effects of interactive computer simulations and tutorials on learners' representational learning with the material as perturbation theory. Results demonstrated that after the combined simulation tutorial, learners proposed more representations, had higher consistency between representations, and had progressed toward higher representational competence levels, showing that learners should be supported to create and make sense of multiple representations while engaging in technology-based laboratories.

However, what kind of assistances should be provided to facilitate science learning in technology-based learning environments? Several principles for appropriate assistances have been proposed (Hsu & Wu, 2016; Kozma, 2003; Wang et al., 2017). First, providing representations to students could make abstract scientific entities and processes explicit. For instance, students could be reminded to select the appropriate variables for the axes when graphing a function in a technology-based laboratory and make sense of the graphs corresponding to the underlying scientific phenomena. Second, to help students make connections across multiple representations. For instance, students could be asked to propose models in a technology-based laboratory and identify how its changes affect the graphs of functions. This principle also corresponded to Moyer-Packenham and Bolyard's (2016) statement that inspecting different simultaneously changing representations allows individuals to make connections and comparisons between representations. Using these principles, Wang et al. (2017) explored the effects of three simulation groups (using two mobile applications and a computer simulation) on students' conceptual understanding and found that all groups made significant progress in understanding physics.

Research Focus

Though the scientific description is essential for every citizen, it is never an easy task to represent real phenomenon by constructing models, especially as learners often encounter challenges in connecting different representations (Ibrahim & Rebello, 2013; Kozma, 2003; Stull et al., 2016). To overcome these challenges, technology-based laboratories offer many advantages (Bernhard, 2018; Hochberg et al., 2020; Wang et al., 2017). With appropriate assistances (e.g., prompts), technology-based laboratories could be scaffoldings for coordinating multiple representations, and thus allowing individuals to propose scientific descriptions differently.

Though the benefits of InduLab for scientific modelling were confirmed in the previous research (Liu et al., 2017), its effect was less pronounced for scientific description in the pilot study. The pilot study was conducted with six undergraduate students. All participants had to complete the experiment using identical materials as in this research, and follow-up interviews were conducted. The results showed that the participants intended to arbitrarily select any two variables as the x- and y-axes when modelling the motion of an object, even though it is conventional in physics to place 'time' along the x-axis (Araujo et al., 2008; Reisslein et al., 2005). Moreover, most students did not propose any mathematical equations to fit the obtained data. Thus, based on the design principles (Hsu & Wu, 2016; Kozma, 2003;

497

Wang et al., 2017), this research developed two prompts as the experimental intervention to remind participants that the time variable should be selected for the *x*-axis and informed them that mathematical modelling is an efficient way to understand scientific phenomena.

For the first research question regarding the participants' performance in scientific descriptions, four hypotheses were developed. The current research predicted that learners, especially those who receive the prompts, could propose scientific descriptions differently. First, according to the phenomenon that was frequently founded in creativity, research people who provide more responses are likely to have responses with greater quality (Silvia et al., 2013), this research suggests that the experimental group that receives the prompts would generate more propositions (Hypothesis 1) and with greater accuracy (Hypothesis 2). Moreover, with the prompts, this research believes that the experimental group would be more able to act as experts that master different representations, as indicated in the studies of Kozma (2003) and Kozma and Russell (2005). Thus, they would adopt more types of representations (i.e., image type; Hypothesis 3) and quantitative descriptions (i.e., quantitative propositional type; Hypothesis 4) in their scientific descriptions after engaging in InduLab.

Hypothesis 1:   After using InduLab, all learners would generate more propositions, especially in the experimental group.

Hypothesis 2:   After using InduLab, learners would get more accurate propositions, especially in the experimental group.

Hypothesis 3:   After using InduLab, more learners would adopt image type as representational type, especially in the experimental group.

Hypothesis 4:   After using InduLab, more learners' propositional type would be quantitative, especially in the experimental group.

For the second research question concerning the participants' spontaneous mathematical-modelling behaviours, one exploratory research question was developed:

Exploratory Research Question: How would the mathematical-modelling behaviours be performed by participants with and without the prompts while engaging in InduLab?

Research Methodology

General Background

The current research was carried out with quantitative research method, which adopted inquiry strategies (e.g., experiments and surveys) to collect data for analysis (Creswell, 2003). Adopting the raw data of a real-world phenomenon, undergraduate students were randomly assigned to experimental and control groups and their scientific descriptions and spontaneous mathematical modelling behaviour were probed before and after using InduLab. The experiment was conducted at a university in northern Taiwan between December 2018 and January 2019.

Participants

A total of 56 undergraduate students with an average age of 21.25 years ($SD = 3.16$) participated in the research. They signed up either through an independent website or via the experimenters' e-mails and received USD\$5 after completing the experiment. Of the participants, 84% were female; and about 11% had taken courses in both calculus and statistics, whereas 36% had taken either of them, indicating at least half of the participants had a basic understanding of using Microsoft Excel and SPSS to solve problems concerning statistics. They were non-science majors (e.g., educational psychology, fine arts, Chinese literature) but had learned linear functions and their graph plots in 7th grade, quadratic functions and their graph plots in 9th grade, and kinematics and mechanics in 10th and 11th grade. They were randomly assigned to the experimental ($n = 28$) and control ($n = 28$) groups.

Ethics

This research received ethics approval from the Research Ethics Committee, National Taiwan Normal University. Before the experiment, the researcher explained the aim of this research to the participants and also clearly stated that the data collection was anonymous and would not be accessed by others.

Instruments

Prior Knowledge Test

This research adapted the Chinese version of the Test of Understanding Graphs in Kinematics (CTUG-K), translated from Beicher (1994), to evaluate participants' prior knowledge of kinematics. The original test comprised seven dimensions with 21 items. Seventeen items with greater difficulty were selected and tested by 94 undergraduate students. After items with rather lower difficulty were removed, 10 items remained in the CTUG-K. The CTUG-K thus comprised 10 items with possible scores ranging from 0–10. For the performance of the 56 participants in the research, the accuracy fell in the range .29–.66 (Md = .54), item difficulty .30–.67 (Md = .50), and item discrimination .47–.77 (Md = .61), while KR_{21} = .72, indicating that CTUG-K has good measurement properties.

Tasks

This research developed tasks on paper worksheets, and every participant had to work on the same sheet of paper twice, as a pre-test (e.g., Appendix A) and a post-test. For the pre-test, participants read a concise explanation and a table involving four variables about a kinematic phenomenon, and then wrote down a scientific description as precisely and in as much detail as possible. For the post-test, with the aid of InduLab, participants revised their descriptions or added more descriptions after manipulating data plots.

To assist participants in understanding this experiment, an example task and a practice task were developed. The example task provided slope motion with its data. For the pre-test, three types of sentences were provided to illustrate how to propose scientific descriptions. The first type was to provide a quantitative description for a variable: 'The basketball is rolling down a hill at a constant velocity of 0.35 mm/ms.' The second type was to describe the linear connection between two variables qualitatively: 'The distance that the basketball rolled down the hill was proportional to the time it rolled.' The third type was to describe the quadratic connection between the two variables qualitatively: 'The distance that the basketball rolled down the hill increased per unit time.' For the post-test, examples to show participants how to revise the descriptions were provided. For example, it should have been said that the basketball rolled down the hill with an acceleration of 0.35 mm/ms². Moreover, the descriptions could also be integrated, such as 'the distance that the basketball rolled down the hill was not proportional to the time. The speed was slower at first for about 0.1 mm/ms, and then became faster at 0.6 mm/ms. This shows that the distance the basketball rolled down the hill increased per unit time.' In addition, the practice task topic was free fall, which had been studied by participants in senior high school.

Regarding the formal task, only an explanation and a table for the kinematic phenomenon were provided with a blank worksheet. The topic and data concerned a weather balloon ascent in Coffeyville, Kansas, USA, in 1991, retrieved from Erickson (2006)'s research. Appendix A shows the worksheet for the formal task. Participants could write their descriptions in the blank area. All materials were adjusted by one professor in educational psychology and one associate professor in science education.

Modelling with InduLab

A technology-based modelling tool called InduLab was developed. After collecting data, students can experience an explicit model building process with InduLab; that is, they can freely propose and revise models to fit the experimental data with immediate feedback including a visual plot and the error in the modelling (Liu et al., 2017). In this research, the raw data of the scientific phenomenon were provided to participants directly, including the variables of time, speed, acceleration, and height. After entering the data in InduLab, a data plot would be shown (the dots in the plots of Appendix B), and then the students were to work on the mathematical models to explain the data. There are two main features in InduLab, axis-changing and formula-finding (i.e., data-fitting). Once a model is committed, InduLab shows the model plot (circles in the plots of Appendix B) in the background of the data plot (dots in the plots). In the top figure of Appendix B, the model $f(t) = 1.5t$ was chosen. The dots were clearly above the circles, showing an error of 195.61 as computed by InduLab and defined as the sum of all vertical absolute distances between all pairs of corresponding dots and circles divided by the sum of the absolute y-values of all experimental data. There are other ways to compute the error measure, but the current measure is chosen due to its simplicity for understanding. The participants could then adjust the formula to improve the model. After several trials, the students entered the model $f(t) = 4t + 300$, and the resulting plot showed that the circles were closer to the dots, with an error of 4.76%. In addition,

participants' manipulations would be recorded in the backstage, including the variables along the x- and y-axes, the time point of modelling, the formula, and the error.

Procedure

The experiment was conducted individually. After obtaining informed consent, participants were administered the CTUG-K for up to 10 minutes. The participants were shown the example task to learn how to write down scientific descriptions, and they then wrote down the descriptions of the practice task and formal task for up to eight minutes each and returned the worksheets. Subsequently, the researcher explained the features of InduLab to both groups, but only the experimental group received two prompts stating that it is common to place 'time' along the x-axis and that mathematical modelling is of great value in physics. Afterwards, participants were instructed on how to revise the scientific descriptions using the example task. Participants were then allowed to enter the data into InduLab and practice its features for about four minutes. After that, participants were given up to eight minutes for each task to revise the practice task and formal task on their worksheet. It took participants about one hour to complete the procedure.

Manipulation Check

To check if the participants in the experimental group followed the prompts, the experimental and control groups' performance on the axis-changing and formula-finding was first examined using the practice task. In axis-changing, the percentage selecting time as the x-axis was significantly higher in the experimental group ($M = 74\%$) than in the control group ($M = 37\%$), $t(54) = -5.52$, $p < .001$. For formula-finding, the number of models was only marginally significantly higher in the experimental group ($M = 11.18$) than in the control group ($M = 5.61$), $t(54) = -1.80$, $p = .078$. Thus, two criteria were then adopted to exclude participants in the experimental group that failed to follow the prompts. The first criterion was the percentage stating the time variable along the x-axis in the first three minutes, which was lower than 80%. The second criterion was the number of models entered in InduLab was zero. These two criteria excluded 4 participants, leaving 24 participants in the experimental group and 28 participants in the control group.

Data Analysis

Regarding the worksheet analysis, a proposition was defined as a unit of meaning that involves sentences/images that convey a particular meaning, and then the number of propositions elicited was calculated. The accuracy of each proposition was examined, and incorrect and correct propositions were scored 0 and 1, respectively. Next, each participant's representational type was identified. Participants who proposed only verbal descriptions were classified as the verbal type, and those who included figures or formulas in the propositions as the image type. Since the scientific phenomenon in this research involved four variables and participants might provide descriptions qualitatively or quantitatively, five propositional types were initially identified as qualitative univariate, quantitative univariate, qualitative multivariate, single sequence multivariate, and sequences/formulas multivariate descriptions. However, several types contained less than five participants, which makes the chi-square analysis unacceptable. Thus, the five types were combined into quantitative–qualitative types (Table 1). A total of 104 responses on the pre-test and post-test were given scores by the first researcher. The other researcher then reviewed 20 of the responses for checking the reliability of the coding. The interrater reliability was perfect, with Kappa value of 1 ($p < .001$).

Table 1

The Scoring Criteria for Two Propositional Types

Types	Scoring	Example of incorrect propositions	Example of correct propositions
Qualitative description	Describe variables or the connections between variables qualitatively.	The acceleration and time are in inverse proportion. The acceleration gets smaller as time goes by.	The weather balloon gets higher as time goes by.
Quantitative description	Describe variables or the connections between variables with sets of data, sequences, or equations.	The weather balloon ascends to 18195 meters within 4193.3 seconds.	The relation between height and time is $f(t)=4.3t$, with an error of 4.

Research Results

The score on CTUG-K showed no significant differences between the experimental and control groups, $t(50) = -0.32$, $p = .975$, confirming the groups' equivalence. The average score was 5.15 ($SD = 2.67$). This result corresponded with the design of recruitment in that the participants were unfamiliar with the conceptions and graphs used in kinematics. In addition, the two groups also showed no significant differences in their response time on CTUG-K, $t(49) = -1.36$, $p = .180$, and the average response time was 8.16 minutes ($SD = 1.92$).

The Participants' Performance of Scientific Descriptions

To explore the performance of scientific description before and after using InduLab between the two groups, the average numbers of their propositions, average accuracy of propositions, representational types, and propositional types were compared.

The average number of propositions on the pre-test was 2.72. A 2 x 2 (group x test) ANOVA of the numbers of proposition was conducted. The main effect was significant for test, $F(1, 50) = 61.20$, $p < .001$, $\eta_p^2 = .55$, and for group, $F(1, 50) = 6.59$, $p = .013$, $\eta_p^2 = .12$, but no interaction effect existed between the two factors, $F(1, 50) = 0.24$, $p = .627$. By test, participants advanced more propositions in the post-test ($M = 4.07$, $SD = 0.20$) than in the pre-test ($M = 2.74$, $SD = 0.14$). By group, the experimental group ($M = 3.79$, $SD = 0.22$) advanced significantly more propositions than the control group ($M = 3.02$, $SD = 0.21$).

For the average accuracy of propositions, the average accuracy of all participants' propositions on the pre-test was about 76%. A two-way ANOVA of the average accuracy of propositions showed no main effect of test, $F(1, 50) = 0.39$, $p = .844$, or group, $F(1, 50) = 0.25$, $p = .619$, nor did interaction effects exist between the two factors, $F(1, 50) = 0.26$, $p = .609$. Overall, the average accuracy of propositions on the post-test for all participants was also 76%.

By representational type, 80% of the participants were of the verbal type and only 20% of the participants were of the image type on the pre-test. Next, a test of homogeneity to examine the differences in representational type between the two groups on the post-test was conducted, but no significant difference was found, $\chi^2 = 0.20$, $p = .657$. Thus, the McNemar's test for all participants was performed, finding that the difference between pre-test and post-test was significant ($\chi^2 = 22.00$) at the 0.01 level. Twenty-two participants of the verbal type on the pre-test changed to the image type on the post-test, and no participant of the image type on the pre-test changed to the verbal type.

For propositional type, the relationships between the two propositional types with the average accuracy of propositions were examined. No significant correlations were found for either the pre-test ($r = -.06$) or the post-test ($r = .19$), indicating that there was no difference in the accuracy of the propositional types that each participant proposed. Afterward, the difference between the two groups on the post-test with tests of homogeneity was examined. Results showed no significant difference in the propositional types between the two groups on the post-test, $\chi^2(1, n = 52) = 1.03$, $p = .310$. Thus, the McNemar's test was conducted to examine whether the changes in the propositional types between pre-test and post-test were statistically significant for all participants. The results were marginally significant, $\chi^2(1, n = 52) = 4.45$, $p = .065$. Nine participants whose propositional type was qualitative type on the pre-test changed to the quantitative type on the post-test, and only two participants whose propositional type on the pre-test was the quantitative type changed to qualitative type.

The Participants' Spontaneous Mathematical-Modelling Behaviours

All participants' spontaneous mathematical-modelling behaviours were examined by the mathematical models they proposed with InduLab. Regarding the time participants spent in proposing the first model, there was no significant difference between the experimental group and the control group, $t(27) = 0.30$, $p = .770$. The average time spent by all participants was 2.65 minutes. The number of models provided by the participants in the experimental group ($M = 21.18$) was marginally significantly higher than in the control group ($M = 12.17$), $t(27) = -1.92$, $p = .066$.

The Preparatory Work of Mathematical Modelling

There were 29 participants proposed models, 12 in the control group and 17 in the experimental group. Most participants proposed models after viewing several graph plots (control/experiment, $n = 8/14$). Table 2 presents
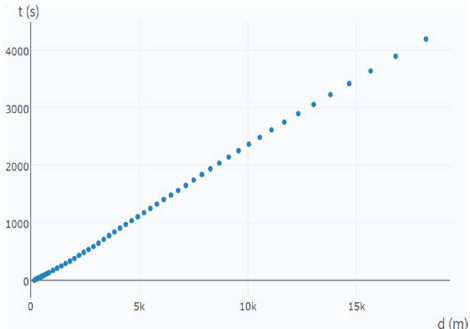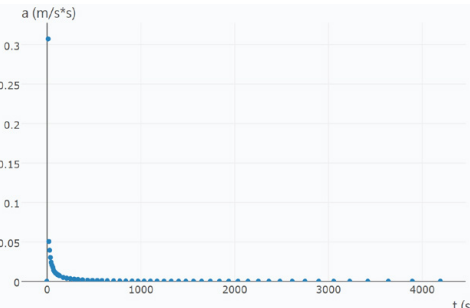
12 graph plots of the data in the order of the number of participants involved in building a model, from the most participants to the least. If a participant proposed only one model, one formula is shown, and if a participant proposed several models, then the first and the last formulas are shown. Finally, the column of error would be 'Infinity' if InduLab could not calculate the error of a given formula.
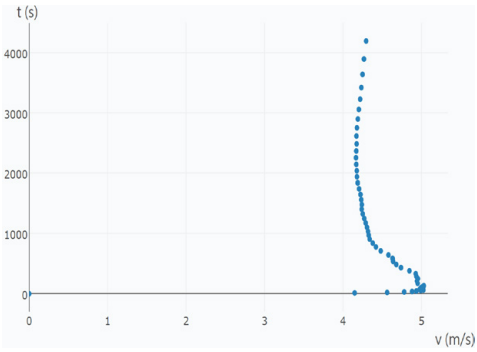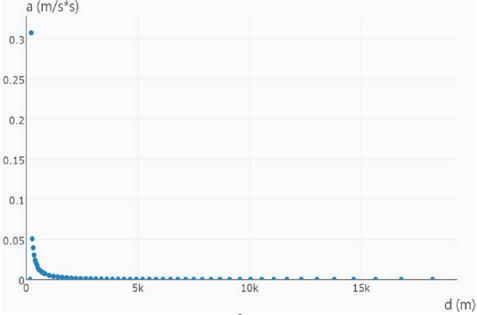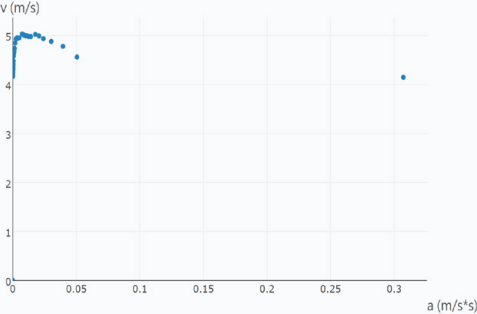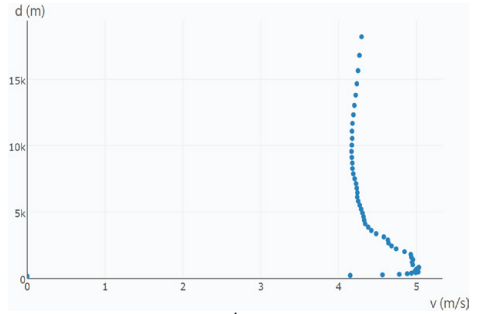
Table 2
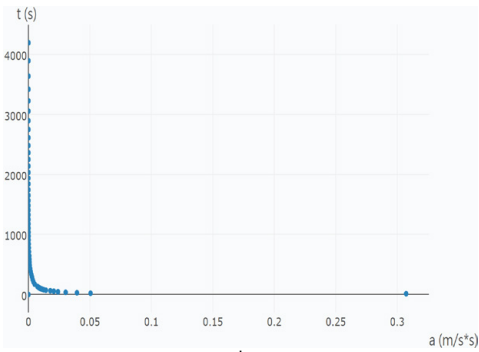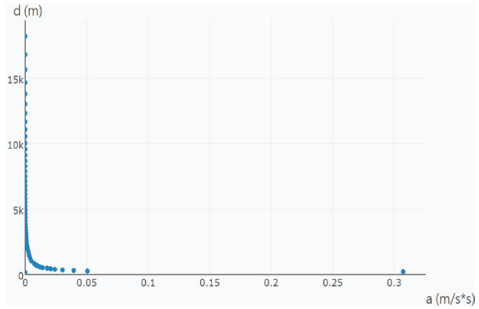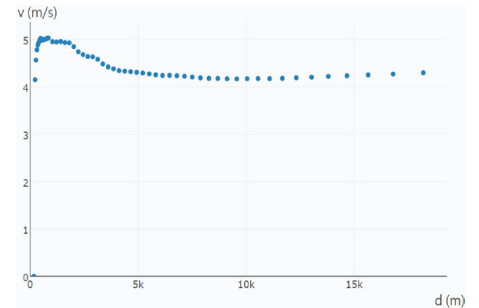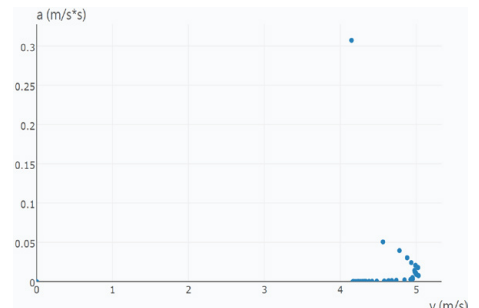*Indulab's Graph Plots and Participants' First and Last Equations*

Graph plots	Participant No.	Formula	Error
a.	4	$f(t) = 3859/838.7$	112843.95
	6	$f(t) = t$	343.41
		$f(t) = 172+0.003t^2$	50.93
	7	$f(t) = 5t$	13.72
		$f(t) = 4.3t$	4.00
	9	$f(t) = 0$	Infinity
		$f(t) = 5t$	13.72
	16	$f(t) = 0.004t$	110749.39
		$f(t) = 4.3t$	4.00
	20	$f(t) = 4t$	10.86
		$f(t) = 4.3t$	4.00
	21	$f(t) = 172/-3.3t$	804019.42
		$f(t) = 95.2t$	95.35
	22	$f(t) = 2t+100$	112.65
		$f(t) = 4.5t+90$	5.89
	25	$f(t) = 0.001t^2$	91.07
		$f(t) = 0.001480t^2$	48.42
	26	$f(t) = 0.5t^2$	99.62
		$f(t) = 0.05t^2$	96.27
	28	$f(t) = 5t$	13.72
		$f(t) = 5t-5$	13.71
	37	$f(t) = 10t$	55.79
		$f(t) = 4.5t$	6.09
	38	$f(t) = 0.5t^2$	99.62
		$f(t) = 0.0025t^2$	47.90
	40	$f(t) = 4t$	10.86
		$f(t) = 4.5001$	8.48
	48	$f(t) = 0.0003t$	1477891.78
		$f(t) = 0.1t^2$	98.11
	51	$f(t) = 5t$	13.72
		$f(t) = 4.3000001t$	4.00

Graph plots	Participant No.	Formula	Error
b.	1	$f(d) = 20d$	98.87
		$f(d) = 0.23d$	4.13
	9	$f(d) = 5d$	95.49
		$f(d) = 50d$	99.55
	18	$f(d) = 2*8 + \mathrm{sqrt}(8) + \mathrm{pow}(8,2)$	1333.30
		$f(d) = 2*30.8 + \mathrm{sqrt}(30.8) + \mathrm{pow}(30.8,2)$	93.37
	21	$f(d) = 172$	615.78
		$f(d) = 2445$	62.95
	29	$f(d) = 0.2d$	15.47
		$f(d) = 0.26d + 0.5$	13.30
	40	$f(d) = 65$	1717.84
	50	$f(d) = 0.5d$	54.90
		$f(d) = 0.235d$	4.12
	52	$f(d) = 300d$	99.92
		$f(d) = 0.232d$	3.89
	57	$f(d) = 172d$	99.87
		$f(d) = 18195m$	Infinity
c.	2	$f(t) = 2s$	Infinity
		$f(t) = 482$	100.00
	4	$f(t) = 3859/838.7$	99.78
	7	$f(t) = \log(t,10)$	Infinity
		$f(t) = t^1$	100.00
	16	$f(t) = 0.0001t$	105.35
		$f(t) = 0.0001t^3$	100.00
	21	$f(t) = -3.3$	100.31
		$f(t) = -3.30t$	100.00
	28	$f(t) = 0.0002t$	102.11
		$f(t) = 0.001t^{-1}$	83083.74
	54	$f(t) = 0.005t^0$	235.61
		$f(t) = 0.0000005t^0$	2060945.52
	56	$f(t) = 0.05t^{-2}$	11039.51
		$f(t) = 0.3t^{11}/100$	100.00
d.	7	$f(t) = .01t^2$	99.98
		$f(t) = 0$	Infinity
	22	$f(t) = t^2 - 5$	100.00
		$f(t) = 2t - 5$	99.81
	28	$f(t) = 0.5t$	99.24
		$f(t) = 3t$	99.87
	33	$f(t) = 0.001t^2$	99.87
		$f(t) = 0.020t$	89.10
	54	$f(t) = 10t + 0.00005t^2$	99.96
		$f(t) = 5 - 0.00032t^1$	6.05
	63	$f(t) = 4$	14.62
		$f(t) = 4.35$	8.48

Graph plots	Participant No.	Formula	Error
e.	21	$f(v) = 4.56221v$	5679.23
		$f(v) = 5.02802v$	5144.45
	22	$f(v) = 1/2v$	52614.82
		$f(v) = v^2 - 5$	7510.09
	33	$f(v) = 100v$	223.06
		$f(v) = 500v$	65.26
	52	$f(v) = 200v^2$	71.70
		$f(v) = 20v^1$	1237.06
f.	29	$f(d) = 0.002d$	99.90
		$f(d) = 0.0001d^2$	100.00
g.	28	$f(a) = 0.05a$	862894.90
		$f(a) = 5a^6$	6128468.69
	50	$f(a) = 5$	11.14
		$f(a) = 4.5$	8.48
h.	3	$f(v) = 3v$	38856.04
	21	$f(v) = 4.14815v$	28073.55

Graph plots	Participant No.	Formula	Error
i.	21	$f(a) = 0$	Infinity
j.	–	–	–
k.	–	–	–
l.	–	–	–

The Process of Mathematical Modelling

Among the participants who proposed models with graph plots that looked more linear, five participants proposed constant models and nine participants estimated the time from the height, indicating that they could not provide appropriate formulas with correct physical meaning to fit this simple linear-looking graph plot. About 30% (9/29) of the participants could not use the parameters of mathematical modelling appropriately, which resulted in 'Infinity' in the error column. In Table 2c, for example, participant No. 2 mistook units for parameters and proposed the model $f(t) = 2s$ for acceleration with time; thus, this error could not be calculated and InduLab responded with Infinity.

In addition, three participants of the experimental group selected the height-time graph for their hypotheses after seeing all the graphs that set the time along the y-axis, and then proposed only quadratic models for graph plots that looked more linear. This is probably because these participants figured out that the connection between time and height is not a linear connection after viewing several graphs that put time along the x-axis, so they persisted in revising a quadratic model to fit the data.

Model Modifying Strategies

Except for the three participants who proposed models less than three times, 96% (25/26) of the participants tended to mildly adjust the coefficients or constants instead of changing the highest degree of the formula. This strategy was also found in the past research (Liu et al. 2017). It demonstrates that most participants revised their models gradually based on the error and corresponding graphs from InduLab to improve the models.

Participants Who Did Not Engage in Modelling

Though the first graph plot in InduLab is simple and looks linear (i.e., Table 2b), up to 23 participants did not propose any models (control/experimental, $n = 16/7$). Among them, 15 participants had viewed the graphs with time along the x-axis, showing that they might be more capable of relating the physical conception to the graph plots. Another eight participants viewed only one or two graphs that put time along the x-axis, and five of them were in the control group with lower scores of prior knowledge. This shows that low-prior-knowledge participants in the control group did not understand the principle that time should be the independent variable along the x-axis.

Among the participants who did not engage in modelling, over 90% of them viewed graph plots with the axis-changing feature; the average number of graphs viewed was 5.77 and the average time spent was 4.75 minutes. Moreover, one-third of the participants were in the experimental group, that is, they had been reminded of this critical feature of mathematical modelling. They still proposed no models to fit the experimental data, probably because these participants failed to connect the formulas to the graph plots when proposing a model, or perhaps because the participants believed that these graphs could not be represented concisely by a mathematical model and thus decided not to propose hypotheses. For instance, one participant in the control group who had a high score on the prior knowledge test viewed the graph plots that set the time along the x-axis for five minutes but proposed no model in the end.

The Performance of Model Builders and Non-Model Builders in Scientific Description

To confirm whether the initial capabilities of model builders and non-model builders were different, their performances in scientific description on the pre-test were compared. Results showed no significant differences between the two groups on CTUG-K, average numbers of their propositions, and average accuracy of propositions, $ts(50) < 1.01$, $p > .319$. There was also no significant difference in the representational type and the propositional types between the two groups, $\chi^2(1, n = 52) < 0.16$, $p > .686$.

On the other hand, the performances of model builders and non-model builders on the post-test were also compared to clarify the impact of using InduLab. The propositional types proposed by the two groups were significantly different, $\chi^2(1, n = 52) = 4.39$, $p = .036$. The percentage of model builders using the quantitative type (86.2%) was significantly higher than among non-model builders (60.9%). However, there were no significant differences between the two groups on the average numbers of their propositions and the average accuracy of propositions, $ts(50) < 1.72$, $p > .092$, as well as the representational type, $\chi^2(1, n = 52) = 0.66$, $p = .416$.

Though model builders proposed more quantitative scientific descriptions, the worksheets showed that only 40% of the model builders provided their equations, and 30% of them just wrote down their equations without any explanation, indicating that most model builders could not make a connection between the formalized language of mathematics and the corresponding graphs and could not provide an explanation for their hypotheses. However, there were a few participants who provided more details and precise data in their scientific descriptions, thus turning to quantitative types after mathematical modelling. For instance, one participant's propositional type changed from the qualitative to quantitative type after proposing models 30 times. On the pre-test, he stated: 'The ascending distance of weather balloon increases as the ascent time gets longer.' On the post-test, he described the phenomenon more precisely: 'The V is getting faster and faster first, decreasing dramatically at about 200 seconds, then diminishing slightly, and finally showing a recovery. According to the distance-time graph, the mathematical model of t is $f(d) = 0.232d$.' Some people further interpreted the data to make sense of the phenomenon after mathematical modelling. Take one participant who proposed models 43 times, a relatively high frequency, as an example. Initially, he stated: 'The acceleration decreases from 0.30727 to 0.00004 after the weather balloon ascends to great altitudes. The speed of 4 m/s is maintained constantly.' After using InduLab, the participant drew two graph plots on his worksheet and described: 'According to the velocity-time graph, the weather balloon does not move at a constant speed at the beginning, which is probably influenced by the airflow of troposphere.'

Discussion

This research adopted raw data from a real physical phenomenon to explore non-science-major undergraduate students' scientific descriptions and spontaneous mathematical-modelling behaviour after using InduLab. For the first research question concerning the participants' performance in scientific descriptions, four hypotheses were examined. Hypothesis 1 was supported that participants generated more propositions after using InduLab, especially in the experimental group. Hypothesis 2 was not supported that the average accuracy of their propositions showed no difference between the test and the group. Hypothesis 3 and 4 were partially supported that 40% of the participants changed the representational type from verbal on the pre-test to image type, and roughly 20% of the participants changed their propositional type from qualitative on the pre-test to the quantitative type.

The main effects of test were significant-participants advanced their performances of scientific description in the post-test than in the pre-test. It indicated that students were able to describe a scientific phenomenon quantitatively and precisely once they have access to a modelling tool to engage with the data of the physical world. The performances of model builders and non-model builders also support the results that the model builders proposed more quantitative propositional types than did the non-model builders. As was found in previous studies (Bernhard, 2018; Hochberg et al., 2020; Wang et al., 2017), this technology-based learning approach can help students visualise experimental data with different representations almost instantaneously, so students can devote greater effort to clarifying the connections between quantitative variables and constructing mathematical models in their scientific descriptions, which was regarded as prioritized skills for mastering scientific knowledge by Hestenes (1987) and Namdar and Shen (2015).

On the contrary, the main effects of group on students' scientific descriptions were much weaker. The limited effects are consistent with those of the studies of Ibrahim and Rebello (2013) and Kozma (2003), who found that novice learners often adopt propositions as the primary mental representations and have trouble focusing on more than one representation. It appears that despite the technology-based InduLab modelling tool, which might assist novices in understanding and describing the connections and variations between variables in a short time, more systematic instructions should be developed. More specifically, it is necessary to design long-term learning activities for students to better familiarize themselves with the tool. For example, Stull et al. (2016) suggested that educators demonstrate how to use models in solving problems and design laboratory activities that require learners to propose models and understand the phenomena. In short, learners would not be able and willing to employ technology-based laboratories until they became familiar with the laboratories and understood their values.

For the second research question regarding participants' spontaneous mathematical-modelling behaviours, one exploratory research question was explored. About half of the participants engaged in the mathematical modelling activity. Regarding the modelling process, among the participants who proposed models with graph plots that looked strongly linear, several of them adopted a constant model to fit the data or estimated height by time, indicating that they did not understand the mathematical meaning of the phenomenon and the physical meaning of their models. Interestingly, three participants from the experimental group viewed the graph plots

that put time on the *x*-axis repeatedly and demonstrated great persistence in proposing a quadratic model, even though the graph plots of height and time looked more linear than quadratic. A possible explanation is these participants constructed the hypothesis that the height and time of the phenomenon were not in a linear connection after they integrated the information from various graph plots. In the classroom, teachers should assist learners in collecting data with different variables, observe the combinations of those variables in mathematical models, and then generate good hypotheses and theoretical models. Regarding the model modifying strategies, the result was corresponding to the previous research (Liu et al. 2017) that most participants tended to revise their models gradually to improve the models. Furthermore, most model builders just copied the formulas on their worksheets without providing explanations, indicating that students could not apply their acquired experiences to connect the formulas and realistic data. The participants had not only learned linear and quadratic functions with their graph plots but had also studied how to adjust the formulas to alter the corresponding graph plots in junior and senior high school. However, these learning experiences would appear not helpful for students in constructing ideal mathematical models and descriptions.

Our earlier studies (Liu et al. 2017) applied InduLab as a modelling tool to assist students in engaging in mathematical-modelling activities after physics experiments. Students participated in experiments that they had learned before and sought an ideal formula relating the two given variables with InduLab. In contrast, the current research adopted a real-world phenomenon that the participants had never learned from textbooks and required them to propose mathematical models in a short time using InduLab. Even under this open-ended problem condition, several participants did not explore the difference between the observed data and the corresponding predicted value to improve the accuracy of their scientific description with mathematical-modelling behaviours. This is consistent with previous research (Erickson 2006). In the mathematics curriculum, symbolic expressions are usually provided without real context, so students have little experience connecting their understanding with real-world phenomena and coping with variability, which causes them difficulty in decoding the mathematical parameters and dealing with data that are not precise and error-free. For example, given a set of authentic data about an ascending weather balloon, the participants could build a mathematical model, such as *distance = 0.4 × time*. However, they failed to connect the quantities and variables in the formula with the actual motion of the weather balloon (e.g., the slope of the graph, 0.4 m/s, is the speed of the balloon).

Conclusions and Implications

This research confirmed that the technology-based laboratory InduLab enabled non-science-major college students to precisely describe facts of a scientific phenomenon with more scientific propositions, image/mathematics-based representations, and quantitative narrations. In addition, this research also determined that many participants tended to adjust their mathematical models by only slightly changing the coefficients or constants to fit the data, rather than applying relevant physics knowledge to revise their models, illustrating their difficulties in connecting mathematical representations with actual phenomena.

In Taiwan, model building is rarely taught or is only demonstrated by teachers in physics classrooms due to time constraints and its practical value vis-à-vis high-stakes tests. Students are told the theory and given equations and rarely deal with the modelling error that arises in a real context. In future research, researchers could clarify how learners deal with various graph plots. Do they realise that the graph plots are influenced by certain critical factors and identify the negligible error in the graph plots? How do their beliefs shape their understanding of scientific phenomena? If learners' understanding and difficulties in the scientific inquiry process could be revealed, then more scientific activities with real context could be developed to assist learners in engaging in science learning.

Although the present research makes several useful contributions, some limitations need to be considered. First, this research involved the task of describing a physical phenomenon instead of a mathematical modelling task (e.g., improving the models for a realistic problem). Future studies may focus specifically on how learners solve various problems with mathematical modelling. Second, since it was difficult to include the participation of high school students, the participants were undergraduate students whose learning experiences were similar to those of high school students. However, some participants conceded that they had already forgotten these conceptions. Future studies should include high school students as participants to clarify learners' scientific descriptions and mathematical modelling behaviours more thoroughly.

Journal of Baltic Science Education, Vol. 21, No. 3, 2022

ISSN 1648–3898 /Print/

ISSN 2538–7138 /Online/

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DESCRIBE SCIENTIFIC PHENOMENA
(PP. 495-512)

Acknowledgements

Declaration of Interest

Authors declare no competing interest.

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0 Journal of Baltic Science Education, Vol. 21, No. 3, 2022

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Appendix A.   Worksheet of the formal task

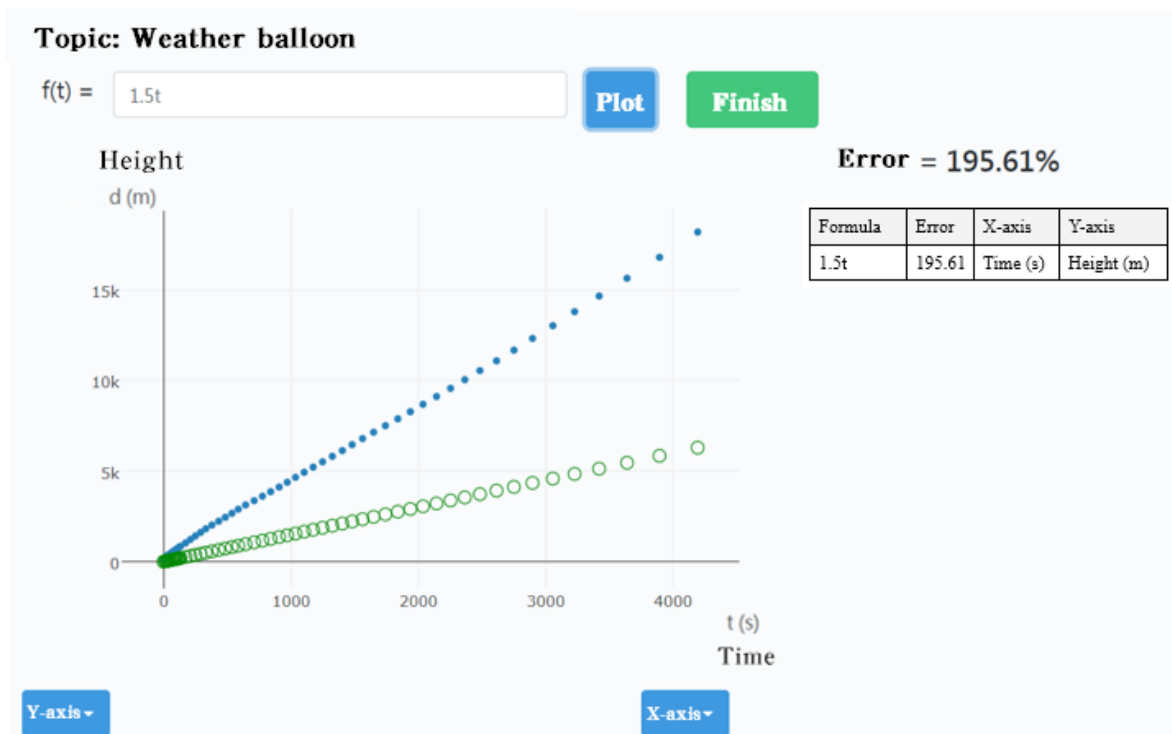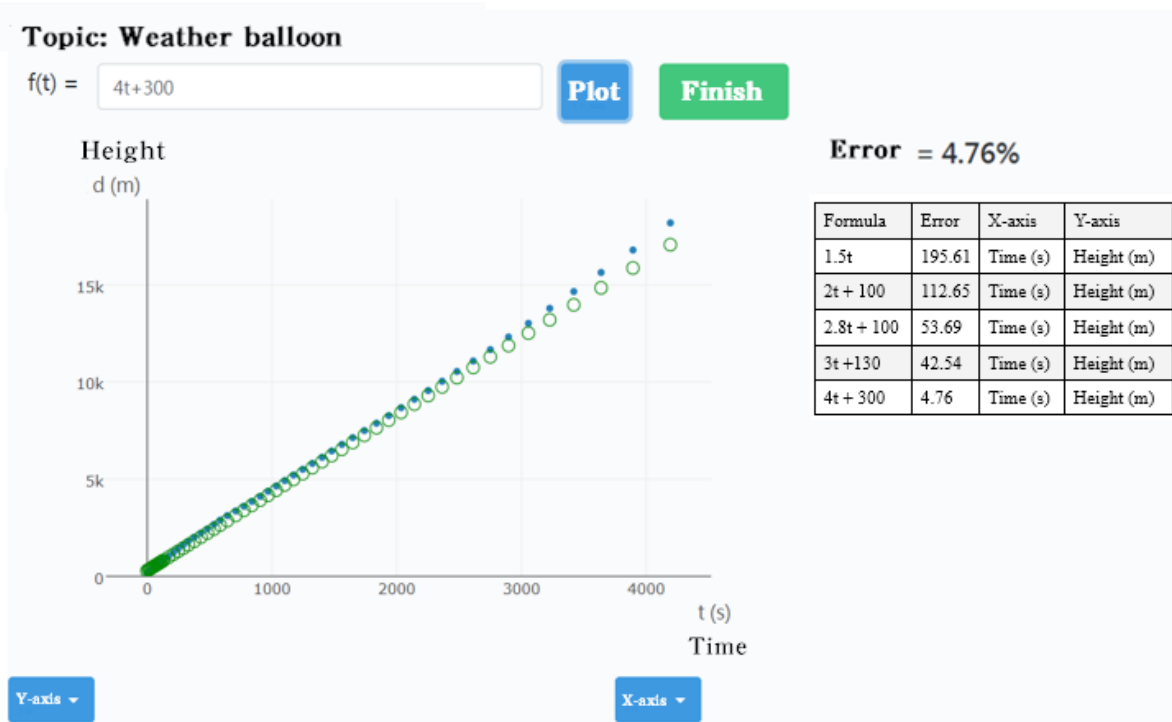### Formal task ⁻ Weather balloon

Instruction

The weather balloons are launched by some weather stations twice a day to observe the change of the weather. The radiosonde carried by a weather balloon could measure many atmospheric parameters (e.g., temperature, air pressure, humidity) and transmits them to a ground receiver every one to two seconds. In addition, the wind direction and speed could be obtain by Global Positioning System.

Notes

Variable	Height	Time	Speed	Acceleration
Unit	m	s	m/s	m/s*s
Abbreviation	d	t	v	a
	172	-3.30	0	0
	228	10.20	4.14815	0.30727
	271	18.40	4.56221	0.05051
	313	26.20	4.77966	0.03948
	356	34.40	4.88064	0.03027
	400	42.90	4.93506	0.02407
	443	51.00	4.99079	0.02066
	487	59.40	5.02392	0.01780
	531	68.80	4.97920	0.01418
	575	77.60	4.98146	0.01237
	620	86.40	4.99443	0.01111
	664	95.20	4.99492	0.00996
	709	104.00	5.00466	0.00913
	754	112.80	5.01292	0.00843
	800	121.60	5.02802	0.00790
	846	130.70	5.02985	0.00732
	1031	170.20	4.95101	0.00502
	1220	208.70	4.94340	0.00401
	1414	247.40	4.95413	0.00340
	1611	288.40	4.93315	0.00282
	1813	329.80	4.92645	0.00244
	2018	377.60	4.84642	0.00190
	2229	430.80	4.73854	0.00140
	2445	482.50	4.67888	0.00112
	2667	534.60	4.63841	0.00094
	2894	584.40	4.63161	0.00084
	3126	641.80	4.57914	0.00068
Data of weather balloon	3365	709.30	4.48077	0.00048
	3609	774.40	4.41944	0.00036
	3859	838.70	4.37886	0.00028
	4116	905.60	4.33931	0.00021
	4380	968.90	4.32833	0.00019
	4652	1034.10	4.31849	0.00017
	4930	1101.90	4.30510	0.00014
	5215	1172.40	4.28936	0.00012
	5510	1246.50	4.27108	0.00010
	5813	1323.50	4.25158	0.00008
	6128	1402.00	4.23824	0.00006
	6456	1478.80	4.23993	0.00006
	6791	1560.60	4.23237	0.00005
	7141	1647.10	4.22261	0.00005
	7504	1740.30	4.20509	0.00003
	7881	1837.70	4.18740	0.00002
	8274	1936.20	4.17737	0.00002
	8684	2035.30	4.17541	0.00001
	9114	2141.60	4.16896	0.00001
	9566	2252.10	4.16511	0.00001
	10042	2364.10	4.16913	0.00001
	10547	2482.50	4.17371	0.00001
	11086	2613.20	4.17122	0.00001
	11673	2749.90	4.17732	0.00001
	12318	2895.90	4.18943	0.00001
	13024	3055.40	4.20179	0.00002
	13797	3226.40	4.21866	0.00002
	14661	3419.20	4.23346	0.00003
	15644	3638.10	4.24892	0.00003
	16799	3893.50	4.26683	0.00003
	18195	4193.30	4.29467	0.00004

Appendix B.   Finding an ideal formula in InduLab

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