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EXPANDING DATA NORMALIZATION METHOD TO CODAS METHOD FOR MULTI-CRITERIA DECISION MAKING

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Abstract:
Data normalization is the conversion of quantities of different dimensions to the same dimensionless form, which is required in multi-criteria decision making (MCDM). The choice of data normalization method has a direct influence on the decision-making results. This study presents the combination of CODAS (COmbinative Distance-based ASsessment) method with six different data normalization methods including Linear normalization, Max - Min linear normalization, Vector normalization, Sum linear normalization, Logarithmic normalization, Max linear normalization . These six combinations have been applied in turn in three different examples. The number of alternatives, the number of criteria, and the method of the weight calculation in these examples are also different. From the results it was reported that only the combination of CODAS and Logarithmic normalization was not suitable. The combination of CODAS with some other data normalization methods not mentioned in this study and it needs to be done in the near future. This task was covered in the last part of this paper.

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1. INTRODUCTION

Consideration to choose one of the many options is a routine work that must be done in many areas. If each solution is described by at least two criteria, then this is called multi-criteria decision making. Multi-criteria decision making is essentially the determination of the ranking of the solutions, from which the best solution will be determined, and away from the worst solutions [1-3]. If the choice of a certain solution is made according to the subjective opinion of the decision maker, it will easily lead to mistakes. Then, the best solution can be ignored and the solution that is not the best one can be chosen, even the worst one can be chosen [1]. The decision of the decision maker can only achieve a certain degree of confidence if it is made according to a certain mathematical method. “Multi-criteria decision making” is the general name for such mathematical methods. Multi-criteria decision making methods are increasingly interested and developed by many researches.

CODAS is a multi-criteria decision making method proposed in 2016 [4]. This method is based on measuring the Euclidean distance and the Taxicab distance from the alternatives to the negative-ideal solution. Because of this feature, when using CODAS method for multi-criteria decision making, high accuracy is always achieved [5,6]. Therefore, this method has been used to rank alternatives in many different cases, such as: To classify workers for a company [7], to evaluate and segment the market [8] , choosing a site for dismantling used vehicles [9], choosing a site for hospital construction [10], to rank steel suppliers [11], to choose a production system flexible production (FMS) [12], to choose investors for emerging companies [13], to choose green suppliers [14], to choose renewable energy

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alternatives [15] , choose the location to build the shopping center [16], etc. The above are just a few of the many recent studies that have applied the CODAS method to multi-criteria decision making. That proves that this method is really highly effective and is gaining a lot of attention.

As with most other multi-criteria decision making methods, data normalization is a must when applying CODAS method. Data normalization helps to convert quantities to dimensionless form, from which the alternatives can be compared [17,18]. Data normalization also creates an opportunity for decision makers to assign the weights to criteria [19]. In each multi-criteria decision-making method, the data normalization method is always mentioned [19]. However, different multi-criteria decision making methods using different data normalization methods. This will result in different ranking results [20-22]. In the next section, several of these cases will be discussed in more detail. Moreover, if choosing an inappropriate data normalization method, it will also cause rank inversion, i.e. causing the phenomenon of wrong ranking of the alternatives. That mean there may be a case of bad solutions. The worst or very bad alternative is ranked as the preferred alternative, causing the best alternative to be ignored [19, 23, 24]. Thus, if only one method of data normalization is used, when making multi-criteria decisions, the best method found is not necessarily the real best method. This can only be solved if multiple data normalization methods are used simultaneously in a single decision. In the scope of this paper, the data normalization method that has been used internally in the CODAS method will be disscussed. The concept of "internal" here is understood as the method of normalizing the data used in the multi-criteria decision-making method itself, used by the inventors of the decision-making method. In addition, the main task of this study is to evaluate the combination of the CODAS method with other data normalization methods.

2. METHODS OF DATA NORMALIZATION

As mentioned above, normalizing data is about bringing the data to a dimensionless form. Data normalization methods also take many different forms. Six common data normalization methods which have been used internally in multi-criteria decision-making methods are listed below.

+ Linear normalization (N1)

$$n_{ij}^{(1C)} = \frac{minx_{ij}}{x_{ij}} \qquad if\ j\ \in C \qquad (1)$$

$$n_{ij}^{(1B)} = \frac{x_{ij}}{maxx_{ij}} \qquad if\ j\ \in B \qquad (2)$$

+ Max - Min linear normalization (N2)

$$n_{ij}^{(2C)} = \frac{x_{ij} - maxx_{ij}}{minx_{ij} - maxx_{ij}} \qquad if\ j\ \in C \qquad (3)$$

$$n_{ij}^{(2B)} = \frac{x_{ij} - minx_{ij}}{maxx_{ij} - minx_{ij}} \qquad if\ j\ \in B \qquad (4)$$

+ Vector normalization (N3)

$$n_{ij}^{(3C)} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \qquad if\ j\ \in C \qquad (5)$$

$$n_{ij}^{(3B)} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \qquad if\ j\ \in B \qquad (6)$$

+ Sum linear normalization (N4)

$$n_{ij}^{(4C)} = \frac{\frac{1}{x_{ij}}}{\sum_{i=1}^{m}\frac{1}{x_{ij}}} \qquad if\ j\ \in C \qquad (7)$$

$$n_{ij}^{(4B)} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \qquad if\ j\ \in B \qquad (8)$$

+ Logarithmic normalization (N5)

$$n_{ij}^{(5C)} = \frac{\frac{1 - \ln(x_{ij})}{ln\left(\Pi_{i=1}^{m} x_{ij}\right)}}{m-1} \qquad if\ j\ \in C \qquad (9)$$

$$n_{ij}^{(5B)} = \frac{\ln(x_{ij})}{ln\left(\Pi_{i=1}^{m} x_{ij}\right)} \qquad if\ j\ \in B \qquad (10)$$

+ Max linear normalization (N6)

$$n_{ij}^{(6C)} = 1 - \frac{x_{ij}}{maxx_{ij}} \qquad if\ j\ \in C \qquad (11)$$

$$n_{ij}^{(6B)} = \frac{x_{ij}}{maxx_{ij}} \qquad if\ j\ \in B \qquad (12)$$

In the formulas from (1) to (12): *C* represents the criterion as small as possible; *B* represents the criterion as large as possible; *i* is the number of alternatives; *j* is the number of criteria, and x_{ij} is the value of criterion *j* in alternative *i*. Table 1 presents several multi-criteria decision making methods along with the data normalization method used internally.

Table 1 shows that N1 method is the most used in multi-criteria decision making methods. However, it is also realized that in practice there will be many cases where this method (N1) will not be able to perform when one of the criteria has $x_{ij} = 0$, then expression (1) will be meaningless. Even formula (2) will be meaningless if $max(x_{ij}) = 0$. Similarly, if there exists a value of $x_{ij} = 0$ then N4 method cannot be used. The method N5 also cannot be used if there exists at least one quantity $x_{ij} \leq 0$. The method N6 will also not be applicable if there exists $max(x_{ij}) = 0$. In such cases, there will have very few options for multi-criteria decision-making methods without another data-normalization method to use. However, even if the decision maker chooses a different method of data normalization, it raises skepticism about the outcome of the decision. That skepticism is understood to be whether using a different data normalization method results in the correct ranking of the alternatives. To solve this problem, a multi-criteria decision must first be made when considering the various methods of data normalization.

Table 1. Data normalization methods have been used internally in various MCDM methods

Multi Attribute Decision Making	Normalization method					
	N1	N2	N3	N4	N5	N6
TOPSIS - Technique for Order Preference by Similarity to Ideal Solution			√			
VIKOR - VIsekriterijumska optimizacijaI KOmpromisno Resenje (in Serbian)		√				
MOORA - Multiobjective Optimization On the basis of Ratio Analysis	√					
COPRAS - COmplex PRroportional ASsessment	√					
COCOSO - COmbined COmpromise SOlution		√				
SAW - Simple Additive Weighting	√					
WASPAS - Weighted Aggregates Sum Product ASsessment	√					
PIV - Proximity Indexed Value	√					
PSI - Preference Selection Index	√					
MABAC - Multi-Attributive Border Approximation area Comparison		√				
PARIS - Preference Analysis for Reference Ideal Solution	√	√	√			
MACONT - Mixed Aggregation by COmprehensive Normalization Technique	√	√				
WPM - Weighted Product Model					√	
WSM - Weighted Sum Model				√		
ROV - Range Of Value		√				
MARCOS - Measurement Alternatives and Ranking according to COmpromise Solution	√					
CODAS - COmbinative Distance-based ASsessment	√					
This paper using CODAS method	√	√	√	√	√	√

From this point of view, several studies have also been conducted on some multi-criteria decision-making methods. Table 2 presents a summary of the combination of several multi-criteria decision making methods with some data normalization methods.

From Table 2, it was note that: (1) For each different multi-criteria decision-making method, the results are not the same when using different method of normalizing the data; (2) When a data normalization method is combined with many different multi-criteria decision-making methods, the ranking results will not be the same. Thus, to choose the best method of normalizing data when combined with a certain multi-criteria decision-making method, it is necessary to use multiple numerical normalization methods simultaneously. Method N1 was used internally within the CODAS method. However, as mentioned above, if $min(x_{ij}) = 0$ or/ and $max(x_{ij}) = 0$ exists, the N1 method cannot be used. In this case, it is necessary to find a new data normalization method to replace the N1 method. However, to date, no studies of this work have been found. This fact prompted a study comparing the combination of the CODAS method with different data normalization methods.

3. CODAS METHOD

Multi-criteria decision making by CODAS method follows the following steps [4]:

Step 1: Build a decision matrix

$$X = [x_{ij}]_{n\times m} = \begin{bmatrix} x_{11} & \cdots & x_{1m} \\ x_{21} & \cdots & x_{2m} \\ \vdots & \cdots & \vdots \\ x_{nm} & \cdots & x_{nm} \end{bmatrix} \quad (13)$$

Where n and m are the number of options and the number of criteria, respectively.

Table 2. Several studies on the combination of MCDM and data normalization methods

MCDM Method	Normalization method	Target	Conclusions	Ref.
ROV	N1, N2, N3, N4, N5	To rank the performance of several companies	The combination of ROV with N1 method is the best, while the combination of ROV and N4 method should be avoided.	[22]
TOPSIS	N1, N2, N3, N4, N5	Choosing the landing method of unmanned aircraft	The N3 method was the best, while the N4 method provided the worst results.	[23]
AHP	N1, N2, N3, N4, N5	Smart parking lot selection	The N4 method cannot be combined with AHP, the combination of AHP and N1 method provided the best results. In contrast, the combination of AHP and N5 methods provided the worst results.	[25]
PROMETHEE II	N1, N2, N3, N4, N5	Airport construction selection	The rank order of alternatives was very different based on the different data normalization methods.	[26]
TOPSIS	N1, N2, N3, N4, N5	Car selection	The N3 method provided the best results	[27]
AHP, Fuzzy AHP, TOPSIS, Fuzzy TOPSIS and PROMETHEE	N3	-	That the result of ranking the alternatives was not the same in those five combinations.	[28]

Step 2. Use formulas (1) and (2) to normalize the data. Note that data normalization by method N1 is the method that has been used internally in CODAS method. Besides, there are no reports about combining methods N2, N3, N4, N5 and N6 with CODAS method.

Step 3. Calculate the normalized value considering the weight of the criteria.

$$r_{ij} = w_j . n_{ij} \quad (14)$$

In (14), w_j in the weight of criteria *j*.

Step 4. Identify the negative-ideal alternative

$$ns = \left[ns_j\right]_{1\times m} \quad (15)$$

$$ns_j = min_i\ (r_{ij}) \quad (16)$$

Step 5. Calculate the Euclidean distance (E_i) and the Taxicab distance (T_i) from the alternatives to the negative-ideal alternative

$$E_i = \sqrt{\sum_{j=1}^{m} \left(r_{ij} - ns_j\right)^2} \quad (17)$$

$$T_i = \sum_{j=1}^{m} \left|r_{ij} - ns_j\right| \quad (18)$$

Step 6. Build a relative rating matrix

$$Ra = \left[h_{ik}\right]_{n\times n} \quad (19)$$

$$h_{ik} = (E_i - E_k) + \varphi(E_i - E_k) \times (T_i - T_k) \quad (20)$$

Where $k \in \{1,2,\dots,n\}$ and φ is a threshold to realize the equality of Euclidean distance between two alternatives and it can be defined as follows.

$$\varphi(x) = \begin{cases} 1 & if \quad |x| \geq \tau \\ 0 & if \quad |x| < \tau \end{cases} \quad (21)$$

In which $\tau = 0.01 \div 0.05$ depends on the decision maker. If the Euclidean distance (E_i) between the two solutions is less than τ, compare the two solutions by the Taxicab distance (T_i). Usually choose $\tau = 0.02$ [4].

Step 7. Calculate the rating for each alternative

$$H_i = \sum_{k=1}^{n} h_{ik} \quad (22)$$

Step 8. The solution with the largest H_i is the best one, and vice versa. That is the principle of ranking alternatives in CODAS.

4. EXAMPLES FOR APLYING

4.1. Example 1

This example re-executes an example of ranking Robots in [4] (Table 3). There are seven types of robots with five criteria including load capacity (*LC*), maximum tip speed (*MS*), repeatability (*RE*), memory capacity (*MC*), and manipulator reach (*MR*). In which, *LC*, *MS*, *MC* and *MR* are criteria of type B (the criterion as large as possible), and *RE* are criteria of type C (the criterion as small as possible).

Table 3. Data of example 1 [4]

Alternatives	Weights of criteria	0.036	0.326	0.192	0.326	0.120
	Robots	*LC (kg)*	*SM (mm/s)*	*RE (mm)*	*MC (steps)*	*MR (mm)*
A1	ASEA-IRB 60/2 Cincinnati	60	0.4	2540	500	990
A2	Milacrone T3-726 Cybotech V15	6.35	0.15	1016	3000	1041
A3	Electric Robot Hitachi America	6.8	0.1	1727.2	1500	1676
A4	Process Robot Unimation PUMA	10	0.2	1000	2000	965
A5	Unimation PUMA 500/600	2.5	0.1	560	500	915
A6	United States Robots Maker 110	4.5	0.08	1016	350	508
A7	Yaskawa Electric Motoman L3C	3	0.1	1778	1000	920

Formula (13) is used to construct the decision matrix, which is the last five columns in Table 3.

In [4], the data were normalized according to the N1 method, and now will be normalized according to the N2, N3, N4, N5 and N6 methods. First, the data were normalized according to the N2 method. The data was normalized according to the Formulas (3) and (4) are used to normalize the data according to the N2 method, the results are presented in Table 4.

Formula (14) is used to calculate the normalized values taking into account the weights of the criteria, the results are also included in Table 4.

The negative-ideal solution (*ns*) has also been determined by formulas (15) and (16), and is also included in the last row of Table 4.

Formulas (17) and (18) are used to calculate E_i and T_i respectively, and the calculated data are also included in Table 4.

Table 4. Some CODAS parameters of example 1 when using N2 method

Alt.	n_{ij}					r_{ij}					E_i	T_i
	LC	*MS*	*RE*	*MC*	*MR*	*LC*	*MS*	*RE*	*MC*	*MR*		
A1	1.000	1.000	0.000	0.057	0.413	0.036	0.326	0.000	0.019	0.050	0.332	0.430
A2	0.067	0.219	0.770	1.000	0.456	0.002	0.071	0.148	0.326	0.055	0.369	0.602
A3	0.075	0.063	0.411	0.434	1.000	0.003	0.020	0.079	0.142	0.120	0.203	0.363
A4	0.130	0.375	0.778	0.623	0.391	0.005	0.122	0.149	0.203	0.047	0.284	0.526
A5	0.000	0.063	1.000	0.057	0.349	0.000	0.020	0.192	0.019	0.042	0.198	0.273
A6	0.035	0.000	0.770	0.000	0.000	0.001	0.000	0.148	0.000	0.000	0.148	0.149
A7	0.009	0.063	0.385	0.245	0.353	0.000	0.020	0.074	0.080	0.042	0.119	0.217
ns						0.000	0.000	0.000	0.000	0.000		

Formulas (19), (20) and (21) are used to build the relative rating matrix. The results are presented in Table 5. Calculate the rating (H_i) for each option according to the formula (22) and the results have also been included in Table 5. The results of the ranking of options have also been performed and are also presented in this Table 5.

Methods N3, N4, N5, and N6 were also applied to normalize the data for example 1, then the ranking of alternatives was also done. The results are presented in Table 6. The results of the ranking of options when normalizing data by the N1 method are also included in this Table 6 [4].

According to the data in Table 6, when using four data normalization methods N2, N3, N4, and N6, the best solution is determined to be option A2. This result also coincides with the ranking when using N1 for data normalization [4].

In contrast, when N5 is the method of data normalization, the best solution is determined to be option A3. This result is not like when using the normalization method N1, N2, N3, N4 and N6. This shows that of the six data normalization methods, N5 is not suitable to combine with CODAS. However, this conclusion is only based on the results of this example. To make the conclusions more reliable, it is necessary to perform a few more examples.

Table 5. Relative rating matrix, H_i values and ratings of alternatives for example 1 when applying N2 method

Alternatives	*A1*	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*	*Hi*	Rank
A1	0	-0.209	0.196	-0.048	0.291	0.465	0.427	1.122	3
A2	0.209	0	0.405	0.161	0.500	0.675	0.636	2.586	1
A3	-0.196	-0.405	0	-0.244	0.095	0.269	0.231	-0.251	4
A4	0.048	-0.161	0.244	0	0.339	0.513	0.475	1.459	2
A5	-0.291	-0.500	-0.095	-0.339	0	0.174	0.136	-0.916	5
A6	-0.465	-0.675	-0.269	-0.513	-0.174	0	-0.039	-2.135	7
A7	-0.427	-0.636	-0.231	-0.475	-0.136	0.039	0	-1.865	6

According to the data in Table 6, when using four data normalization methods N2, N3, N4, and N6, the best solution is determined to be option A2. This result also coincides with the ranking when using N1 for data normalization [4]. In contrast, when N5 is the method of data normalization, the best solution is determined to be option A3. This result is not like when using the normalization method N1, N2, N3, N4 and N6. This shows that of the six data normalization methods, N5 is not suitable to combine with CODAS. However, this conclusion is only based on the results of this example. To make the conclusions more reliable, it is necessary to perform a few more examples.

Table 6. Summarize the ranking of alternatives for example 1 according to six different data normalization methods

Alternatives	Normalization method					
	N1 [4]	*N2*	*N3*	*N4*	*N5*	*N6*
A1	3	3	2	2	7	3
A2	1	1	1	1	5	1
A3	2	4	4	4	1	4
A4	5	2	3	3	6	2
A5	7	5	5	5	4	5
A6	6	7	7	7	2	7
A7	4	6	6	6	3	6

4.2. Example 2

This example also replicates another example of an in-office climate assessment [4]. There are fourteen options with six criteria including the amount of air per head (*TH*), relative air humidity (*RH*), air temperature (*AT*), illumination during work hours (*IH*), rate of air flow (*RF*), and dew points (*DP*). In which, *TH*, *RH*, *AT* and *IH* are criteria of type *B* while *RF* and *DP* are criteria of type *C* (in Table 7).

Table 7. Data of example 2 [4]

Weights of criteria	0.21	0.16	0.26	0.17	0.12	0.08
Alternatives	*TH* (m^3/h)	*RH (percent)*	*AT* (^0C)	*IH (lx)*	*RF (m/s)*	*DP* (^0C)
	7.6	46	18	390	0.1	11
A1	5.5	32	21	360	0.05	11
A2	5.3	32	21	290	0.05	11
A3	5.7	37	19	270	0.05	9
A4	4.2	38	19	240	0.1	8
A5	4.4	38	19	260	0.1	8
A6	3.9	42	16	270	0.1	5
A7	7.9	44	20	400	0.05	6

In [4], the data were normalized according to the N1 method, and now will be normalized according to the N2, N3, N4, N5 and N6 methods. First, the data were normalized according to the N2 method.

The resulting matrix is the last six columns in Table 7 (using formula (13)).

The results of data normalization by the N2 method are presented in Table 8 (using formulas (3) and (4)).

Formula (14) is used to calculate the normalized values taking into account the weighting of the criteria. The results obtained are shown in Table 9.

Table 8. The results of data normalization by the N2 method of example 2

Alternatives	*TH*	*RH*	*AT*	*IH*	*RF*	*DP*
A1	0.881	0.778	0.400	0.938	0.000	0.143
A2	0.381	0.000	1.000	0.750	1.000	0.143
A3	0.333	0.000	1.000	0.313	1.000	0.143
A4	0.429	0.278	0.600	0.188	1.000	0.429
A5	0.071	0.333	0.600	0.000	0.000	0.571
A6	0.119	0.333	0.600	0.125	0.000	0.571
A7	0.000	0.556	0.000	0.188	0.000	1.000
A8	0.952	0.667	0.800	1.000	1.000	0.857
A9	1.000	0.667	0.800	0.875	1.000	0.857
A10	0.143	0.778	0.400	0.500	0.000	0.714
A11	0.429	0.889	0.800	0.500	1.000	0.143
A12	0.310	0.889	0.800	0.438	1.000	0.143
A13	0.762	0.944	0.600	0.250	0.000	0.000
A14	0.714	1.000	0.000	0.063	1.000	0.286

Table 9. Some CODAS parameters of example 2 when applying the N2 method

Alternatives	r_{ij}						*Ei*	*Ti*
	TH	*RH*	*AT*	*IH*	*RF*	*DP*		
A1	0.185	0.124	0.104	0.159	0.000	0.011	0.293	0.584
A2	0.080	0.000	0.260	0.128	0.120	0.011	0.324	0.599
A3	0.070	0.000	0.260	0.053	0.120	0.011	0.300	0.515
A4	0.090	0.044	0.156	0.032	0.120	0.034	0.226	0.477
A5	0.015	0.053	0.156	0.000	0.000	0.046	0.172	0.270
A6	0.025	0.053	0.156	0.021	0.000	0.046	0.174	0.301
A7	0.000	0.089	0.000	0.032	0.000	0.080	0.124	0.201
A8	0.200	0.107	0.208	0.170	0.120	0.069	0.378	0.873
A9	0.210	0.107	0.208	0.149	0.120	0.069	0.374	0.862
A10	0.030	0.124	0.104	0.085	0.000	0.057	0.194	0.401
A11	0.090	0.142	0.208	0.085	0.120	0.011	0.306	0.657
A12	0.065	0.142	0.208	0.074	0.120	0.011	0.296	0.621
A13	0.160	0.151	0.156	0.043	0.000	0.000	0.273	0.510
A14	0.150	0.160	0.000	0.011	0.120	0.023	0.251	0.464
ns	0.000	0.000	0.000	0.000	0.000	0.000		

The *ns* values of the negative-ideal alternative have also been determined by formulas (15) and (16), and are also included in the last row of Table 9.

E_i and T_i values have been calculated and entered in Table 9 (using formulas (17) and (18)).

Formulas (19), (20) and (21) are used to build the relative rating matrix. The obtained results are presented in Table 10. Calculate the evaluation score (H_i) for each option according to the formula (22) and the results obtained are described in Table 10. This Table 10 also shows the results of ranking the options according to the value of H_i.

With the same method, methods N3, N4, N5, and N6 were also applied to normalize the data for example 2. Then the ranking of alternatives was also performed and the results were presented in Table 11. The results of ranking options when normalized data by the N1 method are also included in this Table 11 [4].

Table 10. Relative rating matrix, H_i values and ratings of alternatives for example 2 using the N2 method

Alt.	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	Hi	Rank
A1	0	0.045	0.063	0.175	0.436	0.402	0.553	-0.373	-0.359	0.283	-0.085	-0.040	0.095	0.163	1.359	6
A2	0.045	0	0.108	0.220	0.481	0.447	0.598	-0.328	-0.314	0.328	-0.040	0.005	0.140	0.208	1.899	4
A3	-0.063	-0.108	0	0.112	0.373	0.339	0.490	-0.437	-0.422	0.220	-0.148	-0.103	0.032	0.100	0.383	7
A4	-0.175	-0.220	-0.112	0	0.261	0.227	0.378	-0.549	-0.534	0.108	-0.260	-0.215	-0.080	-0.012	-1.183	10
A5	-0.436	-0.481	-0.373	-0.261	0	-0.034	0.117	-0.809	-0.794	-0.153	-0.520	-0.476	-0.341	-0.273	-4.833	13
A6	-0.402	-0.447	-0.339	-0.227	0.034	0	0.151	-0.775	-0.761	-0.119	-0.487	-0.442	-0.307	-0.239	-4.361	12
A7	-0.553	-0.598	-0.490	-0.378	-0.117	-0.151	0	-0.926	-0.912	-0.270	-0.638	-0.593	-0.458	-0.390	-6.474	14
A8	0.373	0.328	0.437	0.549	0.809	0.775	0.926	0	0.015	0.656	0.289	0.334	0.468	0.536	6.495	1
A9	0.359	0.314	0.422	0.534	0.794	0.761	0.912	-0.015	0	0.641	0.274	0.319	0.453	0.521	6.288	2
A10	-0.283	-0.328	-0.220	-0.108	0.153	0.119	0.270	-0.656	-0.641	0	-0.367	-0.323	-0.188	-0.120	-2.691	11
A11	0.085	0.040	0.148	0.260	0.520	0.487	0.638	-0.289	-0.274	0.367	0	0.045	0.180	0.247	2.453	3
A12	0.040	-0.005	0.103	0.215	0.476	0.442	0.593	-0.334	-0.319	0.323	-0.045	0	0.135	0.203	1.825	5
A13	-0.095	-0.140	-0.032	0.080	0.341	0.307	0.458	-0.468	-0.453	0.188	-0.180	-0.135	0	0.068	-0.060	8
A14	-0.163	-0.208	-0.100	0.012	0.273	0.239	0.390	-0.536	-0.521	0.120	-0.247	-0.203	-0.068	0	-1.011	9

From Table 11 it can be seen that when using four methods of data normalization N2, N3, N4, and N6, the best solution is determined to be A8. This result also coincides with the ranking when using the N1 method [4]. In contrast, when using the N5 method, the best alternative was determined to be option A9, and this result is not the same as when using the normalized methods N1, N2, N3, N4 and N6. This shows that of the six data normalization methods, N5 is not suitable to combine with CODAS. This is the second time that it has been found that N5 is an unsuitable method to combine with CODAS. However, in the two examples that have been implemented, the weights of the criteria are determined by only one method. From this, a question arises as to what the ranking results when using different normalization methods will be if different weighting methods are used. For this reason, another example will be made below.

Table 11. Summarize the ranking results of the options for example 2 according to six data normalization methods

Alternatives	Normalization method					
	N1 [4]	N2	N3	N4	N5	N6
A1	3	6	3	3	3	4
A2	7	4	7	7	7	6
A3	9	7	10	10	10	9
A4	10	10	9	9	9	10
A5	14	13	14	14	13	14
A6	13	12	12	13	12	12
A7	12	14	13	12	14	13
A8	1	1	1	1	2	1
A9	2	2	2	2	1	2
A10	11	11	11	11	11	11
A11	4	3	4	4	6	3
A12	6	5	6	6	8	5
A13	8	8	8	8	4	8
A14	5	9	5	5	5	7

4.2. Example 3

This example uses the test results of a turning process which is shown in Table 12 [29].

In this example, *Ra* is the surface roughness, belonging to the C-type criterion. *MRR* is the material removal rate, belonging to the B-type criterion. The multi-criteria decision making problem aims to choose the best alternative out of sixteen given alternatives that simultaneously ensures minimum *Ra* and maximum *MRR*. In [29], four multi-criteria decision making methods including MAIRCA, EAMR, MARCOS and TOPSIS along with two weighting methods, including

Entropy and MEREC, were used to perform this task.

Table 12. The data of example 3 [29]

Alternatives	*Ra* (μm)	*MRR* (mm^3/s)
A1	0.572	36.255
A2	1.395	98.717
A3	2.704	230.144
A4	2.897	297.531
A5	0.532	68.441
A6	1.166	82.824
A7	2.662	362.046
A8	2.602	299.555
A9	0.542	102.724
A10	1.372	233.083
A11	2.301	163.018
A12	2.502	252.902
A13	0.455	161.855
A14	1.082	235.041
A15	2.221	308.228
A16	2.211	212.522

In this section, the ranking of alternatives with the above purpose will be performed using the CODAS method with both weight determination methods including Entropy and MEREC.

The weights of *Ra* and *MRR* are 0.7174 and 0.2816 respectively when determined by the Entropy method, and equal to 0.7042 and 0.2958 when using the Merec method [29]. First of all, decision making is performed when N1 is used as the data normalization method, and the weights of the criteria have been determined by the Entropy method.

Formula (13) is used to construct the decision matrix, which is the last two columns in Table 12.

Normalized data according to the N1 method are presented in Table 13. The weighted normalized values of the criteria are included in Table 13 (using (14)). The *ns* values have also been included in the last row of Table 13 (formulas (15) and (16)). The E_i and T_i values that have been determined by formula (17) and (18) have also been included in Table 13

Table 13. Several CODAS parameters when applying method N1 in example 3

Alternatives	n_{ij}		r_{ij}		*Ei*	*Ti*
	Ra	*MRR*	*Ra*	*MRR*		
A1	0.019	0.044	0.013	0.012	0.014	0.014
A2	0.008	0.056	0.006	0.016	0.007	0.010
A3	0.000	0.066	0.000	0.019	0.006	0.007
A4	-0.001	0.070	-0.001	0.020	0.007	0.007
A5	0.019	0.052	0.014	0.015	0.015	0.017
A6	0.010	0.054	0.007	0.015	0.008	0.011
A7	0.000	0.072	0.000	0.020	0.008	0.009
A8	0.001	0.070	0.000	0.020	0.007	0.008
A9	0.019	0.057	0.014	0.016	0.015	0.018
A10	0.008	0.067	0.006	0.019	0.009	0.013
A11	0.002	0.062	0.001	0.018	0.006	0.007
A12	0.001	0.068	0.001	0.019	0.007	0.008
A13	0.021	0.062	0.015	0.018	0.017	0.021
A14	0.011	0.067	0.008	0.019	0.011	0.015
A15	0.002	0.070	0.002	0.020	0.008	0.010
A16	0.002	0.065	0.002	0.019	0.007	0.008
ns			-0.0005	0.0124		

Formulas (19), (20) and (21) are used to build the relative rating matrix. The obtained results are presented in Table 14.

The H_i scores for each alternative are shown in Table 14 (using formula (22)). This Table 14 also shows the ranking results of the alternatives.

With the same implementation method, the ranking of options when data normalization was performed according to methods N2, N3, N4, N5 and N6 was performed. The results obtained are presented in Table 15. To facilitate the discussion, the ranking results of the alternatives using the MAIRCA, EAMR, MARCOS and TOPSIS methods are also summarized in this Table 15 [29]. From Table 15, it can be seen that A13 is determined to be the best option when using five data normalization methods including N1, N2, N3, N4 and N6. This result also coincides with the ranking results when using the MAIRCA, EAMR, MARCOS, and TOPSIS methods [29]. Besides, these results are not consistent with the cases where the N5 method is used to normalize the data. This once again reinforces the idea that N5 is not suitable in combination with the CODAS method.

Table 14. Relative rating matrix, H_i values and ratings of the options of example 3 when applying the N1 method

Alt.	*A1*	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*	*A8*	*A9*	*A10*	*A11*	*A12*	*A13*	*A14*	*A15*	*A16*
A1	0	0.615	0.605	0.508	-0.112	0.541	0.397	0.492	-0.118	0.439	0.685	0.559	-0.399	0.327	0.454	0.601
A2	-0.615	0	-0.010	-0.107	-0.727	-0.074	-0.218	-0.124	-0.733	-0.176	0.070	0.003	-1.015	-0.288	-0.161	-0.014
A3	-0.605	0.010	0	-0.097	-0.717	-0.064	-0.208	-0.113	-0.723	-0.166	0.080	-0.046	-1.004	-0.278	-0.151	-0.004
A4	-0.508	0.107	0.097	0	-0.620	0.033	-0.111	-0.016	-0.626	-0.069	0.177	0.051	-0.907	-0.181	-0.054	0.093
A5	0.112	0.988	0.717	0.620	0	0.653	0.509	0.603	-0.006	0.550	0.796	0.671	-0.288	0.439	0.566	0.713
A6	-0.541	0.074	0.064	-0.033	-0.653	0	-0.144	-0.050	-0.659	-0.102	0.144	0.018	-0.941	-0.214	-0.087	0.060
A7	-0.397	0.218	0.208	0.111	-0.509	0.144	0	0.095	-0.515	0.042	0.288	0.162	-0.797	-0.070	0.057	0.204
A8	-0.492	0.385	0.113	0.016	-0.603	0.050	-0.095	0	-0.610	-0.053	0.193	0.067	-0.891	-0.165	-0.037	0.110
A9	0.118	0.994	0.723	0.626	0.006	0.659	0.515	0.610	0	0.557	0.802	0.677	-0.282	0.445	0.572	0.719
A10	-0.685	0.438	0.166	0.069	0.102	0.102	-0.042	0.053	-0.557	0	0.246	0.120	-0.838	-0.112	0.015	0.163
A11	-0.685	0.192	-0.080	-0.177	-0.144	-0.144	-0.288	-0.193	-0.802	-0.246	0	-0.126	-1.084	-0.357	-0.230	-0.083
A12	-0.559	0.056	0.046	-0.051	-0.018	-0.018	-0.162	-0.067	-0.677	-0.120	0.126	0	-0.958	-0.232	-0.105	0.042
A13	0.399	1.015	1.004	0.907	0.941	0.941	0.797	0.891	0.282	0.838	1.084	0.958	0	0.727	0.854	1.001
A14	-0.327	0.288	0.278	0.181	0.214	0.214	0.070	0.165	-0.445	0.112	0.357	0.232	-0.727	0	0.127	0.274
A15	-0.454	0.161	0.151	0.054	0.087	0.087	-0.057	0.037	-0.572	-0.015	0.230	0.105	-0.854	-0.127	0	0.147
A16	-0.601	0.014	0.004	-0.093	-0.060	-0.060	-0.204	-0.110	-0.719	-0.163	0.083	-0.042	-1.001	-0.274	-0.147	0

Table 15. Ranking of alternatives in example 3 when using Entropy method for weight calculation

Alt.	Normalization method						Rank [29]			
	N1	*N2*	*N3*	*N4*	*N5*	*N6*	*MAIRCA*	*EAMR*	*MARCOS*	*TOPSIS*
A1	4	4	4	4	4	4	6	16	4	6
A2	15	8	8	13	9	8	9	15	15	9
A3	14	16	16	16	16	16	16	12	13	15
A4	10	15	15	12	14	15	14	11	10	14
A5	3	3	3	2	3	3	4	9	3	5
A6	12	7	7	9	7	7	8	14	14	7
A7	7	10	10	8	10	11	10	7	6	10
A8	9	12	11	10	11	12	11	8	9	11
A9	2	2	2	3	1	2	3	3	2	3
A10	6	6	6	6	6	6	5	4	8	4
A11	16	13	14	15	15	13	15	13	16	16
A12	11	14	13	11	13	14	13	10	11	13
A13	1	1	1	1	2	1	1	1	1	1
A14	5	5	5	5	5	5	2	2	5	2
A15	8	9	9	7	8	9	7	6	7	8
A16	13	11	12	14	12	10	12	5	12	12

Proceeding in the same way as above, the results of ranking the alternatives when the weights of the criteria are determined by the Merec method are presented in Table 16. The results of ranking the alternatives when using the MAIRCA methods, EAMR, MARCOS and TOPSIS are also summarized in this Table 16 [29]. From the results in Table 16, A13 is also determined to be the best option when using methods N1, N2, N3, N4 and N6. This result also coincides with the ranking results when using the MAIRCA, EAMR, MARCOS, and TOPSIS methods [29]. Using the N5 method to normalize the data again gave different results than using N1, N2, N3, N4 and N6.

In both cases where two different weighting methods are used (Entropy and Merec), A13 is always determined to be the best solution if methods N1, N2, N3, N4 and N6 are used to normalize the data. It has also been found that the A13 is also the best option if the MAIRCA, EAMR, MARCOS, and TOPSIS methods are used [29]. In addition, it shows that the best solution is determined regardless of the weighting method.

Table 16. Ranking of alternatives in example 3 when using MEREC method for weight calculation

Alt.	Normalization method						Rank [29]			
	N1	*N2*	*N3*	*N4*	*N5*	*N6*	*MAIRCA*	*EAMR*	*MARCOS*	*TOPSIS*
A1	4	4	4	4	4	5	5	16	4	5
A2	15	8	8	14	11	10	8	15	14	8
A3	14	16	16	16	16	16	16	12	15	16
A4	10	15	15	12	14	14	15	11	10	14
A5	3	3	3	2	3	3	3	9	3	4
A6	12	7	7	9	8	8	7	14	12	7
A7	6	10	10	8	9	9	10	7	7	10
A8	9	12	11	10	10	11	12	8	9	12
A9	2	2	2	3	1	2	2	3	2	3
A10	7	6	6	6	6	6	6	4	6	6
A11	16	13	14	15	15	15	14	13	16	15
A12	11	14	13	11	13	13	13	10	13	13
A13	1	1	1	1	2	1	1	1	1	1
A14	5	5	5	5	5	4	4	2	5	2
A15	8	9	9	7	7	7	9	6	8	9
A16	13	11	12	13	12	12	11	5	11	11

In the above three examples, the number of options and the number of criteria in each example are not the same, but the best solution is always determined uniformly if methods N1, N2, N3, N4 and N6 are used to data normalization. In contrast, also in all three examples, it is shown that combining N5 with CODAS is not appropriate.

5. CONCLUSIONS

The combination of CODAS method with all six data normalization methods (N1, N2, N3, N4, N5, and N6) that was the first time was performed in this study for multi-criteria decision making. Through the performing the multi-criteria decision making in three examples from three different fields, several conclusions are drawn as follows:

- Methods N1, N2, N3, N4, and N6 were determined to be suitable to combine with CODAS method for multi-criteria decision making. However, it should be noted that the combination of N5 and CODAS should be avoided.

- If there is a case where there exists at least one value of $x_{ij} = 0$, then methods N1 and N4 will not be applicable, or when $max(x_{ij}) = 0$ exists, then option N6 cannot be applied either. In this case, we can completely use N2 and/or N3 methods to combine with CODAS.

- When the CODAS method is used together with the N1, N2, N3, N4 and N6 data normalization methods, the best alternative is determined to be consistent with the use of the MAIRCA, EAMR, MARCOS, MAIRCA, and TOPSIS methods.

- When using methods N1, N2, N3, N4 and N6 to standardize the data, the best solution is determined regardless of the number of options, the number of criteria as well as the weight calculation methods.

- In addition to the six data normalization methods used in this study, to evaluate the combination of CODAS with other data normalization methods such as Jüttler-Körth normalization, Stopp normalization, Nonlinear (Peldschus) normalization further studies need to be performed.

REFERENCES

[1] C. Zopounidis, M. Doumpos, Multiple Criteria Decision Making - Applications in Management and Engineering. *Springer*, 2017. https://doi.org/10.1007/978-3-319-39292-9

[2] D.D. Trung, A combination method for multi-criteria decision making problem in turning process. *Manufacturing review*, 8(26), 2021: 1-17. https://doi.org/10.1051/mfreview/2021024

[3] D.D. Trung, Application of TOPSIS and PIV methods for multi-criteria decision making in hard turning process. *Journal of Machine Engineering*, 21(4), 2021: 57-71. https://doi.org/10.36897/jme/142599

[4] M. Keshavarz Ghorabaee, E.K. Zavadskas, Z. Turskis, J. Antucheviciene, A new combinative distance-based assessment (CODAS) method for multi-criteria decision-

making, *Economic Computation and Economic Cybernetics Studies and Research*, 50(3), 2016: 25-44.

[5] I.A. Badi, A. M. Abdulshahed, A.G. Shetwan, Supplier Selection Using Combinative Distance-Based Assessment (CODAS) Method for Multi-Criteria Decision-Making, *Proceedings of the 1st International Conference on Management, Engineering and Environment (ICMNEE 2017)*, 28 september, 2017, Belgrade, Serbia, pp.395-407. http://dx.doi.org/10.2139/ssrn.3177276

[6] F. Ruvalcaba, A. Azalia, P. Domingueze, Luis, G. Villalba, L.A. Almaraz Duran, Sara, A comparison between MOORA and CODAS methods under Pythagorean Fuzzy Sets. *Revista de Innovación Sistemática*, 3(10), 2019: 9-19. http://dx.doi.org/10.35429/JSI.2019.10.3.9.19

[7] N. Yalcin, N.Y. Pehlivan, Application of the Fuzzy CODAS Method Based on Fuzzy Envelopes for Hesitant Fuzzy Linguistic Term Sets: A Case Study on a Personnel Selection Problem. *Symmetry*, 11(4), 2019: 1-27. https://doi.org/10.3390/sym11040493

[8] M.K. Ghorabaee, M. Amiri, E.K. Zavadskas, R. Hooshmand, J. Antucheviciene, Fuzzy extension of the CODAS method for multi-criteria market segment evaluation. *Journal of Business Economics and Management*, 18(1), 2017: 1-19. https://doi.org/10.3846/16111699.2016.1278559

[9] S. Karagoz, M. Deveci, V. Simic, N. Aydin, U. Bolukbas, A novel intuitionistic fuzzy MCDM-based CODAS approach for locating an authorized dismantling center: a case study of Istanbul. *Waste Management & Research: The Journal for a Sustainable Circular Economy*, 2020: 1-13. https://doi.org/10.1177/0734242X19899729

[10] E.A. Adali, A. Tus, Hospital site selection with distance-based multi-criteria decision-making methods. *Internationla journal of healthcare management*, 14(2), 2021: 534-544. https://doi.org/10.1080/20479700.2019.1674005

[11] I. Badi, A.M. Abdulshahed, A. Shetwan, A case study of supplier selection for a steel making company in Libya by using the combinative distance based assessment (CODAS) model, *Decision Making: Applications in Management and Engineering*, 1(1), 2018: 1-12. https://doi.org/10.31181/dmame180101b

[12] M. Mathew, J. Thomas, Interval valued multi criteria decision making methods for the selection of flexible manufacturing system. *International Journal of Data and Network Science*, 3, 2019: 349–358. https://doi.org/10.5267/j.ijdns.2019.4.001

[13] L. Chen, X. Gou, The application of probabilistic linguistic CODAS method based on new score function in multi-criteria decision-making. *Computational and Applied Mathematics*, 41(11), 2022: 1-25. https://doi.org/10.1007/s40314-021-01568-6

[14] S. Gul, A. Aydogdu, Novel Entropy Measure Definitions and Their Uses in a Modified Combinative Distance-Based Assessment (CODAS) Method Under Picture Fuzzy Environment. *Informatica*, 32(4), 2021: 759–794. https://doi.org/10.15388/21-INFOR458

[15] K. Deveci, R. Cin, A. Kagızman, A modified interval valued intuitionistic fuzzy CODAS method and its application to multi-criteria selection among renewable energy alternatives in Turkey. *Applied Soft Computing Journal*, 96, 2020: 1-18. https://doi.org/10.1016/j.asoc.2020.106660

[16] A.I. Maghsoodi, H. Rasoulipanah, L.M. Lopez, H. Liao, E.K. Zavadskas, Integrating Interval-valued Multi-granular 2-tuple Linguistic BWM-CODAS Approach with Target-based Attributes: Site Selection for a Construction Project. *Computers & Industrial Engineering*, 139, 2019: 1-30. https://doi.org/10.1016/j.cie.2019.106147

[17] E.K. Zavadskas, J. Antucheviciene, P. Chatterjee, Multiple-Criteria Decision-Making (MCDM) Techniques for Business Processes Information Management. *MPDI*, 2019. https://doi.org/10.3390/books978-3-03897-643-1

[18] A. Jahan, K. L. Edwards, A state-of-the-art survey on the influence of normalization techniques in ranking: Improving the materials selection process in engineering design. *Materials & Design*, 65, 2015: 335–342.

https://doi.org/10.1016/j.matdes.2014.09.022

[19] A. Aytekin, Comparative Analysis of the Normalization Techniques in the Context of MCDM Problems, *Decision Making: Applications in Management and Engineering*, 4(2), 2021: 1-27. https://doi.org/10.31181/dmame210402001a

[20] Z. Wen, H. Liao, E.K. Zavadskas, MACONT: Mixed Aggregation by Comprehensive Normalization Technique for Multi-Criteria Analysis, *Informatica*, 31(4), 2020: 857-880. https://doi.org/10.15388/20-INFOR417

[21] N. Vafaei, R.A. Ribeiro, L.M. Camarinha-Matos, Data normalisation techniques in decision making: case study with TOPSIS method. *International Journal of Information and Decision Sciences*, 10(1), 2018: 19-38. https://doi.org/10.1504/IJIDS.2018.090667

[22] N. Ersoy, Selecting the Best Normalization Technique for ROV Method: Towards a Real Life Application, *Gazi University Journal of Science*, 34(2), 2021: 592-609. https://doi.org/10.35378/gujs.767525

[23] P. Chatterjee, S. Chakraborty, Investigating the effect of normalization norms in flexible manufacturing system selection using multi-criteria decision-making methods. *International Journal of Information and Decision Sciences*, 7(3), 2014: 141-150.

[24] E. Mokotoff, E. G. J. Perez, Normalization procedures on multicriteria decision making - An Example on Environmental Problems. *The 12th International Conference on Enterprise Information Systems - Artificial Intelligence and Decision Support Systems (ICEIS 2010)*, Madeira, Portugal, 2010, pp. 206-211. https://doi.org/10.5220/0002896102060211

[25] N. Vafaei, R.A. Ribeiro, L.M. Camarinha-Matos, Normalization Techniques for Multi-Criteria Decision Making: Analytical Hierarchy Process Case Study, *Doctoral Conference on Computing, Electrical and Industrial Systems*, Costa de Caparica, Portugal, 2017: 261-269. https://doi.org/10.1007/978-3-319-31165-4_26

[26] K. Palczewski, W. Sałabun, Influence of various normalization methods in PROMETHEE II: an empirical study on the selection of the airport location. *Procedia Computer Science*, 159, 2019: 2051-2060. https://doi.org/10.1016/j.procs.2019.09.378

[27] T.M. Lakshmi, V.P. Venkatesan, A Comparison of Various Normalization in Techniques for Order Performance by Similarity to Ideal Solution (TOPSIS). *International Journal of Computing Algorithm*, 3(3), 2014: 255-259.

[28] R. Sarraf, M.P. McGuire, Effect of Normalization on TOPSIS and Fuzzy TOPSIS. *2021 Proceedings of the Conference on Information Systems Applied Research*, Washington DC, 2021, pp.1-11.

[29] D.D. Trung, H.X. Thinh, A multi-criteria decision-making in turning process using the MAIRCA, EAMR, MARCOS and TOPSIS methods: A comparative study. *Advances in Production Engineering & Management*, 16(4), 2021: 443-456. https://doi.org/10.14743/apem2021.4.412