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Understanding mathematical abstraction in the formularization of Galileo's law

Abstracts. *Galileo's revolution in science introduced an analytical method to science that typifies the overall modern thinking of extracting, abstracting, and grasping only critical aspects of the target phenomena and focusing on "how", which is a quantitative relationship between variables, instead of "why". For example, to him, the question of 'why does an object fall' is of no significance; instead, only the quantitative relationship between distance from the falling object and time is important. Yet, the most fundamental aspect of his idea is that he introduced a quantified time t. When an object is projected horizontally, the distance travelled at some time in the horizontal direction is summed up as* $d \propto t$*, whereas the distance falling at some time in the vertical direction is summed up as* $d \propto t^2$*. Here, the distance, which is a spatial attribute, is expressed as a function of time, t. That is, time is identified as a homogeneous amount that can be reduced to an algebraic number. It is now possible to calculate the laws of motion of things using functions of time. In this respect, mathematical time was a decisive variable in making mathematisation of physical nature practical. Because, according to atomic theory, vacuum exists between an atom and an object composed of atoms or between objects – ignoring factors that interfere with motion, such as friction – the space for absolute time, which is a mathematical time, can be geometrically defined. In order to justify this mathematical abstraction strategy, thought experiments were conducted rather than laboratory experiments, which at that time were difficult to perform.*

Keywords*: Galileo; quantitative relationship; atomic theory; mathematical abstraction; thought experiment*

Introduction.

There were a number of great practitioners of science in the 17th century, including Galileo, Descartes, Newton and Leibniz. The development of quantum mechanics and relativity in time would be nearly impossible without the crucial role that thought experiments play. In terms of thought experiments, Einstein and Galileo were arguably the most influential thinkers of all time.

In "Dialogue on the Two Chief Systems", Galileo's motion frame is a ship that flattens the sea like glass. Galileo broke Aristotle's centuries-old concept of universe, but his concept of motion was circular rather than linear. The reason is that Galileo insists on continuity with Greek ideas that emphasize circular motion. Galileo was a 16th-century scientist. At the time, theology viewed the ever-continuing linear movement as a creepy phenomenon that was difficult to accept. Galileo, according to Koyre, would have to shift from finite space to infinite space to include the notion of linear motion. Removing Earth from the center of the universe and allowing it to revolve around the sun is obviously different from allowing it to wander in infinite space (Miller, 1996, pp. 128–129).

It is a concise statement of relativity that "all determined places" are lost due to infinite linear motion (inertial motion) and since every point can be the center in an infinite universe, there is no center. For this reason, it was natural for Galileo in the Renaissance to insist on circular inertia, not straight inertia.

The idealization introduced by Galileo played a role in defining the characteristics of modern science (McMulin, 1985). However, when Galileo tried to present a new theory that would contribute to the development of theory of dynamics, the theory had not yet been proven, and it was later verified by an accurate laboratory experiment. It is never surprising that his efforts were focused on thought experiments, analogies, and exemplary metaphors, not on detailed laboratory experiments. A concept begins as a vague idea, but gradually becomes clear as the theory containing it becomes more concrete and accurate (Chalmers, 1999, p. 106). Science does not simply develop by induction or deduction (Oh, 2021), and the creativity of scientists can be demonstrated through abstraction and scientific thought experiments. In other words, a revolution in thought occurs.

Our study aims to explore the strategy that Galileo used in the thought experiment process in which a concept becomes solidified. Focusing on inertia, which is his most important discovery, the following research questions are presented for this research's purpose. Taking focus on inertia, where he made his most notable discovery, we devised the following research questions.

First, preliminarily, establish a possible relationship between thought experiment, abstraction, and idealization.

Second, how did Galileo use abstraction and idealization to convince people of his scientific ideas?

Third, how did these strategies contribute to scientific development?

Thought experiment.

Mach wrote in his 1905 essay, "On Thought Experiments", that "Our thoughts are much more manageable than physical facts. Thought experiments are inexpensive". In his time and today, Mach explained clearly to people what thought experiments are: it is the idealization or abstraction of physical conditions that exist.

Galileo's thought experiment, in particular, is intended to prove an already proposed hypothesis, as other scientists do later. However, Einstein's thought experiment is different, as it was not intended for verifying the hypothesis, but for the insight of discovery. These are the two great thought experiments from 1895 and 1907. The thought experiment in 1895 became a special relativity theory 10 years later through thought experiments. In the early stages, all experiments are thought experiments. In 1905, Mach said, "Our thoughts are much more manageable than physical facts. In other words, thought experiments are inexpensive". Mach described a thought experiment correctly, describing it as "an idealization or abstraction of existing physical conditions". Galileo's thought experiment was intended to prove the hypothesis already proposed. Einstein's thought experiment, however, is not a test of hypothesis, but an insight into discovery (Miller, 1996, p. 371).

Based on the above, thought experiments apparently serve several purposes: first, verifying an already established hypothesis; second, constructing a hypothesis; and third, destroying and constructing an existing theory. Brown contends that Plato's thought experiments are a priori, which is both 'constructive' and 'destructive', and that Galileo's thinking experiments are the very ones that can be destructive as well as constructive (Brown, 1991a, p. 41. Brown, 1991b, p. 124. Oh, 2016).

In the author's view, Galileo aspired to prove the law of inertia by justifying and reconstructing the hypothesis that had already been proven. Einstein's thought experiments on free fall, on the other hand, have the function of dismantling and reconstructing existing theories. However, both engaged in mathematical abstraction at first.

Abstraction and idealization.

Abstraction: To give deep thoughts to a certain aspect of the reality they study, scholars will "abstract" the rest. This does not mean that the rest does not exist. The scholars temporarily abandon the rest of them. It is a form of ontological thinking. Mathematics is a study that firmly secures universal laws existing in this world and systematically organizes rules established between them. For example, it is a study that prepares 'the theorem of geometry', which is a rule of algebra or a law on shapes, and allows someone to get the right answer for any purpose at any time. To achieve such universality, "mathematics" abstract. Numbers are highly abstract representations of what exists in reality. According to Butland Russell, the level of abstraction of the "number concept" was heightened by expressing both a pair and two days as number 2. In a sense, it is an epistemological expression, as it emphasizes only the amount that can be expressed mathematically and omits the rest. The higher the universality, the higher the degree of abstraction (Oh, 2016).

The remarkable achievements of science have been made possible by human abstraction ability and mathematics based on it. Being able to abstract is essential to our survival as we evolve. Individuals who possess excellent qualities will have an advantage in natural selection. The strategy of mathematical abstraction in the early stages of theory formation is an important step that demonstrates human mental science, that is, creativity.

In addition, it is on the basis of a specific socio-cultural and ideological phase that mathematics, which is the first science, emerges from history and that practical abstraction takes place. Likewise, Galileo in the modern era accepted Plato's mathematical ideas, rather than Aristotle's biological ideas, as the socio-cultural and ideological basis.

Idealization: Reality is more than simply abstracted - it is also “idealized”. The concept of straight lines is created by idealizing the tight-pulled thread in our heads. By idealizing the surface of a table or the surface of a pond without wind, the concept of a “plane” was created, but in reality, a “straight line” that has no width or thickness cannot exist. The same is true for horizontal planes with no friction at all. Concepts like these are derived from idealizing anything in the world inside the human mind. Nevertheless, it is still meaningful since it is modeled after reality. It is not arbitrarily imagined, but because it relies on what exists in reality, it would be considered to fall under an ontological realm (Oh, 2016). This “thought experiment” can be best described as “an experiment of what cannot be realized in real life based on thoughts only under idealized conditions”; but in reality, it still takes extreme values by using a strategy that gets smaller and smaller in value.

Though Kuhn (1970) stressed the rebuttal significance of Thinking Experiences (TEs) and some of the current episodes do support this, in other cases, TEs may serve a corroborating function. TEs can also play a role in generating theories. Brown (1991a) identified characteristics associated with the relationship between destructive and constructive thought experiments. As part of this study, we intend to examine Galileo's thought experiments in terms of their evaluative role.

Using Galileo's abstraction and idealization strategically.

Scientists believe Galileo was the first to discover a mathematical law through abstract reasoning and devised a thought experiment or practical experimental device under idealized conditions to justify it (Crease, 2003, p. 86).

Therefore, our study first attempted mathematical abstraction to devise a law, and then, in steps 2 and 3, in order to justify such a law, we used thought experiments aimed at idealization conditions.

Stage 1: Mathematical abstracts.

Scientists start with a big picture of the way the world exists, or a big picture of how the data is revealed, while exploring a theory. These big pictures can be derived from metaphysics, mathematics, theology, and other areas. Metaphysics, therefore, can be incorporated into the very structure of scientific methodology by providing criteria that guide from above (Moreland & Craig, 2003, p. 33).

For example, Plato's metaphysical view that "The universe has a perfect mathematical order. Also, God intervened in such a mathematical order", becomes a guide to modern scientists. Actually, it provides an opportunity to escape from the metaphysical question of "why" and move toward a mathematical and abstract world. As Galileo recognized Aristotle's biological theory has a problem with symmetry, he impliedly stretched the meaning of Plato's knowledge to address the problematic situation.

(Galileo) According to Plato, to a Greek astronomer the phrase "relieve the phenomenon" or "find a mathematical model of celestial bodies that conforms to the Greek ideal of order and beauty" means to describe a phenomenon in terms of a circle - a perfect form of Greek geometry. Plato also said, "God is always geometrizing". He is quoting from the Pythagorean school, as he praises the circle. For the Pythagorean school, which explored the "Principle of Unity in Nature", numbers contain the shape of things and are both real and abstract (Gillispie, 1990, pp. 14–15). Mathematics was not only free from the errors of our sensory senses, but we humans were able to use the mathematical language given to us by God.

<Galileo's kinematic explanation: "how?" instead of "why?" >

While Aristotle focused on the question of "why", Galileo saw that the question of "how", which has been subjected to scientific empirical tests, is more important, asserting that it is meaningless if not verified. As such, the historical paradigm of science has shifted from source to method (Dolnick, 2011).

There are all examples of projectile motion. Which Galileo can describe as taking in two dimensions. Galileo first accurately described projectile motion. He showed that it could be understood by analysis, the horizontal and vertical components of the motion separately. This was an innovative analysis, not done in this way by anyone prior to Galileo (Cohen, 1985, p. 117).

Mathematical abstraction in the vertical direction: Free Failing Motion.

All objects consist of homogeneous materials. <**The World of Quantitative Mathematics**> Atomic theory states that all matter is made of atoms, and if the vacuum between atoms is recognized, i.e., if air resistance is ignored, all objects falling vertically in the air must have the same falling rate and acceleration, regardless of time. <**uniformly accelerated motion**>

In addition, as space is geometricized according to mathematical time, quantified time ***t*** (T in figure) is introduced. For instance, the horizontal distance travelled is summarized as **d (area of a triangle, or the rectangular)** $\propto$ **t** and the distance at which an object falling in the vertical direction falls at a particular time as $\mathbf{d \propto t^2}$**.** <**Mathematical Abstraction**>

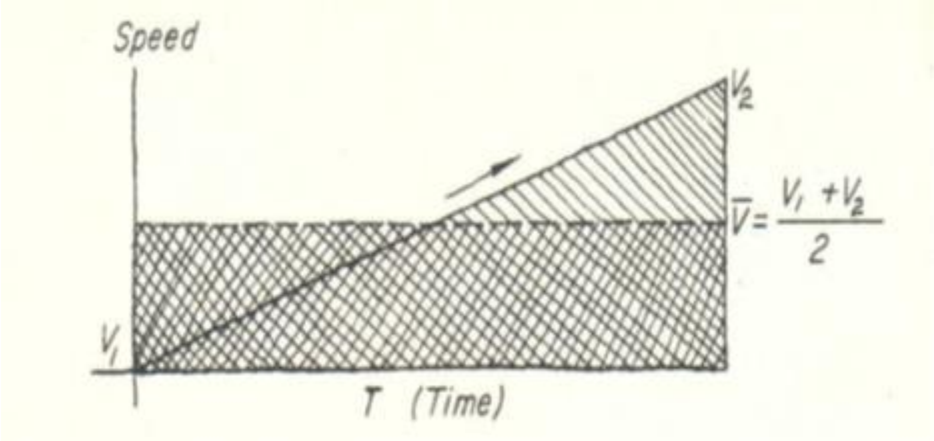

Figure 1. Galileo's Geometry of Space through Time (Cohen, 1985, p. 104).

Mathematics abstraction in horizontal direction: inertial movement.

If vacuum is recognized according to atomic theory, the same speed can be maintained in the celestial sphere as well as on earth, as long as there is no force interfering with motion in the horizontal direction, such as friction. <**circular inertia**>

If, according to simplicity, natural celestial motion is circular (**Plato's hypothesi**s), all matter on earth and celestial sphere are composed of the same material, and all forces that interfere with motion have been removed (**Auxiliary hypothesis or initial condition**), what would happen?

Then, it can be said that objects have a natural property of moving in a constant circular motion, just like in the celestial sphere. In other words, movement precedes stopping in heliocentrism, so according to the atomic theory in which movement is possible in a vacuum, the surrounding environment of an object has become a target that must be removed and ignored.

And, the earth is approximately an inertial system. That is, the nature of an object to move at a constant velocity (circular motion) even on the ground by integrating the celestial sphere and the earth is completely reasonable. Therefore, rotational inertia is a natural motion in the horizontal direction even on the ground.

Although Aristotle limited Plato's ideal world to the celestial sphere in reality, Plato's ideal world was applied even to the earth in the idealized world beyond the observed phenomenon. In this way, the teleological idea of Aristotle's natural tendency was introduced, and hence it cannot be said that it completely deviated from Greek philosophy. Above all, the circular orbit cannot be viewed as radically diverging from Greek teleological ideas, since it illustrates the tendency of celestial bodies to emulate philosophical deities.

Thus, time was abstracted and absolutized on the basis of the entire physical phenomenon. Time has established itself as an independent variable that is vital to the cohesion of the universe and the world.

In the end, Galileo's ideology, which characterized modern science by mathematizing nature and seeking possibility for calculation, transforms space into an algebraic space and sets time as the absolute criteria for all movements over such a space. Mathematics, in this regard, created an objective world in the most original sense from time and space, which are universal forms of the living world. The scale of time

Galileo assumed was a homogeneous and divisible way of measuring time. It was considered that some partial time was no different from others, and that it could be combined and multiplied.

Stage 2: Idealization process (justification of abstraction).

From the **empirical world** of common sense to the **world of idealization** where every power is removed (Fig. 2).

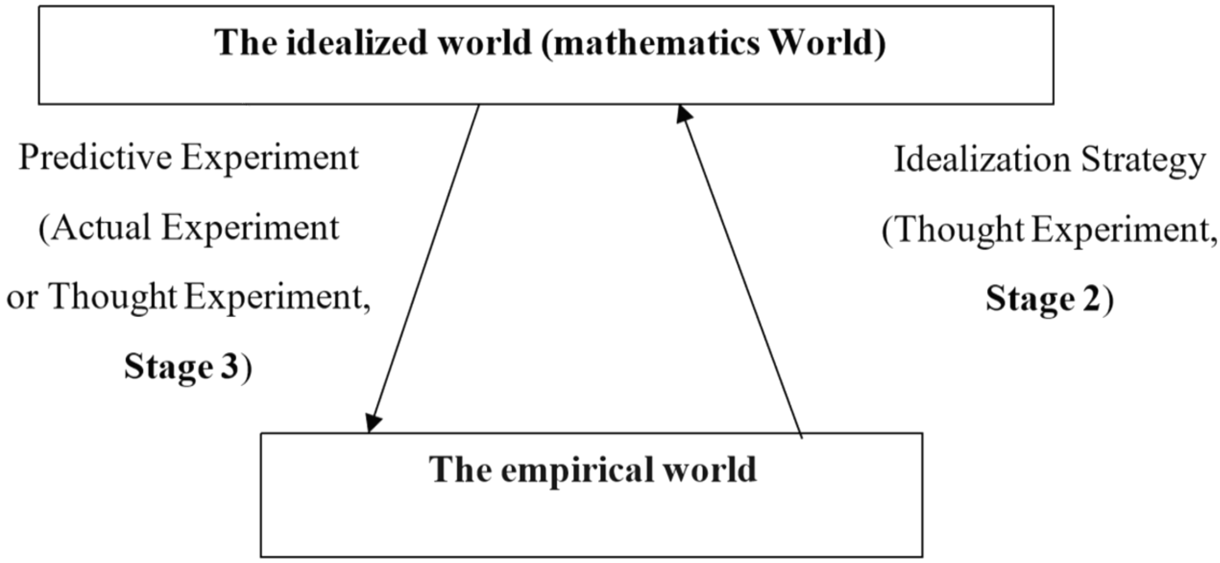

Figure 2. The process of Galileo's scientific inquiry method (Oh, 2016).

<Thought experiment to remove friction from the horizontal plane: Justification of the abstracted hypothesis>

<Justification of New Inertial Hypothesis>

If it is correct that friction force in a plane interferes with the motion of an object (new hypothesis **which is abstracted**), and if the contact surface of the horizontal plane of an object that is freely moving at the same height is smoothed and the friction force interrupting the motion is gradually reduced thereby changing the space over time (**Planned test**), then, can we see what will happen to the motion of that object in the end? (Thought experiment) Perhaps it is reasonable to expect that in this case, the object's motion distance and motion time are getting longer (**$d \propto t$**)? (**a new hypothesis situation**)

And in fact, it is reasonable to imagine that as the object's exercise distance and exercise time become longer and longer, the motion eventually continues. (**Evaluation of a new hypothesis situation)**

So what?

Therefore, it is possible to imagine that all objects have inertia that allows for continuous motion in the idealized horizon where friction forces that interfere with motion in the empirical world are removed. **(The conclusion from the evaluation)**

To support the inertia derived from abstraction, an idealization situation in which friction was removed actually using a hypothesis-deductive method could not be obtained. These thought experiments were used to prove the previously proposed hypothesis (Miller, 1996, p. 371). In addition, Aristotle's teleological and hierarchical laws of physics (assumptions) are destroyed, and a new mathematical law (convergence) in which the celestial sphere and the earth are integrated by reason is constructed. <Platonic Thought Experiment>.

A scenario for an idealized world created by spatial change and scale change over time is being implemented. It is observed that friction force was removed as the spatial change transformed to scale change (Trickett & Trafton, 2007) spatial transformation. Here, the scale change refers to an expansion of space where motion continues as compared to stationary space. In addition, in the evaluation process, scenario results are compared to the problem situation to confirm the comparison result, and the problem situation resolution system mechanism is restated.

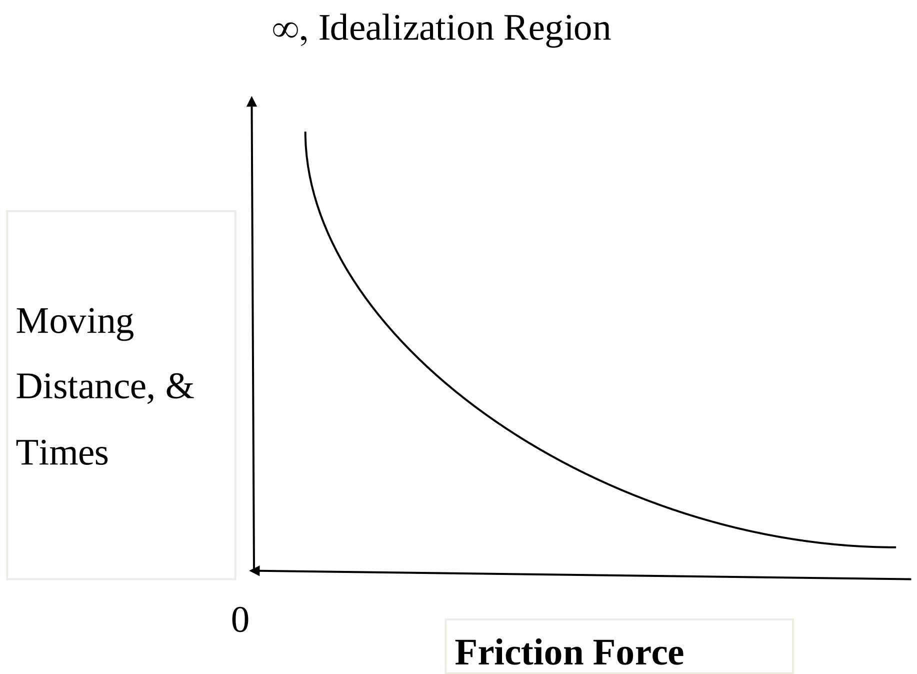

Figure 3. The relationship between the moving distance and friction force during moving of an object.

Figure 3 illustrates how idealization plays a vital role when extrapolating from empirical data obtained through experiments to calculate results for extreme cases. Galileo could infer a phenomenon (objects are in perpetual motion) of an idealized realm, a realm that cannot be obtained through data derived from everyday life or from a narrow range of observation (experiencing how an object in motion will stop farther when friction force is reduced). This is referred to as a limiting case analysis in thought experiments (Nersessian, 1992; Park, Chung, Kwon, & Song, 1998).

However, Galileo did not completely deviate from Aristotle's doctrine, and was circular inertia as a perfect circular motion in the heavens. However, it can be seen that the infinite number of regular polygons is a circle of infinite mathematics.

Stage 3: A predictive experiment of probable real-world phenomena

From **(the idealized world)** to (**the empirical world**)

Norton stated that conclusions reached by the (successful) thought experiment should be proved by actual experiments, not thought experiments themselves (Norton, 1991, p. 131; Norton, 1996, p. 336). In other words, in order to materialize the abstracted theory, an experiment called practice must be accompanied.

Galileo distinguished himself from other philosophers by not only conducting thought experiments, but also by performing real experiments. In fact, the experiments he performed were the beginning of the type of experiment carried out by scientists today. In the meantime, he avoided criticism by claiming that his experiment was unpredictable and that its results could only be approximate.

Experiment in a possible empirical world on a horizontal direction (Fig. 4).

Predictive experiments: If the kinetic component is mathematically broken down and it is observed that only the earth's surface and horizontal components are inertial, independent of the natural tendency to move towards the center of the earth move (mathematically abstracted **hypothesis**), (and) if we drop a heavy object on the mast of a ship that travels at a constant speed when there is no air resistance that interferes with the motion (the **planned experiment** in which space changes over time), (then) the object would fall directly below the mast **(prediction**).

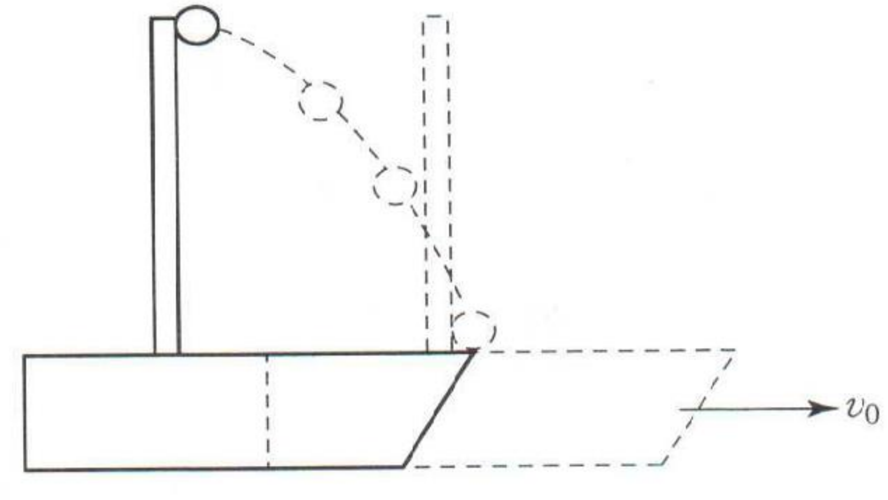

Figure 4. Possible experiments of the law of inertia (Cushing, 1998, p. 106).

Actual experimental results: And, in fact, it falls approximately just below the mast.

(**Therefore**), on a horizontal plane such as the earth's surface, the hypothesis that a moving object continues to move even when a constant force, which is responsible for the movement, is applied is valid.

Actual experiment and thought experiment on vertical free falling motion.

In Galileo's inclined experiment, the fact that the distance rolled down is proportional to the square of time holds regardless of the inclined angles of the surface, so the thought experiment that this proportional relationship holds even in the extreme case, that is, when the inclined angle gradually increases and stands upright is very reasonable. In a letter to his friend Baliani, Galileo describes a method for calculating the acceleration of an object in free fall from an inclined plane experiment (Cohen, 1985, p. 98). In this process, it can be said that the actual experiment and the thought experiment were performed simultaneously (Fig. 5).

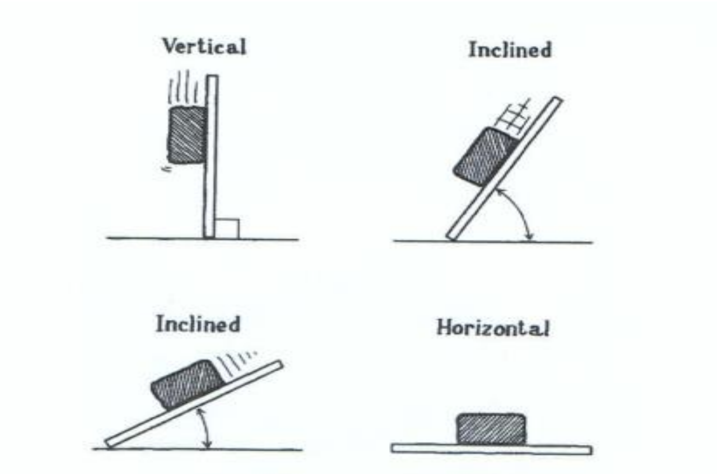

Figure 5. Thought experiment of free fall from an inclined plane (Brown, 2011, p. 3).

It is also the concept of terminal speed from a modern point of view. This thought experiment was already thought of by Galileo (Cohen, 1985, p. 109). If an object falls in dense air, if the object has the same volume and has the same sweeping area, the air resistance increases as the falling speed of the object increases and becomes equal to the weight of the object, the object will have the same speed. Also, if the density of the air is getting smaller and smaller, we have to wait for the terminal velocity to reach a higher falling velocity. After all, without the extreme value of air resistance, the rate of fall of any object will continue to increase, and the increase in velocity will be the same. In other words, taking the limit means that we have progressed to an idealized world and a mathematical world.

Galileo's abstraction strategy.

Aristotle failed to achieve the same level of mechanics as Newton or Galileo through imagination, and it was not because he disregarded facts as the critics claimed in the 17th century. Rather, his desire to stick to the facts too closely is a more pertinent reason. For example, Aristotle did not recognize (abstract) that some aspects of common sense had to be discounted to create a complete theory of motion, and that is why Aristotle’s physics could not be developed into modern science (Gottlieb, 2000, p. 242).

In other words, Aristotle strictly adhered to only common-sense observations. For example, Aristotle, unlike Galileo, did not consider an idealized world like friction-free motion that occurs in a vacuum, as it does according to atomic theory.

Galileo's great achievement was in detecting that in Aristotle's universe, a systematic mathematical discussion of motion theory was impossible. In order to solve this situation, Galileo proposes to separate <first property> from the <second property>. The first property is appropriate as a scientific theory because it can be described mathematically. Examples include speed and acceleration. The second property does not allow for mathematical conversion. Examples of the second properties, which are subjective properties, include special properties such as propensity, scent, and smell. Through this distinction, the first property was used as a material for mathematical theory that can be predicted over time.

Since something "beneficial" for humans is under the control of natural laws, mankind has expanded the scope of accessible objects within the natural world, thanks to the predictive ability of science. Aristotle's physics could not predict the trajectory of an object's motion because it did not contain the appropriate concept of time. However, Galileo abstracted time so that it could also be used in equations dealing with phenomena such as non-periodic falls. He believed that time was mathematically equivalent to space and thus could be used as coordinates. The only way to formulate physics predictably in time and space is in this way (Miller, 1996, pp. 144–145).

If the Earth is at the center of the universe, it consists of heavy soil, water, air, and fire, from bottom to top. Therefore, stones with a lot of soil components must move on their own toward increasing density to reach their target point, the ground.

However, according to the heliocentric theory, such distribution by component becomes meaningless. The cause of Earth's movement is also attributed to universal gravitation. Therefore, in situations where remote forces act, the surrounding environment becomes a collateral situation where there is a vacuum state with no factor hindering motion, so it should be removed or ignored.

Conclusion and suggestion.

Aristotle attempted to teleologically explain why objects on earth fall, while Galileo tried to mathematically describe how they fall. Contrary to Aristotle's argument, Galileo discovered that all objects fall at the same speed regardless of their weight. Methodologically speaking, the way he explained motion did not take into account the 'why' of objects moving. He simply ignored the qualitative nature of objects Aristotle emphasized. Galileo was instead interested in quantities, that is, the mathematical and abstract properties of objects. By excluding the properties of objects such as shapes, colors, and components, Galileo was able to present an abstract mathematical explanation of the behavior of objects (Oh, 2016).

However, with regards to celestial motion, Galileo still maintained a Pythagorean-Platonic position that asserted that nature is written with geometric symbols.

In contrast, Democritus, a descendent of Ionic philosophers, approached differently and established the following principles: every visible thing is made up of small, non-divisible particles called atoms that are moving, especially in an infinite empty space where these particles are in constant motion (Frank, 2011, p. 95).

This was a change in the Ionian concept of theoretical knowledge in the Pythagorean concept. Only the mechanical properties of things were primary. Numbers and geometries were no longer independently inherent in nature. In Newton's mechanics, the formula governing a cosmic mechanical gas is a differential equation, not a numerical rule or a symmetry. Geometry became a theory dealing with empty space, and hermeneutics was merged into geometry by Descartes and separated from realms beyond experience. If Galileo brought abstract time *t* into nature's phenomena, Newton laid the foundation for reducing all motion to time.

Scientists may believe that ideas resulting from abstraction during scientific activities are impressive and revolutionary (Welling, 2007). In the author’s view, the geometricization of space and time through mathematical abstraction that is performed by Galileo, as well as the idealization strategy that justifies the laws subsequently obtained should also be introduced into modern science education.

Moreover, high creativity is not the result of a single action, but the result of utilizing several factors in the discovery process that occur over a long period of time in the history of science (Runco, 2014).

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The authors declare no conflict of interest.

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Республіка Корея

Розуміння математичної абстракції у формулюванні закону Галілея

Анотація. *Революція Галілея в науці ввела в науку аналітичний метод, типовий для загального сучасного мислення, що полягає у добуванні, абстрагуванні та розумінні лише критичних аспектів цільових явищ та зосередженні уваги на тому, "як", що є кількісним зв'язком між змінними, а не на "чому". Наприклад, для нього питання "чому предмет падає" немає значення; натомість важливий лише кількісний зв'язок між відстанню від падаючого об'єкта та часом. Тим не менш, найбільш фундаментальний аспект його ідеї полягає в тому, що він запровадив кількісну оцінку часу t. Коли об'єкт спроектований горизонтально, відстань, пройдена в якийсь момент часу у горизонтальному напрямку, підсумовується як* $d \propto t$*, тоді як відстань, що падає у якийсь момент часу у вертикальному напрямку, сумується як* $d \propto t^2$*. Тут відстань, що є просторовим атрибутом, виражається як функція часу t. Тобто час ідентифікується як однорідна величина, яку можна звести до алгебраїчного числа. Тепер можна визначати закони руху речей, використовуючи функції часу. Щодо цього математичний час був вирішальним фактором у практичній реалізації математизації фізичної природи. Оскільки, згідно з атомною теорією, між атомом і об'єктом, що складається з атомів, або між об'єктами існує вакуум — якщо не брати до уваги фактори, що заважають руху, такі як тертя, — простір для абсолютного часу, який є математичним часом, може бути визначений геометрично . Щоб виправдати цю стратегію математичної абстракції, проводилися уявні, а не лабораторні експерименти, які тоді було важко виконати.*

Ключові слова: *Галілей; кількісні стосунки; атомна теорія; математична абстракція; уявний експеримент*
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