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MATHEMATICAL MODEL AND CONTROL DESIGN OF A FUNCTIONALLY STABLE TECHNOLOGICAL PROCESS

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Abstract. The paper suggests an approach to modeling of industrial enterprises providing production according to the set standard with admissible tolerances and requirements. The mathematical model has the form of a discrete control system. We use the properties of generalized inverse matrices to design the control. We present an algorithm of the control of a production process providing release of production. This approach allows to simulate the technological processes (including metallurgical, chemical, energy, etc.) and gives the operating conditions under the constant influence of internal and external destabilizing factors.

Key words: Functional stability, mathematical model of technological process, control design, generalized invertion.

2010 Mathematics Subject Classification: 93C55, 93C95, 03C45, 03H10.

1. Introduction

Nowadays the problem of improving the efficiency of the production enterprise management is of constant interest to the researchers. The problem is related to the improvement of the operations management and production planning system at the enterprise level. The main purpose of organization of the planning processes is to ensure thorough fulfillment of the production tasks together with maximal utilization of the production resources. This allows timely fulfillment of the obligations to produce outputs by the time they are required at the next production line, guarantees the optimal duration of the production cycle, and leads to the reduction of work in progress and to the minimization of shortages.

It is advisable to use the methods of mathematical modeling and simulation to create a technology of planning in production. Approaches to the mathematical modeling of production processes in an enterprise as a whole and in its individual production centers are underdeveloped. Therefore, it is necessary to create the techniques which allow describing the production processes in the strict mathematical terms. The authors suggest an approach which can be practically integrated into wide classes of the information platforms for manufacturing enterprises: MRP

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II (Manufacturing Resource Planning), ERP (Enterprise Resource Planning), APS (Advanced Planning & Scheduling Systems) and MES (Manufacturing Execution Systems). Given the high level of the automation of production, the introduction of effective tools in the form of production planning system class APS (Advanced Planning & Scheduling Systems) in combination with MES - systems (Manufacturing Execution Systems) provides high-precision production process planning in real time [1, 2].

2. Mathematical Model of the Production Process Management System of an Industrial Enterprise

Considering the problem of designing the control which ensures the execution of the production process in accordance with the established standards the authors have reviewed the mathematical model of the production control system of an industrial enterprise. Having analyzed the problems of the automation of the enterprise management systems the authors intend to suggest an algorithm for the automation of the production process using the technique of genelized inversion [2].

The production process of a modern enterprise consists of a set of measures to produce finished, semi-finished or other products. One of the main tasks of industrial development is the introduction of the new products, machine and equipment designs, automation tools, the latest technologies, etc. Each product industry has its own specifics depending on the type of production, purpose, size and accuracy of the machines, level of production and technical equipment. In general case, automation of the production is a stage of machine production, characterized by the release of the human factor from the direct performance of the management functions in the production processes and the delegation of these functions to the information and computing systems [3]. Control is a purposeful action on the object to ensure its operation in the optimal or specified mode within acceptable tolerances.

Automation of the production processes does not exclude a person completely from the value chain. Automation rather means the most rational distribution of computing and production load at each production center. The proportions of such distribution depends on the specific enterprise and the goals of automation. The enterprise automation processes are subject to certain requirements, without which they become inefficient and difficult to implement.

Firstly, a process management model is a must. At present, significant number of the enterprises operate on the basis of a system-functional approach, which preceded the process approach. The complexity of the transition to the process management model depends on the scale and specifics of an enterprise.

Secondly, compliance of the current model of the enterprise processes with the technical criteria used in their automation is a very important requirement.

Nowadays at modern enterprises it is impossible to organize a serial production of the quality products without automation of the process of control over the parameters of production processes. We suggest a mathematical model for solving the similar problems of the manufacturing plants, to ensure the stability of production processes with real-time control of key production parameters. This model can be integrated into an automated enterprise management system. The property of functional stability of complex technical systems must be realized. It means that the technological process must perform its main technological tasks as intended under the influence of external and internal destabilizing factors [4].

The automation stage allows re-engineering of the processes. The purpose of reengineering is to find and overcome the bottlenecks in the enterprise. It is necessary to monitor the production potential, to identify the opportunities for expansion of the production system of the enterprise, the resources that are not used rationally, and so on. It is important to prepare the companies for the re-engineering process. It is necessary to bring the structures of its processes in the most efficient configuration and in the most efficient form. It is necessary to ensure strict compliance with the requirements for the technological process at each production center in accordance with the standards, with the deviations only within the permissible tolerance standards.

In practice, the principal characteristics of integrated automated enterprise management systems are widely implemented for this purpose. These systems automate a wide range of the management functions, including the tasks of strategic, production and financial planning, operational management of supply, procurement, and inventory. In addition, they automate the tasks of design, technological and technical preparation of production, etc.

Automation of the modern technological processes leads to the creation of the complex dynamic models. The behavior of such models has a fractal structure [5]. Many processes are described by nonlinear dynamical systems of complex structure that have global attractors [6–8]. At the same time problems of control arise in such systems [9–13].

Producing usually consists of a number of stages, at each of which there are certain requirements for the parameters and characteristics of the raw materials, semifinished and finished products. Denote by $x(i)$ the vector of parameters at i -th stage, $i = 1, 2, \dots N$ (2.1). At each stage there is an external influence $u(i)$ on the production process to obtain the desired parameters (work effect, energy effect, chemical or other technological influences at each stage). It is clear that the final quality of the product, as well as the intermediate quantity at each stage, depends on the strict adherence to the technology and ensuring the endurance of the necessary parameters at each previous step. We assume that this requirement holds.

Also we denote by $A(i)$ a matrix of dependence of product quality indicators at $i+1$-st stage on the indicators at i-th stage, and by $C(i)$ a matrix that determines the structure of influence on the production process $u(i)$. Then the mathematical model of the technological process can be written as follows

$$x\,(t+1) = A\,(t)\,x\,(t) + C\,(t)\,u\,(t)\,,\ t = 0, 1, \dots, N-1, \tag{2.1}$$

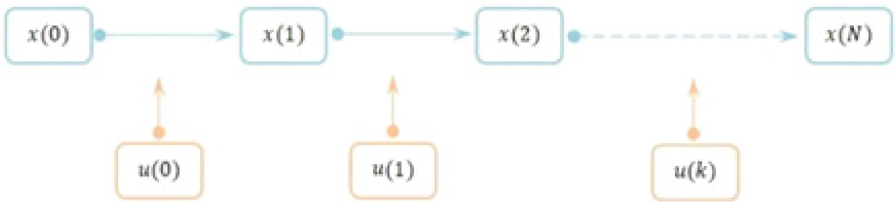

Fig. 2.1. Topology of linear technological production process

$$x(t) \in \mathbb{R}^n,\ A(t) \in \mathbb{R}^{n\times n},\ C(t) \in \mathbb{R}^{n\times m},\ u(t) \in \mathbb{R}^m.$$

Here we denote by $\mathbb{R}^n$ the n-dimensional Euclidean space with the Euclidean norm $\|\cdot\|$ in it, $x = (x_1, x_2, \ldots, x_n)^T$ is a state vector of system (2.1), $u = (u_1, \ldots, u_m)^T$ is a control vector, $A(t)$ is an $n \times n$ matrix, $C(t)$ is an $n \times m$ matrix, $t = 0, 1, \ldots, N-1$. Let $I_N = \{0, 1, \ldots, N\}$, $x(t, x_0, u)$ be a solution of system (2.1), $t \in I_N$ under the control $u(t)$, $t \in I_{N-1}$.

We admit that there is an accurately defined set of certain works and a number of criteria in order to fulfill during realization of process. It means that we know characteristics of the process at the initial stage, requirements for products at the end of the process, and intermediate characteristics of products at control points at stages of this process. At the same time, in the automation of such processes in practice it is necessary to set control tasks describing the design conditions for the control function u providing controlled purposeful execution of the process. In addition, it is advisable to provide conditions for practical stability for these processes [13–18].

3. Control Design

The main problem we analyze consists of finding the control function providing execution of the process, so that the result of the process ensures ultimately in $x(N)$ products that meet all the quality characteristics required by current standards for it.

The purpose of designing a control function u is to ensure that the process is performed in such a way that we end up with a product that meets all the characteristics required by the standards. If at the end of the process the product has deviations from the specified standard parameters, then such deviations are guaranteed to fall into the set of permissible tolerances, which are defined by current standards for such products [2]. This means that there is a desired final state $x_N \in \mathbb{R}^n$ and a positive parameter $\varepsilon > 0$ such that

$$\|x(N) - x_N\| < \varepsilon.$$

Let us define a set of admissible controls. To do this, we consider space $\ell_2^{(m)}$ of sequences of vectors from $\mathbb{R}^m$ such that if $u \in \ell_2^{(m)}$ then $\sum_{t=0}^{\infty} \| u(t) \|^2 < \infty$,

$u(t) \in \mathbb{R}^m$, $t = 0, 1, 2, \ldots$. $\ell_2^{(m)}$ is a real Hilbert space with inner product

$$\langle u, v \rangle_{\ell_2} = \sum_{t=0}^{\infty} \langle u(t), v(t) \rangle, \ u, v \in \ell_2^{(m)}$$

and norm

$$\| u \|_{\ell_2} = \sqrt{\sum_{t=0}^{\infty} \|u(t)\|^2} < \infty.$$

We assume that a control function u is admissible if $u \in \ell_2^{(m)}$ and $u(t) = 0$, $t = N+1, N+2, \ldots$.

Denote $\Theta(t) = A_{t-1}A_{t-2}\ldots A_1A_0$, $\Theta(t,s) = A_{t-1}A_{t-2}\ldots A_s$, $\Theta(t,t) = E$, where E is the identity matrix, $t, s \in I_N$. The problem is to find a control that moves the system (2.1) from the initial state $x(0) = x_0$ to the nearest point $x(N)$ to a given state x_N [12]. This way we get the problem of minimization

$$I(u) = \|x(N, x_0, u) - x_N\| \tag{3.1}$$

on the solutions of system (2.1) with the initial condition $x(0) = x_0$. Here $x(N, x_0, u)$ denotes the value of the solution of system (2.1) with the initial condition $x(0) = x_0$ for an admissible control $u \in \ell_2^{(m)}$ at the moment $t = N$. Since

$$x(N, x_0, u) = \Theta(N)x_0 + \sum_{k=0}^{N-1} \Theta(N,k)C(k)u(k),$$

then the substitution of the last equality in (3.1) gives

$$\begin{aligned} I(u) &= \|x(N, x_0, u) - x_N\| = \|\Theta(N)x_0 + \sum_{k=0}^{N-1} \Theta(N,k)C(k)u(k) - x_N\| \\ &= \|\sum_{k=0}^{N-1} W(k)u(k) - c\|, \end{aligned} \tag{3.2}$$

where $c = x_N - \Theta(N)x_0$,

$$W(t) = \Theta(N,t)C(t), \ W^T(t) = (w_1(t) \ w_2(t) \ \ldots \ w_n(t)),$$

$w_j(t) \in \mathbb{R}^m$, $t \in I_{N-1}$, $j = 1, 2, \ldots, n$ are vectors describing the matrix $W(t)$ rows, $t \in I_{N-1}$. At the same time, one can observe that

$$H = \{w_1(\cdot), w_2(\cdot), \ldots, w_n(\cdot)\} \subset \ell_2^{(m)}$$

and $w_j(t) = 0$, $t = N, N+1, \ldots$, $j = 1, 2, \ldots, n$.

We define a linear manifold $L = Lin\ H$. Since $\ell_2^{(m)}$ is Hilbert space then $\ell_2^{(m)}$ decomposes into a direct sum

$$\ell_2^{(m)} = L \oplus L^{\perp},$$

where $L^{\perp}$ is an orthogonal complement to L. Any control $u \in \ell_2^{(m)}$ can be represented as follows

$$u(t) = u_0(t) + v(t), \ t = 0, 1, \ldots. \tag{3.3}$$

Here $u_0 \in L$, $v \in L^{\perp}$. Therefore

$$\langle u_0, v \rangle_{\ell_2} = 0$$

for $u_0 \in L$. Since $u_0 \in Lin\ H$ there exists a vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$ such that

$$u_0(t) = \lambda_1 w_1(t) + \lambda_2 w_2(t) + \lambda_n w_n(t) = W^T(t)\lambda \in Im(W^T(t)).$$

Here $Im(\cdot)$ denotes an image of a linear operator. From (3.3) we have

$$u(t) = W^T(t)\lambda + v(t), \quad t = 0, 1, \ldots. \tag{3.4}$$

Observe that $\sum_{k=0}^{N-1} W(k)\, v(k) = 0$ since $v \in L^{\perp}$. Substituting (3.4) in (3.2) transforms problem (3.1) into

$$J(\lambda) = \| \sum_{k=0}^{N-1} W(k)\, W^T(k)\, \lambda - c \| \to \min_{\lambda \in \mathbb{R}^n}, \tag{3.5}$$

$c = x_N - \Theta(N)x_0$. We see that $v \in L^{\perp}$ does not affect the solution (3.1), it plays the role of an invariant and it can be considered zero. Using the properties of generalized inversion [19] we obtain the solution of problem (3.5)

$$\widehat{\lambda} = \Phi^{+}(N)c + z, \tag{3.6}$$

where $\Phi(N) = \sum_{k=0}^{N-1} W(k)\, W^T(k)$, $\Phi^{+}(N)$ is a generalized inverse matrix to the matrix $\Phi(N)$, $z \in Ker(\Phi(N))$ is an arbitrary vector [19].

Since $Ker(\Phi(N)) = Z_N\mathbb{R}^n$, $Z_N = Z(\Phi(N)) = E - \Phi^{+}(N)\Phi(N)$ is the projection operator onto $Ker(\Phi(N))$, then (3.6) has the following representation

$$\widehat{\lambda} = \Phi^{+}(N)c + Z_N p, \tag{3.7}$$

where $p \in \mathbb{R}^n$, $c = x_N - \Theta(N)x_0$ [19]. Formula (3.7) describes the set of all solutions of problem (3.4). Note that among the vectors that solve the problem (3.5), the vector

$$\widehat{\lambda} = \Phi^{+}(N)c$$

has the smallest norm. This follows from the properties of generalized inverse matrices. Substituting (3.7) in (3.4) at $v(t) = 0$, $t \in I_{N-1}$ gives

$$u(t) = W^T(t)\, \Phi^{+}(N)\, (x_N - \Theta(N)x_0) + W^T(t)\, Z_N p. \tag{3.8}$$

$p \in \mathbb{R}^n$, $t \in I_{N-1}$. Formula (3.8) solves problem (3.1). If $p = 0$ then

$$u(t) = W^T(t)\, \Phi^{+}(N)\, (x_N - \Theta(N)x_0), \ t \in I_{N-1}.$$

Substituting (3.8) in (3.5) we obtain

$$I(u) = J(\widehat{\lambda}) = \|\Phi(N)\widehat{\lambda} - c\| = \|\Phi(N)\Phi^{+}(N)c + \Phi(N)Z_N p - c\|.$$

Since $Z_N p \in Ker(\Phi\,(N)$ then $\Phi(N)Z_N p = 0$. Therefore

$$I(u) = \|\Phi(N)\Phi^{+}(N)c - c\| = \|(I - \Phi(N)\Phi^{+}(N))c\|.$$

Since $Y(\Phi(N)) = \Phi(N)\Phi^{+}(N)$ is a projector onto the image $Im(\Phi(N))$ of the matrix $\Phi(N)$, $Z(\Phi^T(N)) = E - \Phi(N)\Phi^{+}(N)$ is a projector onto the kernel $Ker\left(\Phi^T\,(N)\right)$ of the matrix $\Phi^T(N)$ [19], then

$$I(u) = \|Y(\Phi(N))c - c\| = \|Z(\Phi^T(N))c\|. \tag{3.9}$$

Formula (3.9) shows how accurately we can move system (2.1) from the point $x\,(0) = x_0$ to the state $x\,(N) = x_N$. From (3.9) it follows, that $I(u) = 0$ if and only if $(\Phi(N))c = c$ or $Z(\Phi^T(N)) = 0$. Thus the following statement is true.

Theorem 3.1. *The control function*

$$u\,(t) = W^T\,(t)\,\Phi^{+}(N)\,(x_N - \Theta(N)x_0) + W^T\,(t)\,Z_N p \tag{3.10}$$

moves system (2.1) *from the initial state* $x(0) = x_0$ *to the nearest point* $x(N)$ *to a given state* x_N*. Here* $p \in \mathbb{R}^n$*,* $t \in I_{N-1}$*,* $Z_N = Z\,(\Phi(N)) = E - \Phi^{+}\,(N)\,\Phi\,(N)$*. Moreover*

$$\|x(N) - x_N\| = \|Y(\Phi(N))c - c\| = \|Z(\Phi^T(N))c\|$$

describes Euclidean distance from $x\,(N)$ *to* x_N*, where* $c = x_N - \Theta(N)x_0$*. If* $Y(\Phi(N))c = c$ *so that* $c = x_N - \Theta(N)x_0$ *belongs to the image* $Im(\Phi(N))$ *of the matrix* $\Phi(N)$*, then* (3.10) *moves system* (2.1) *from* $x(0) = x_0$ *to the point* $x\,(N) = x_N$*.*

Note that control function (3.10) solves the problem for arbitrary x_0, x_N if and only if $Z(\Phi^T(N)) = 0$ or in equivalent form $\Phi(N)\Phi^{+}(N) = E$. Since the matrix $\Phi(N)$ is symmetric of $n \times n$, this means that $\Phi^{+}(N) = \Phi^{-1}(N)$.

Theorem 3.2. *Among the control functions that moves system* (2.1) *from* $x\,(0) = x_0$ *to the nearest state* $x\,(N)$ *to the point* x_N*, the function*

$$u_*\,(t) = W^T\,(t)\,\Phi^{+}(N)\,(x_N - \Theta(N)x_0)\,,\ t \in I_{N-1} \tag{3.11}$$

has the smallest norm in $\ell_2^{(m)}$*.*

Proof. From the proof of theorem (3.1) it follows that the admissible control u moving system (2.1) from $x\,(0) = x_0$ to the nearest point to x_N $x\,(N)$ satisfies (3.3), where $u_0 = u_* + z_0 \in L$, u_* is determined by (3.11), $z_0(t) = W^T\,(t)\,Z_N p$, $p \in \mathbb{R}^n$, $t \in I_{N-1}$, $v \in L^{\perp}$. Then

$$\begin{aligned}\|u\|_{\ell_2}^2 &= \langle u, u\rangle_{\ell_2} = \langle u_0 + v_0, u_0 + v_0\rangle_{\ell_2}\\ &= \langle u_0, u_0\rangle_{\ell_2} + \langle v_0, v_0\rangle_{\ell_2} + 2\langle u_0, v_0\rangle_{\ell_2} = \langle u_0, u_0\rangle_{\ell_2} + \langle v_0, v_0\rangle_{\ell_2}\\ &\geqslant \langle u_0, u_0\rangle_{\ell_2} = \langle u_* + z_0, u_* + z_0\rangle_{\ell_2} = \langle u_*, u_*\rangle_{\ell_2} + \langle z_0, z_0\rangle_{\ell_2} + 2\langle u_*, z_0\rangle_{\ell_2}.\end{aligned}$$

Since $Z_N p \in Ker\Phi(N)$ we have $\Phi(N)Z_N p = 0$ and

$$\begin{aligned}\langle u_*, z_0\rangle_{\ell_2} &= \sum_{t=0}^{N-1} \left\langle W^T(t)\,\Phi^+(N)\,(x_N - \Theta(N)x_0)\,, W^T(t)\,Z_N p\right\rangle \\ &= \left\langle \Phi^+(N)\,(x_N - \Theta(N)x_0)\,, \sum_{t=0}^{N-1} W(t)\,W^T(t)\,Z_N p\right\rangle \\ &= \left\langle \Phi^+(N)\,(x_N - \Theta(N)x_0)\,, \Phi(N)Z_N p\right\rangle = 0.\end{aligned}$$

Finally, we obtain

$$\|u\|_{\ell_2}^2 \geqslant \langle u_*, u_*\rangle_{\ell_2} + \langle z_0, z_0\rangle_{\ell_2} \geqslant \langle u_*, u_*\rangle_{\ell_2} = \|u_*\|_{\ell_2}^2.$$

The last inequality proves the theorem. □

4. Algorithm of Control of Production Process

We offer an algorithm of control design of the production process, which ensures the production according to a standard in compliance with the permissible standards of tolerances at a production plant.

Step 1. Given the initial state $x(0) = x_0$ and the final state $x(N) = x_N$, the parameter $\varepsilon > 0$ determining the set of possible deviations (tolerances) for the product from the requirements of the standard, the matrices $A(t)$, $C(t)$, $t = 0, 1, \ldots N-1$.

Step 2. Find the matrices

$$\Theta(N) = A_{N-1}A_{N-2}\ldots A_1 A_0,\ \Theta(N,t) = A_{N-1}A_{N-2}\ldots A_t,$$
$$W(t) = \Theta(N,t)\,C(t),\ t = 0, 1, \ldots N-1.$$

Step 3. Find the matrix $\Phi(N) = \sum_{k=0}^{N-1} W(k)\,W^T(k)$.

Step 4. Find the generalized inverse matrix $\Phi^+(N)$.

Step 5. Find the control function

$$u(t) = K(t)\,(x_N - \Theta(N)x_0)\,,$$

where $K(t) = W^T(t)\,\Phi^+(N)$ for all $t = 0, 1, \ldots N-1$.

Step 6. Find the matrix $Z_N = E - \Phi^+(N)\Phi(N)$. If the condition

$$\|Z_N\,(x_N - \Theta(N)x_0)\| < \varepsilon$$

is true then the control $u(t)$ solves the problem with specified tolerances

$$\|x(N) - x_N\| < \varepsilon.$$

Otherwise the control $u(t)$ ensuring the producing of products with given tolerances does not exist. End of the algorithm description.

5. Conclusion

In this paper we have analyzed the modern approaches to the automation of production process management in the industrial enterprises, we have solved the problem of designing of a control function that ensures the implementation of the production process. As a result of the process, we get a finished product that meets all the characteristics required by the current standards. We propose an algorithm for automating the atomic process of production.

The research results are important for the design, modernization and integration of the enterprise information systems into one generalized enterprise information system. This will ensure their high efficiency in operation. The lack of such solutions in our country and abroad makes research results a priority.

We see the prospects for further research in the design and improvement of the models and methods for constructing functionally stable technological processes that are integrated into the information system of the enterprise. This approach ensures the efficiency of the information infrastructure during the time required to perform the technological processes and sustainable operation of the enterprise as a whole. In doing so, we will take into account the specific needs of the enterprises operating in the sectors with continuous production cycle, such as metallurgy, energy, chemical industry and so on.

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