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A Note on Discrete Fourier Transform

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Abstract: This paper contains basic concept of DFT, The Inverse Discrete Fourier Transform, Computation of DFT directly via the FFT, DFT (or FFT) for spectral analysis, FFT of the sum of two sine-waves, Verification of Zero-padding, The even symmetry by Extension.

Keywords: Fourier Transform, Zero-padding, Spectral Analysis.

1. Introduction

The sequence $\{x[r]\}$ of discrete time Fourier transform (DTFT) is:

$$X\left(e^{j\Omega}\sum_{r=-\infty}^{\infty} x\left[r\right]e^{-jr\Omega}\right)$$

as $\Omega = \omega\ T$ radians/sample.

By sampling with suitable limited band $\{x[r]\}$ is obtained by a signal $x_a\left(j\omega\right)$, i.e.,

$$x\left(e^{j\Omega}\right) = \left(\frac{1}{T}\right)x_a\left(j\omega\right) \quad for\ \pi < \Omega = \omega T < \pi$$

By computing the DTFT means $x_a(j\omega)$ with integral. There are 2 practical variations:

1. Infinite range of summation.

2. Continuation in $X(e^{j\Omega})$ of Ω function.

The main variation in the 'windowing' / finite block of restricting the sequence $\{x[r]\}$ with non-zero samples; i.e., for n is o to $R-1$. The sequence resulted: $\{\ldots, 0, \underline{x[0]}, x[1], \ldots, x[R-1]\}$ denoted by $\{x[r]\}_{0,R-1}$. The 2^{nd} variation is 'frequency-domain sampling' should be used.

By inverse DTFT formula it becomes normal by $X(e^{j\Omega})$ for Ω from the range $-\pi$ to π. So that in the real signals it can be considered as range $0 \leq \Omega \leq \pi$ from the value of $X(e^{j\Omega})$.

Many applications we need to form the DTFT complex signals and in other ways $X(e^{j\Omega})$ for Ω from range $-\pi$ to π. While the $X(e^{j\Omega})$ repeating at 2π intervals of frequency, similar values we obtained if $X(e^{j\Omega}) = X(e^{j[\Omega+2\pi]})$ so the range from $-\pi$ to o is repeated for Ω from sequence π to 2π.

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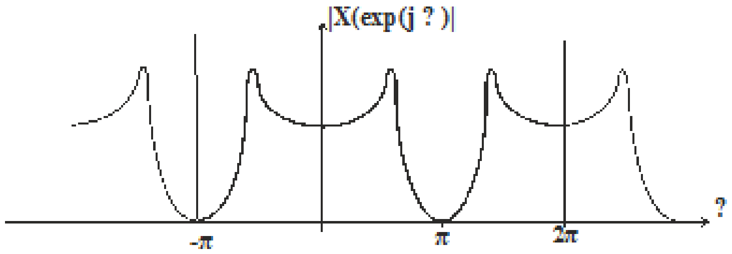
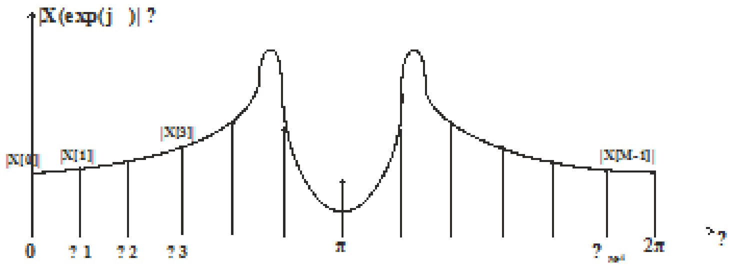


Dividing the above into M equal frequency-domains in the range $0 \leq \Omega \leq 2\pi$ numbers:

$$\left\{x\left(e^{j\Omega_k}\right)\right\}_{0,M-1} = \left\{X\left(e^{j\Omega_0}\right), X\left(e^{j\Omega_1}\right), \ldots, X\left(e^{j\Omega_{M-1}}\right)\right\}$$

Therefore $\Omega_k = 2\pi k/M$ for $k = 0, 1, \ldots, M-1$.



The DTFT produces the windowing and frequency-domain samples for the equation given:

$$x\left(e^{j\Omega_k}\right) = \sum_{r=0}^{R-1} x\left[r\right] e^{-j\Omega_k n} - j\Omega_{k^n} \quad where \quad \Omega_k = 2\pi k \ / \ M$$

This can be evaluated as $k = 0, 1, 2 \ldots, M-1$, such that Ω_k becomes graphs. To obtain unambiguous representation of quick signals, it shown as R frequency-domain samples. since, the case, an reverse transform exists to cover the original time-domain signals from the R DTFT signals. By compute R frequency so main samples are help to draw graphs. Due to this, it may reduce the computing process. From the Equation 3 the Discrete Fourier Transform (DFT) of the sequence $\{x[n])_{0,R-1}\}$ can be obtained by $M = R$.

2. The Inverse Discrete Fourier Transform (DFT)

Introducing notation: $X\left(e^{j\Omega_k}\right) = X\left[k\right]$, therefore DFT can be defined as:

$$\{x\left[r\right]\}_{0,R-1} \rightarrow \{x\left[k\right]\}_{0,R-1}$$

by

$$X\left[k\right] = \sum_{r=0}^{R-1} x\left[r\right] e^{-j\Omega_k r} \quad since \quad \Omega_k = 2\pi k/R$$

where as $k = 0, 1, 2, \ldots, R-1$. It is normal to consider $\{x[r]\}_{0,R-1}$ because the complex sequence though practice the imaginary part of samples are often is set to zero. Therefore inverse DFT is

$$\{X[k]\}_{0,R-1} \rightarrow \{x[r]\}_{0,R-1},$$

May be performed using the formula:

$$X[r] = \left(\frac{1}{R}\right) \sum_{k=0}^{R-1} X\left[k\right] e^{jr\omega_k}$$

Let $r = 0, 1, 2, \ldots, R-1$ through $\Omega_k = 2\pi k/R$.

3. Computation of DFT Directly via the FFT

The comparison among equations exploited through computer programs capable to perform DFT or the inverse by means of fundamentally similar code. These types of programs implement equations to direct manner shown in the above program be little slow, however we try to make it speed up using Fast Fourier Transform technique. So this is fairly motivating to learn how to the improvement of speed achieve through fast Fourier transform technique this is implement by DFT. so might be present and found most of DSP books, other than it is outside of the course outline in this section. so we are trying to implement the concept of DFT also understand the results and in how the programmed proficiently. To additional demonstrate straight DFT program through the MATLAB version is existing or presented in the following Table 3. The table tries to use to find capability in MATLAB to manipulate and define complex information, other than it is more complicated to some extent the fact that every MATLAB bunch of array it start by means of file 1. After that as an alternative of storing samples as in x[o] array content $x(0)$ as might desire therefore it store this value in $x[1]$. The above direct DFT MATLAB program takes of academic curiosity only MATLAB to provide the resourceful FFT procedure about the signal processing tool box. The concept of MATLAB program which study the music from file and split up into 512 samples and it make performs of the DFT scrutiny on every section.

3.1. DFT (or FFT) for spectral analysis

By data $\{x[r]\}_{0,R-1}$ the DFT or FFT gives $\{X[k]\}_{0,R-1}$ through R complex spectral sample from 0 to *fs*. Where *fs* sampling frequency while $\{x[r]\}$ real, hence we require only to plot magnitudes of $X[k]$ for $k = 0$ to $R/2$. When $R = 512$, $k = R/2 = 256$ corresponds to $fs/2$.

In natural to describe signals blocks preliminary from $r = 0$ unfortunately, arrays of MATLAB cannot begin from zero. Condition the signal $\{x[r]\}_{(0,R-1}$ may wish to store this in array of the MATLAB say x or like better to adopt a dissimilar names like as 'array for x'. In our convenience shorter names is the better choice. The above notes often take the similar names for the sake of array and signal. In better to avoid the be short of MATLAB array constituent through index is zero, then store $x[0]$ at $x(1), x[1]$ at $x(2)\ldots$ generally we store $x[r]$ as in $x(1 + r)$. Likewise $\{X[k]\}_{(0,R-1)}$ stored in $X(1)\ldots X(R)$. Now we demonstrate by the use of the DFT or FFT to spectrally examine segments of signal containing cosine waves.

3.2. FFT of the sum of two sine-waves

Let us begin with analyze $\{x[r]\}_{(0,63)} = \{100 * \cos(\pi/4)r) + 100 * \cos((\pi/2)r)\}_{(0,63)}$ through the FFT in of order of $R = 64$.

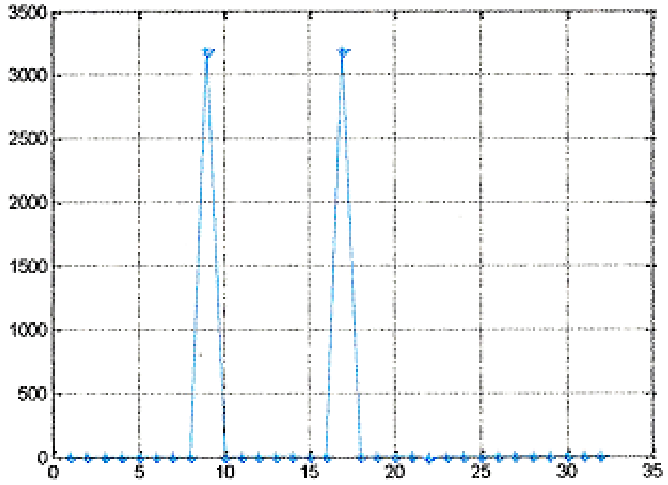
For $r = 0 : 63 \quad X(1 + r) = 100 * \cos(0.25 * pi * r) + 100 * \cos(0.5 * pi * r);$

End;

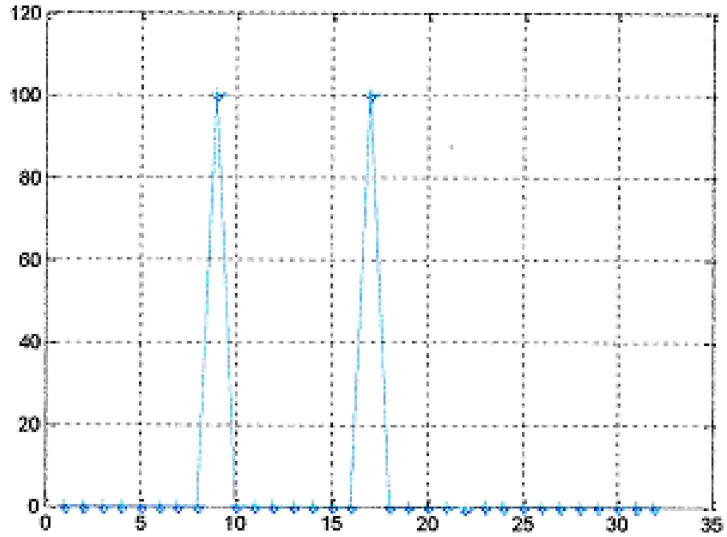
$X = FFT(x);$

Plot(abs(X(1:32)));

grid on;

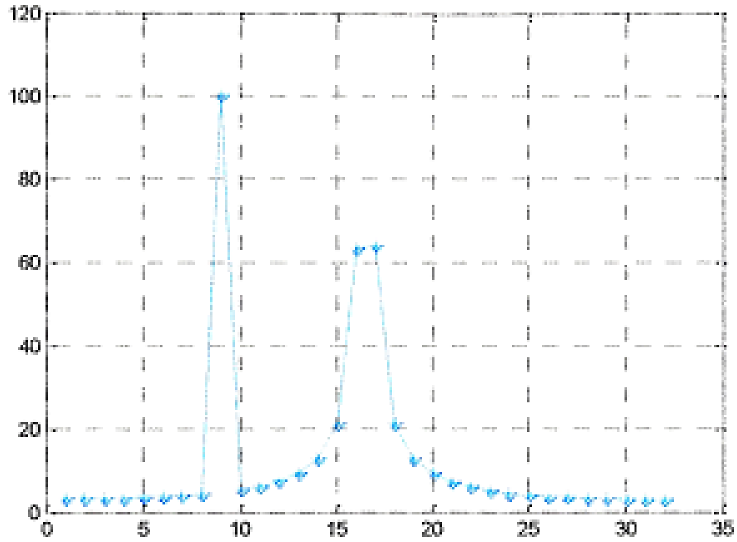


Condition that if we are tries to modify the above program is little slightly to plot FFT spectrum is divided R/2 that is in 32 then we can interpret amplitude of the sine-wave (both 100) in a straight line from graph.



The number of time-domain is increase sample from begin with 64 to 128 it give us more points about in the frequency-domain also after that read the amplitudes as well as the frequencies more precisely. Other than it cannot for all time possible to acquire more time-domain sample. Its spectrum may be gradually changing and the change but the signals may not be 'stationary' over the range must be 128 of samples.

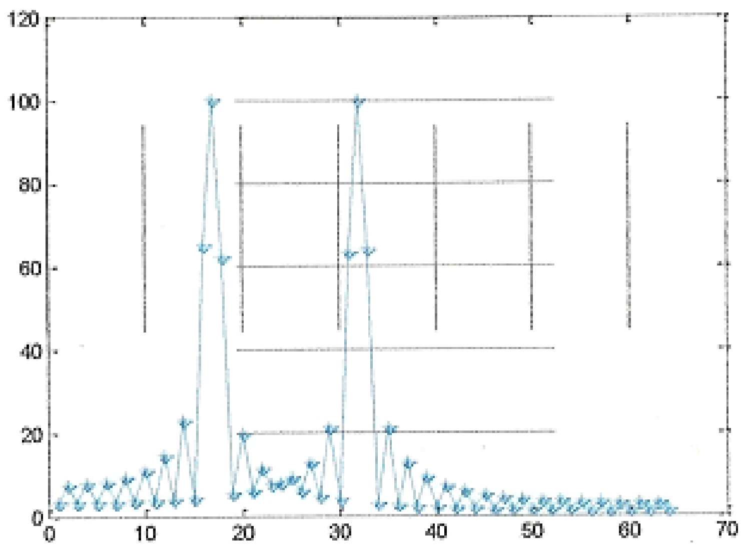
Fixed $R = 64$, then we consider what happen condition is that one of the sine wave frequency is higher therefore it can say that, it is changes a little. In the First observe it becomes $\pi/2$ corresponding to $(fs/4)$ therefore which coincides accurately through $(16\pi/32)$, that is one of the sampling points of frequency domain, here we observe that. the one by way of $k = 16$. now we decrease the frequency of the sine wave from $16\pi/32$ to $15.5\pi/32 \approx 1.522$. We observe the situation What it happens condition Therefore the sampling points lies between $k = 15$ to $k = 16$. For instance that no frequency sampling point in 1.522. Therefore the graph of magnitude spectral obtain as follows.



Now we don't get the correct amplitude for the sinusoid at 1.522 therefore the frequency sampling which is does not correspond to one of points. Thereafter it will become 35% error of the concept amplitude analysis. Now we observe the overcome of this problem? The possible of condition that we increase R from 64 it would be improve some matters to this case would solve problem completely when 1.522 it would be coincide through one of new frequency sampling points are to be formed by means of doubling the R. However the above solution not be possible therefore only 64 samples are obtainable. Of course signal might be change quickly say non-stationary it is just may about the possible to consider as in quasi-stationary for $R = 64$ other than not for $R = 128$.

3.3. Verification of Zero-padding

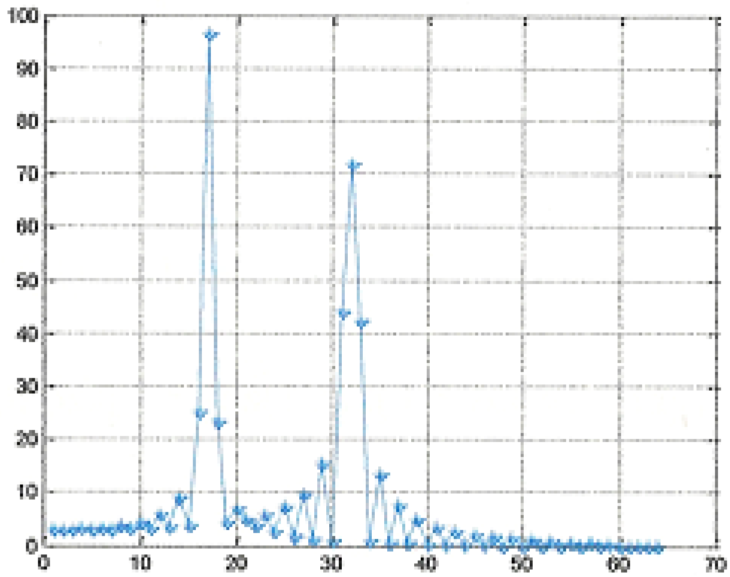
Zero-padding is the possible solution to apply by means of appends the zero samples after or before the signals to unnaturally enlarge of numeral of samples. The enlarge or increase numeral of frequency sample produced by means of FFT and insertion of 64 samples after the 64 or in front of samples already analyzed previously then enchanting FFT which will get the spectrum whose magnitude as follows below.



The following case the technique is reasonably efficient case of interpolating among about the frequency domain sample previously produced in 64 points of the FFT also it gives more precise value of indication of amplitude in the highest frequency about the sine-wave. This ripple was very attractive also we will be discussing later on.

3.4. The even symmetry by Extension

The doubling the number of frequency-domain samples is to extend the block of R samples by even symmetry it is alternative to 'zero padding'. To exemplify for the segment or set of presently 4 samples, {6124} becomes moreover {61244216} or {42166124}. The above even order extension are satisfactory, other than the second is pre-extension to be favored. Is the condition of 64 sample to be analyze before clearly even order then the pre-extended to produce samples of 128 after that the samples are FFT analyzed and the spectrum obtained as a magnitude is shown below.



In this case the improvement is insignificant so not as victorious the zero-padding. In other one of the important benefit about this to this method of doubling number of frequency-domain samples. This leads to the discrete cosine transform. Consider the example of simple where {6 1 2 4} be pre-extended to becomes {4 2 1 6 6 1 2 4}. DFT of the 8-point extended sequence for $k = 0, 1, \ldots 7$. Let

$$X[k] = \{4 + 2e^{-j\Omega_k} + e^{-2j\Omega_k} + 6e^{-3j\Omega_k} + 6e^{-4j\Omega_k} + e^{-5j\Omega_k} + 2e^{-6j\Omega_k} + 4e^{-7j\Omega_k}\}$$

$$\begin{aligned} X[k] &= \{4 + 2e^{-j\Omega_k} + e^{-2j\Omega_k} + 6e^{-3j\Omega_k} + 6e^{-4j\Omega_k} + e^{-5j\Omega_k} + 2e^{-6j\Omega_k} + 4e^{-7j\Omega_k}\} \\ &= \{e^{-3.5j\Omega_k}[4e^{3.5j\Omega_k} + 2e^{2.5j\Omega_k} + e^{1.5j\Omega_k} + 6e^{0.5j\Omega_k} + 6e^{-0.5j\Omega_k} + e^{-0.5j\Omega_k} + 2e^{-2.5j\Omega_k} + 4e^{-3.5j\Omega_k}]\} \\ &= \{2e^{-3.5j\Omega_k}\{4\cos(3.5\Omega_k + 2\cos(2.5\Omega_k) + \cos(1.5\Omega_k) + 6\cos(0.5\Omega_k))\}\} \\ &= 2e^{-3.5j\Omega_k} DCT[k] \end{aligned}$$

where $\Omega_k = 2\pi k/8 = \pi k/4$ and $DCT[k] = \{6\cos(0.5\Omega_k) + \cos(1.5\Omega_k) + \cos(2.5\Omega_k) + \cos(3.5\Omega_k)\}$.

To evaluate DFT for $k = 0, 1, 2, 3, 4, 5, 6, 7, \ldots$ it give the frequency-sample range from 0 to (fs) then if we analyze the real signal examine the first 4 of the above. So $\{DCT[k]\}$ that is captivating the first 4 out of 8 DFT samples this is the 'discrete cosine transform of $\{x[k]\}$ and this is also called since pre-extension by means of even symmetry to be produced the DFT it is represented the sum of the cosine series. Therefore the enormous benefit is the expression is purely real but with the absence of complex numbers while already in the DFT.

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