Int. J. Math. And Appl., 9(1)(2021), 119–126
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

Temperature Dependent Viscosity Effects on Unsteady MHD Accelerating/Decelerating Flow over a Wedge

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Abstract: The present work brings into focus the unsteady MHD two dimensional accelerating/decelerating boundary layer flows and heat transfer of an incompressible electrically conducting fluid above a moving wedge in the existence of variable viscosity. Suitable transformation is used to form a system of coupled non linear partial differential equations for governing both the flow and heat transfer. These equations have been carried out numerically, which is done by utilizing an implicit finite difference method in combination with quasilinearization technique. The obtained numerical results have been presented graphically in terms of local nusselt number, skin friction, temperature distribution, and velocity distribution for different values of magnetic field (M) and variable viscosity (ε) parameters along with the fixed Prandtl number (Pr) and wedge angle (m). It is found that there exist a unique solution for accelerating flow and dual solutions for decelerating flow.

MSC: 76W05, 76R10.

Keywords: Accelerating flow, Decelerating flow, Heat transfer, Skin friction, Variable Viscosity.

1. Introduction

Boundary layer theory is important in many fields of engineering and real world setbacks. The main application of this theory is all about measuring the skin friction drag. Its main focus is on the way the friction drag acts on a body while it is moving through a fluid, for instance the flow over an airplane wing or past an entire ship. This led to Falkner-Skan [1] developing a model known as wedge flow based on the Prandtl boundary layer theory. Following it, a great deal of work has been done over the most recent couple of years by many investigators [2–12] on the Falkner-Skan problem which is considered a classic. It is carried out by employing various numerical and analytical methods for various types of flow including heat transfer conditions.

Of the previously mentioned investigations, the properties of the fluids were assumed to be constant. In many technical applications in the field of engineering, however, this assumption cannot be conformed to. Assumption of variable viscosity becomes necessary in considering such problems. It is accepted knowledge that there may be a major change in the physical properties of the fluid, whenever the temperature changes. (Take for example the fact that viscosity of water is seen to decrease by about 24% whenever the temperature is seen to increase from 10^0 to 50^0c). Herwing [13] made the principal endeavor to work out the Falkner-Skan problem by having the variable viscosity also taking into account temperature. Later many investigators [14–16] started to work on the effects of variable viscosity on flow past a moving wedge.

There has been extraordinary enthusiasm for the investigation of MHD flow and heat transfer present in any of the available mediums because of the impact of the applied magnetic field present on the boundary layer flow control as also on the

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performance of many of the systems utilizing fluids capable of conducting electrically. This sort of flow has grabbed the attention of several investigators because of its applications in MHD generators, plasma studies, and in the design for cooling of nuclear reactors. Therefore the works on the study of magnetic convection at a wedge and as well as cone started by Vajravelu [17]. Further some researchers [18–22] chose to study on MHD Falkner-Skan boundary layer flow problems. Recently some research work has done on MHD Boundary Layer Flow along with variable viscosity [23, 24]. But the effect of unsteadiness is not considered in all the above published works. In the present study, the unsteady MHD accelerating/decelerating flow above a moving wedge having variable viscosity shall be analyzed.

2. Mathematical Analysis

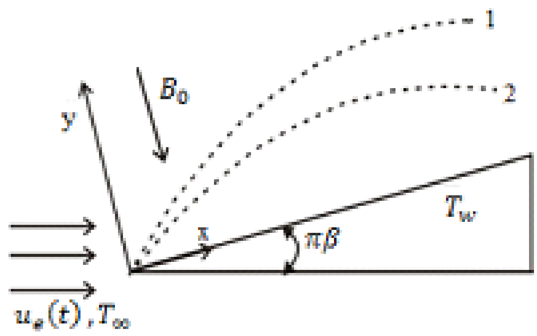

Figure 1. Co-ordinate system & Flow configuration for unsteady MHD Falkner-Skan wedge flow, here edge of thermal and momentum boundary layers represents as 1 & 2, respectively.

Figure 1, shows an unsteady and incompressible viscous electrically conducting fluid above a wedge that is moving in a two dimensional manner. In this the measurement of x is taken along the wedge surface with y being normal to it. Let u_e be the free stream velocity, which introduces the unsteadiness of the flow field and this is seen to vary inversely with time. The free stream temperature (T_∞) is lesser than the wall temperature (T_w) which is uniform and constant. A transverse magnetic field (B_0) is applied in a y-direction that is normal to the body surface in which it is assumed that the magnetic Reynolds is minute, hence the induced magnetic field can be ignored. With the exception of fluid viscosity (μ) which is taken to be an inverse linear function of the temperature (T) the fluid is assumed to have physical properties that are constant.

According to the aforesaid assumption, the unsteady forced convection boundary layer flow equations over a moving wedge are,

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial x} + v\frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\sigma B_0^2}{\rho}(u - u_e) \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

The initial and boundary conditions are specified by

$$\begin{aligned} &at\ \ y = 0: && u = v = 0 \ \text{ and } T = T_w \\ &as\, y \to \infty: && u \to U(x) = u_\infty(x/L)^m \text{ and } T = T_\infty \\ &at\ x = 0: && u = u_\infty \text{ and } T = T_\infty \end{aligned} \tag{4}$$

In the present study, the viscosity of the form is shown using a semi-empirical formula

$$\frac{\mu}{\mu_\infty} = \frac{1}{1 + \gamma(T - T_\infty)}$$

Which is created by Ling and Dybbs [25] and has been embraced, where γ is a constant and $\mu_\infty$is the ambient fluid viscosity. Further,

$$u = \frac{\partial \psi}{\partial y} \qquad \text{and} \qquad v = -\frac{\partial \psi}{\partial x}$$
$$f(\eta) = \sqrt{\frac{1+m}{2}\frac{L^m}{\upsilon u_\infty}}\left(\frac{\psi}{x^{(1+m)/2}}\right) \tag{5}$$
$$\eta = \sqrt{\frac{1+m}{2}\frac{u_\infty}{\nu L^m}}\left(\frac{y}{x^{(1-m)/2}}\right); \qquad G(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

Substituting the above transformations from equation (1) to (4), we acquire:

$$(1+\varepsilon G) F'' - \varepsilon G' F' + (1+\varepsilon G)^2 \left[\left(\frac{2m}{1+m}\right)(1-F^2) + \lambda\left(\frac{2}{1+m}\right)\left(1 - F - \frac{\eta}{2}F'\right) + fF' + M(1-F)\right] = 0 \tag{6}$$
$$\overset{-1}{\Pr} G" + fG' - \lambda\eta\left(\frac{1}{m+1}\right) G' = 0 \tag{7}$$

Where

$$\frac{u}{u_e} = f' = F; f = \int_0^\eta F d\eta$$
$$v = -\sqrt{\frac{2}{1+m}}\sqrt{\frac{\upsilon u_\infty}{L^m}}\,(1-\lambda t^*)^{-1/2} x^{(m-1)/2}\left[\frac{m+1}{2} f + \eta f" \frac{m-1}{2}\right] \tag{8}$$
$$\Pr = \frac{\upsilon}{\alpha}$$

It can next be seen that, in (6) & (7), the apex angle $\pi\beta$ is associated with parameter m by the relation $m = \beta/(2-\beta)$ or $\beta = 2m/(m+1)$. The corresponding boundary conditions are:

$$F = 0; \quad G = 1 \quad at \quad \eta = 0$$
$$F = 1; \quad G = 0 \quad as \quad \eta \to \infty \tag{9}$$

Here, $\varepsilon = (T_w - T_\infty)\gamma$ is named as the variation of viscosity parameter. ψ - dimensional stream function and f - dimensionless stream function; m - Falkner-Skan parameter; M - dimensionless magnetic field parameter; F - dimensionless velocity & G - dimensionless temperature of the fluid; λ - unsteady parameter ; Pr - Prandtl number; η - similarity variable. Here prime ($'$) means derivative with respect to η. Respectively, the Skin friction and heat transfer coefficients as nusselt number, can be communicated, as

$$C_f\,(Re_L)^{1/2} = \frac{\tau_w}{\frac{1}{2}\rho u_e^2} = \frac{2\sqrt{\frac{1+m}{2}}\left(F'\right)_{\eta=0}}{1+\varepsilon G}$$
$$Nu(Re_L)^{-1/2} = -k\frac{\left(\frac{\partial T}{\partial y}\right)_{y=0}}{(T_w - T_\infty)} = -\sqrt{\frac{1+m}{2}}\,(G')_{\eta=0} \tag{10}$$

Here the wall shear stress τ_w is given by $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ where μ is dynamic viscosity, k is thermal conductivity and $Re_L = \frac{u_\infty L}{\nu}$ called the local Reynolds number.

3. Results and Discussions

The coupled nonlinear partial differential Equations (6) and (7) are solved alongside the boundary conditions (9) by utilizing an implicit finite difference method. This is agreed out in combination with a quasilinearization technique. Since the technique is described in Inoue and Tate [26] and Ajay [27], in the interests of brevity, the description of the same has been omitted here. The numerical computations as shown in graphical representations of this paper have been done for different values of temperature dependent viscosity (ε), magnetic field parameter (M), unsteady parameter (λ), and Falkner-Skan parameter (m) for both the accelerating, as well as decelerating flow. To validate the accuracy of our numerical technique, we have compared skin friction ($F_w^{'}$) and heat transfer ($G_w^{'}$) parameters with those given by Watanabe [7] ranging from $0 \leq m \leq 1.0$ [as shown in Table 1] taking $Pr = 0.72$.

m	$F_w^{'}$		$G_w^{'}$	
	Present	Watanabe[7]	Present	Watanabe[7]
0.0	0.46961	0.46960	0.41511	0.41512
0.014	0.50460	0.50461	0.42050	0.42051
0.0425	0.56899	0.56898	0.42988	0.42984
0.0909	0.65499	0.65498	0.44124	0.44125
0.1429	0.73201	0.73200	0.45041	0.45042
0.2	0.80214	0.80213	0.45827	0.45826
0.3333	0.92768	0.92765	0.47084	0.47083
1.0	1.23259	1.23258	0.49570	0.49571

Table 1. A comparison of results for the various values of m for steady state ($\lambda = 0$) when $\varepsilon = 0.0$ with those given by Watanabe [7]

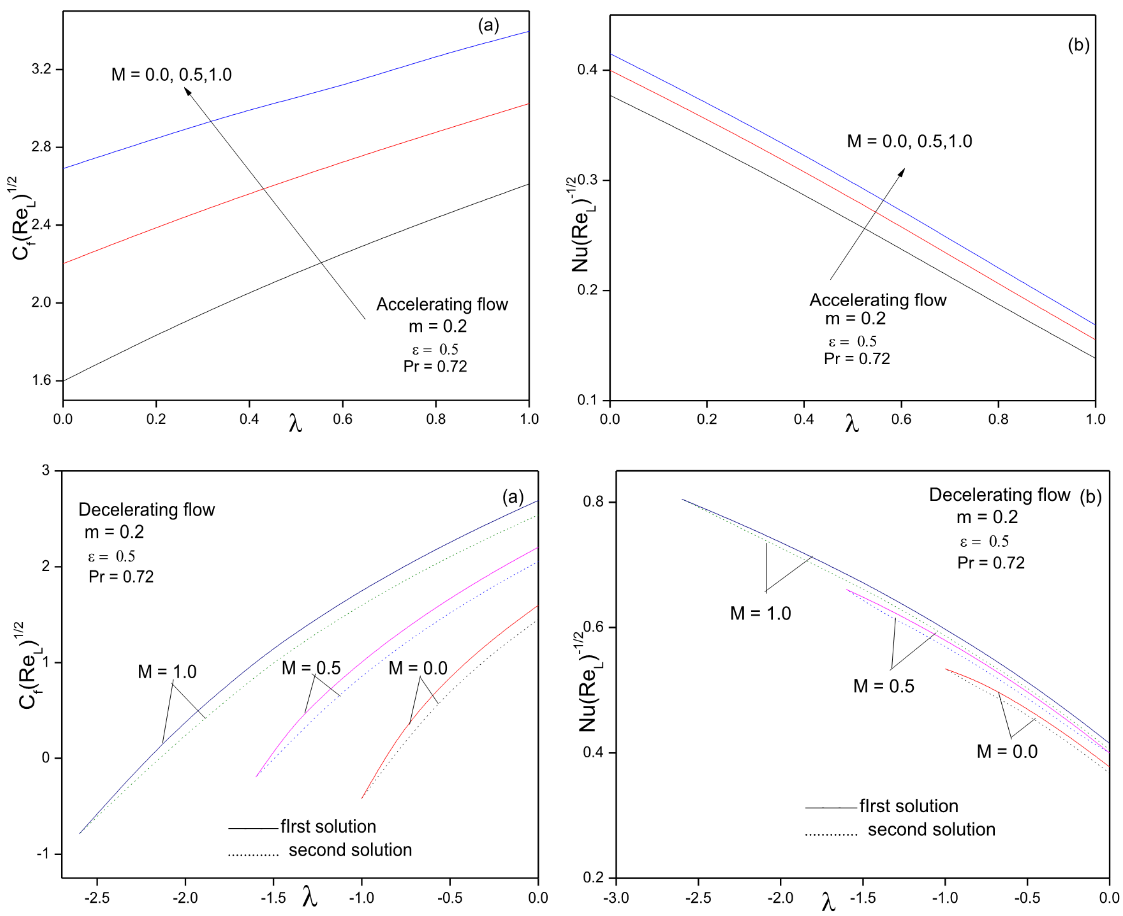

Figure 2. (a) skin friction and (b) heat transfer coefficients for various values of ε for accelerating and decelerating flow respectively.

The effect of magnetic field parameter (M) on skin friction $[C_f(Re_L)^{1/2}]$ and heat transfer $[N_u(Re_L)^{-1/2}]$ coefficients when $m = 0.2$ (60^o), $Pr = 0.72$ and $\varepsilon = 0.5$ for $\lambda > 0$ (accelerating flow) and for $\lambda < 0$ (decelerating flow) is as shown in Figure 2. It is noticed that as M increases, both $[C_f(Re_L)^{1/2}]$ and $[N_u(Re_L)^{-1/2}]$ are seen to increase quantitatively. The percentage of increase in $[C_f(Re_L)^{1/2}]$ is about 93.98% and in $[N_u(Re_L)^{-1/2}]$ is found to be 3.61% in the range $0 \leq \lambda \leq 1.0$ foe accelerating flow and similarly for decelerating flow the percentage of enhance in $[C_f(Re_L)^{1/2}]$ is about 132.39% and in $[N_u(Re_L)^{-1/2}]$ is found to be 4.49% in the range $-2.6 \leq \lambda \leq 0.0$. It is intriguing to see that there is unique solution for $\lambda > 0$ and dual solutions for $\lambda < 0$ for both $[C_f(Re_L)^{1/2}]$ and $[N_u(Re_L)^{-1/2}]$, in the vary of $\lambda\,(\lambda_c < \lambda < 0)$ and there is no result for $\lambda < \lambda_c$, here λ_c is a critical value of λ. Hence, the result is seen to exist up to a critical value $\lambda = \lambda_c < 0$, further than, the boundary layer isolates from the wedge surface and the result depend on the boundary layer approximation is beyond the realm of imagination. Based on our calculation, the estimations of λ_c are $(\lambda_c = -1.05, -1.61, \& -2.62)$ correspondingly for $[C_f(Re_L)^{1/2}]$ and $[N_u(Re_L)^{-1/2}]$. As magnetic field increases $(M = 0.0, 0.5, 1.0)$ the critical value λ_c (which separates the wedge surface) are seen to be pushed.

Figure 3 demonstrates the significant velocity and temperature distributions for different values of the magnetic field parameter $(M = 0.0, 0.5, 1.0)$. Here the second solution profile confirms the presence of dual solutions in a decelerated flow. Increasing the magnetic field with the presence of variable viscosity $(\varepsilon = 0.5)$ causes an increase in the velocity profile but decreases the temperature profile in both accelerating and decelerating flow. So the magnetic field can hence be utilized to control the flow characteristics. It is commented that, the first outcomes are steady and physically feasible; where as the second of the solutions are most certainly not. Solutions such as these, though lacking physical significance are however seen to possess mathematical significance [28].

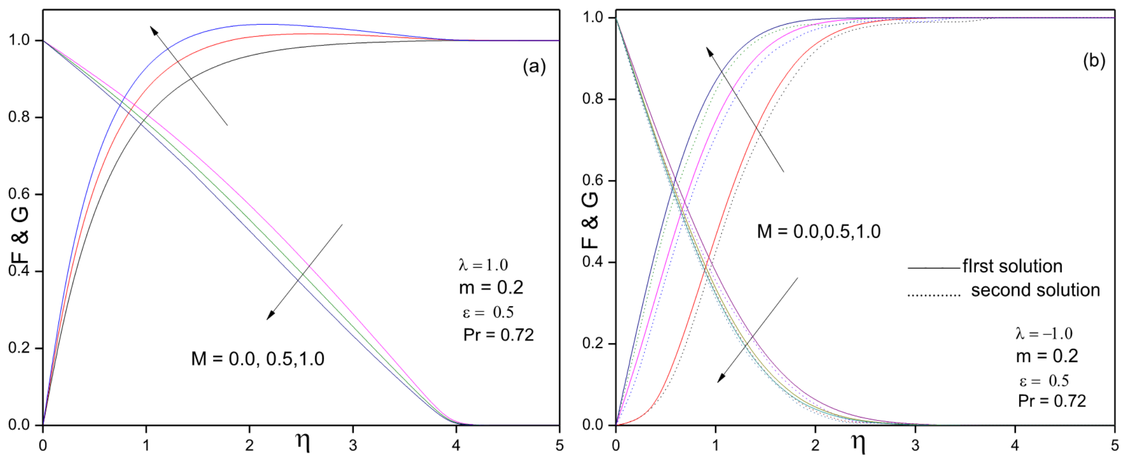

Figure 3. Velocity (F) and Temperature (G) distributions for various values of M for (a) accelerating and (b) decelerating flow respectively.

Figure 4 displays the variation of $[C_f(Re_L)^{1/2}]$ and $[N_u(Re_L)^{-1/2}]$ for different values of variable viscosity parameter $(\varepsilon = 0.0, 0.5, 1.0)$ for various unsteady parameters $(\lambda = 1.0$ and $\lambda = -1.0)$ corresponding to wedge angle $m = 0.2(60^0)$ and magnetic field $M = 0.5$. It is clear that both $[C_f(Re_L)^{1/2}]$ and $[N_u(Re_L)^{-1/2}]$ increase with the expansion of variable viscosity. The level of enhance of skin friction is about 110.3 % & 88.3%, and heat transfer is around 3.9 % and 3.41% for an increase of ε in $\lambda > 0$ and $\lambda < 0$ respectively.

Figure 5 depicts the effects of variable viscosity (ε) on velocity [F] and temperature [G] distributions. Here the velocity distribution is seen to increase while the temperature distribution decreases with an enhancement in variable viscosity for a fixed magnetic field $(M = 0.5)$. As the variable viscosity increases, it is seen unmistakably that the thicknesses of thermal and momentum boundary layers diminishes. Slower movement of fluid leads to a decrease in the rate of heat transfer, thus decreasing the thermal boundary layer thickness.

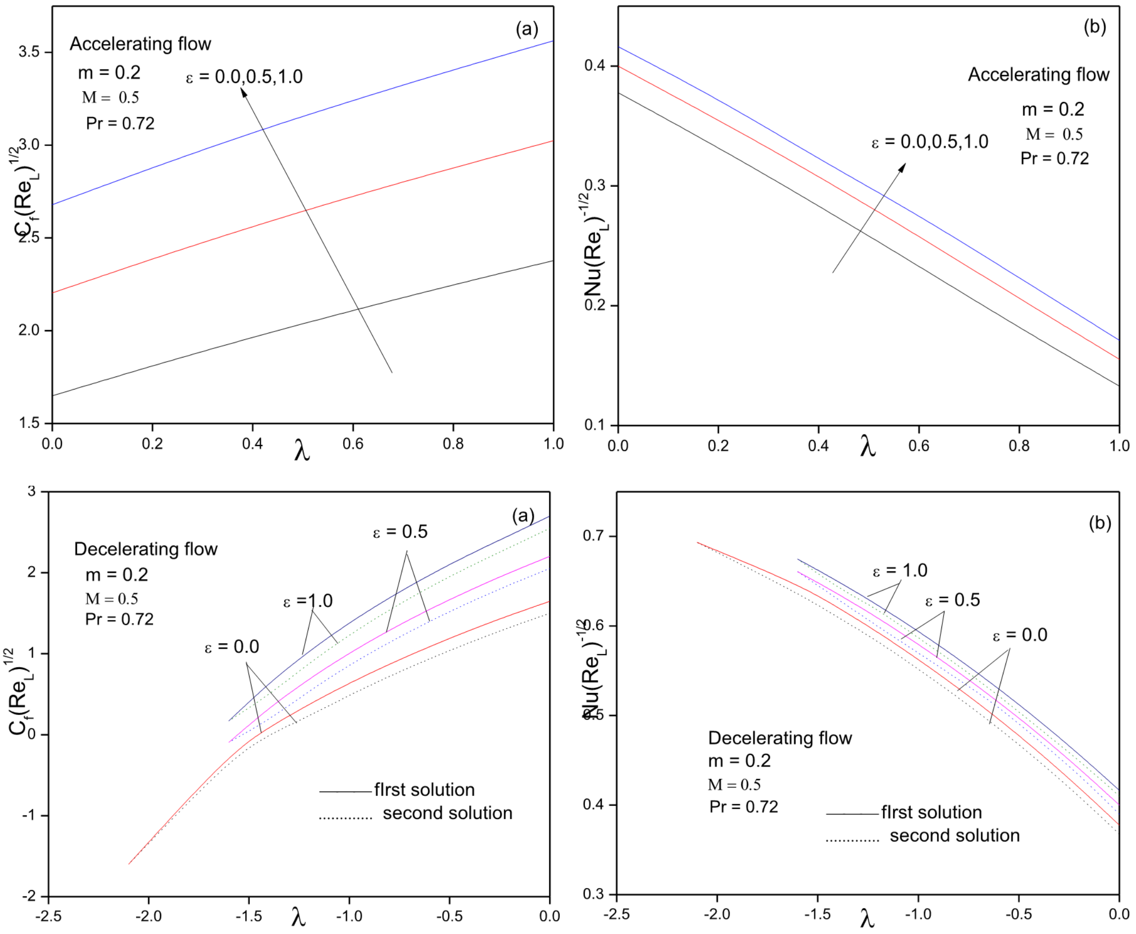

Figure 4. (a) skin friction and (b) heat transfer coefficients for various values of ε for accelerating and decelerating flow respectively.

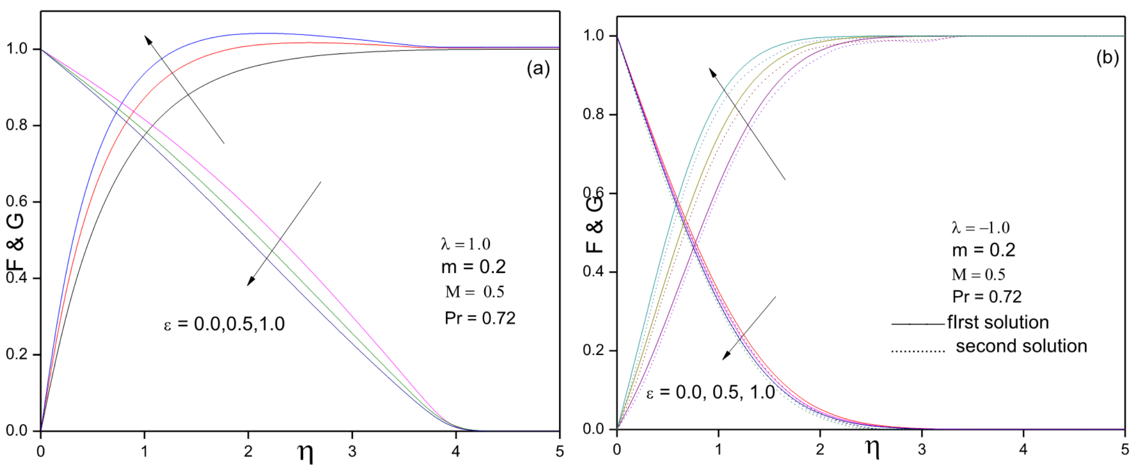

Figure 5. Velocity (F) and Temperature (G) distributions for various values of ε for (a) accelerating and (b) decelerating flow respectively.

4. Conclusions

In the present study, the impact of variable viscosity and magnetic field on the unsteady accelerating/ decelerating flow of an incompressible fluid capable of conducting electrically above a moving wedge have been investigated.

- The skin friction and heat transfer co efficient increases with an enhance of magnetic field parameter ($M = 0, 0.5, 1.0$) and the temperature distribution decreases but the opposite trend in velocity distribution for the fixed Prandtl number

($Pr = 0.72$), variable viscosity ($\varepsilon = 0.5$) and the wedge angle ($m = 0.2$) in both accelerating and decelerating case was observed.

- Increasing the variable viscosity parameter with the fixed Prandtl number ($Pr = 0.72$), magnetic field parameter ($M = 0.5$) & wedge angle ($m = 0.2$) leads to an increase in both the coefficients of skin friction and heat transfer, where as velocity profiles increases & temperature profile decreases.

Acknowledgement

The authors would like to express their gratitude to Principal and Management of the Maharaja Institute of Technology, Mysore-571477, for their benevolent support.

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