Int. J. Math. And Appl., 9(1)(2021), 35–39
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

# To Study Linear with Integer Programming and Solving Multi-Objective Optimization

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**Abstract:** Here we present a new exact approach for solving Multi-Objective Optimization with. Linear programs and integer programs are optimization problems with linear impartial functions and linear constraints. These problems are actual general: linear programs can be used to express an inclusive range of problems such as two-part corresponding and network stream, while integer programs can express an even larger range of problems, including all of those in model of operation research. We define each of these formalisms in turn.

**Keywords:** Linear Programming, Integer Programming, multi-objective Optimization, approximation algorithms.



## 1. Introduction

In (1947) George Dantzing and his associates, while working in the U.S. Department of Airborne Militaries, detect that a vast numeral of aggressive programming and research disobedient capacity be expressed as exploit/minimize a linear from of revenue/rate role whose variables were controlled to value satisfying a classification of linear equation.

A model which is used for optimum allocation of limited resources to competing activity under such as assumption as certainly, linearity, fixed technology, and constant profit per unit, is linear programming. Linear programming is one of the most multipurpose, influential and suitable technique for creation decision-making conclusion. Linear programming technique may be used for solving board range of problems arising in commercial, administration, hospices, public library etc. when we want to apportion the available limited resource for various contending activity for achieving our desired objective, the technique that helps us in Linear programming. As a decision-making tool, it has demonstrated its value in various fields such as production, finance, marketing, research and development. Determination of optimal product mix, transportation schedules, assignment problem and many more [1].

As the name implies Integer linear programming's are the special class of linear programming problems where all or some of the variable in the optimal solution are restricted to non-negative integer value such problems are called as all integer or mixed integer problems depending are respectively whether all or some of the variable are restricted to integer values [2]. In (1956), R. E. Gomory suggested first of all systematic method to obtain an optimum integer solution to all integer programming problem; Later, he extended the method to deal with more complicated case of mixed integer programming problem when only some of the variable are required to be integer. These algorithms are proved to converge to the optimal

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integer solution in a finite number of iterations making use of familiar dual simplex method. This is called the "cutting plane algorithms" because mainly introduce the clever idea of constructing secondary constraints which, when added to the non-integer solution, will effectively cut the solution space toward the required result. Successive application of these constraint should gradually force the non-integer optimum solution toward the desired "all integer" or "mixed integer" solution [3].

## 2. Defining Linear Programs, Preliminaries and Problem Formulation

A linear program is defined by: a set of real-valued variables; a linear objective function (i.e., a weighted sum of the variables); and a set of linear constraints (i.e., the requirement that a weighted sum of the variables must be less than or equal to some constant). Let the set of variables be $\{x_1 \ldots x_n\}$, with each $x_i \in R$. The objective function of a linear program, given a set of constants $w_1 \ldots w_n$, is

$$Maximize \sum_{i=0}^{n} w_i x_i$$

Linear agendas can also express minimization problems: these are just maximization problems with all hefts in the impartial purpose annulled. Constraints express the requirement that a weighted sum of the variables must be greater than or equal to some constant. Explicitly, given a set of constants $a_{1j} \ldots a_{nj}$ and a constant $b_j$, a constraint is an expression

$$Maximize \sum_{i=0}^{n} a_{ij} x_i \leq b_j.$$

This form actually allows us to express a broader range of constraints than might immediately be apparent. By negating all constants, we can express greater than-or-equal constraints. By providing both less-than-or-equal and greater-than or-equal constraints with the similar constants, we can express equivalence constraints by situation some constants to zero, we cannister direct constraints that do not encompass all of the variables. Additionally, even problems with piecewise-linear constraints (e.g., concerning occupations similar a max of linear terms) can sometimes be expressed as linear lineups by adding both new restrictions and new variables. Observe that we cannot always write strict dissimilarity constraints, though occasionally such constraints can be compulsory finished changes to the neutral function. Attractive it all together, if we have m different restrictions, we can write a linear program as follows.

$$\begin{aligned}
&\text{Maximize} \sum_{i=0}^{n} w_i x_i \\
&\text{Subject to} \sum_{i=0}^{n} a_{ij}\ x_i \leq b_j \quad \forall \quad j = 1 \ldots m \\
&\qquad\qquad\qquad x_i \geq 0 \quad \forall \quad i = 1 \ldots n
\end{aligned}$$

Observe that the requirement that each $x_i$ must be nonnegative is not restrictive: problems involving negative variables can always be reformulated into equivalent problems that satisfy the constraint [4]. A linear program can also be written in matrix form. Let $w$ be an $n \times 1$ vector containing the weights $w_i$, let $x$ be an $n \times 1$ vector containing the variables $x_i$, let $A$ be an $m \times n$ matrix of constants $a_{ij}$, and let $\mathbf{b}$ be an $m \times 1$ vector of constants $b_j$. We can then write a linear program in matrix form as follows.

$$\begin{aligned}
&\text{Maximize } w^T x \\
&\text{Subject to} \qquad Ax \leq b
\end{aligned}$$

$$x \geq 0$$

In some cases, we care to satisfy a given set of constraints, but do not have an associated objective function; any solution will do. In this case the LP reduces to a constraint fulfilment or feasibility problem, but we will sometimes still refer to one as an LP with an empty objective function (or, equivalently, the trivial one). Original problem Finally, every linear program (a so-called primitive problem) has a corresponding dual problem which shares the same optimal solution. For the linear program dual problem given earlier, the dual program is as follows.

$$\begin{aligned} &\text{Minimize } b^T y \\ &\text{Subject to} \quad A^T y \geq w \\ &\qquad\qquad\qquad y \geq 0 \end{aligned}$$

In this linear program our variables are $\mathbf{y}$. Variables and constraints successfully trade places: there is one variable $y \in y$ in the dual problem for every constraint from the primal problem and one constraint in the dual problem for every variable $x \in x$ from the primal problem.

## 2.1. Solving Linear Programs

In order to solve linear programs, it is useful to observe that the set of feasible solutions to a linear program corresponds to a convex polyhedron in $n$-dimensional space. This is true because all of the constraints are linear: they correspond to hyperplanes in this space, and so the set of feasible solutions is the region bounded by all of the hyperplanes [5]. The fact that the objective function is also linear allows us to conclude two useful things: any local optimum in the feasible region will be a global optimum, and at least one optimal solution will exist at a vertex of the polyhedron. (More than one optimal solution may exist if an edge or even a whole face of the polyhedron is a local maximum). The most popular algorithm for solving linear programs is the simplex algorithm. This algorithm works by identifying one vertex of the polyhedron and then taking uphill (i.e., objective-function-improving) steps to neighboring vertices until an optimum is found. This algorithm requires an exponential number of steps in the worst case, but is usually very efficient in practice. Although the simplex algorithm is not polynomial, it can be shown that other interior-point algorithms called interior-point methods solve linear programs in worst-case method polynomial time. These algorithms get their name from the fact that they move through the interior region of the polyhedron rather than jump from one vertex to another. Surprisingly, although these algorithms dominate the simplex method in the worst case, they can be much slower in practice.

## 2.2. Defining Integer Programs, Preliminaries and Problem Formulation

Integer programs are linear programs in which one additional constraint holds: the variables are required to take integral (rather than real) values. This makes it possible to express combinatorial optimization problems such as satisfiability or set packing as integer programs [6]. A useful subclass of integer programs is 0-1 integer program, in which each variable is constrained to take either the value 0 or the value 1. These programs are sufficient to express any problem in NP. The form of a 0–1 integer program is as follows.

$$\begin{aligned} &\text{Maximize } \sum_{i=0}^{n} w_i x_i \\ &\text{Subject to } \sum_{i=0}^{n} a_{ij}\ x_i \leq b_j \quad \forall \quad j = 1 \ldots m \end{aligned}$$

$$x_i \in \{0,1\} \quad \forall \quad i = 1 \ldots n$$

mixed-integer Another useful class of integer program is mixed-integer programs, which program involve a combination of integer and real-valued variables. Finally, as in the LP case, both integer and mixed-integer programs can come without an associated objective function, in which case they reduce to constraint satisfaction problems.

### 2.3. Solving Integer Linear Programs

The introduction of an integrality constraint to linear programs leads to a much harder computational problem: in the worst-case integer programs are undecidable. When variables' domains are finite sets of integers, they are NP-hard. Thus, it should not be surprising that there is no efficient procedure for solving integer programs. branch-and the most commonly used technique is branch-and-bound search [7]. The space of bound search variable assignments is explored depth-first: first one variable is assigned a value, then the next, and so on; when a constraint is violated or a complete variable project is accomplished, the search backpedals and tries other projects. The best achievable solution originate so far is recorded as a minor bound on the value linear program of the optimal solution. At each search node the linear program slackening of relaxation the integer program is solved: this is the linear program where the remaining variables are allowed to take real rather than integral values between the minimum and maximum values in their domains.

It is easy to see that the value of a linear program reduction of an integer program is an upper bound on the worth of that integer program, since it encompasses a relaxing of the concluding problem's constrictions [8]. Branch-and-bound search diverges from ordinary depth-first exploration because it sometimes thins the tree. Unambiguously, branch-and-bound backtracks whenever the upper bound at a search node is less than or equal to the lower bound. In this way it can skip over large parts of the search tree while still promising that it will find the optimal solution. Other, more composite techniques for solving integer programs include branch and-cut and branch-and-price search. These methods offer no advantage over branch-and-bound search in the worst case, but often outclass it in practice. Although they are computationally obstinate in the wickedest case, sometimes integer programs are provably easy. This occurs when it can be shown that the solution to the linear programming relaxation is integral, meaning that the integer program can be solved in polynomial time. One important example is when the total constraint matrix is totally unimodular and the vector **b** is integral. A unimodular unimodularity matrix is a square matrix whose determinant is either $-1$ or 1; a totally unimodular matrix is one for which every square submatrix is unimodular. This definition implies that the entries in a totally unimodular matrix can only be $-1$, 0, and 1.

## 3. Conclusions

This paper investigated and we present a new exact method combining the well-known principle of branching in linear with integer programming new efficient cut is described to generate all integer efficient solution of MOILP problem. It can be considered as a general method dedicated to MOLP problems with integer as well as zero-one decision variable can be solved by the method.

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