ASSET ALLOCATION AND PORTFOLIO OPTIMIZATION PROBLEMS WITH METAHEURISTICS: A LITERATURE SURVEY

Bilel JARRAYA
University of Sfax, Sfax, Tunisia
bilel.jarraya@gmail.com

Abstract
The main objective of Markowitz work is seeking optimal allocation of wealth on a defined number of assets while minimizing risk and maximizing returns of expected portfolio. At the beginning, proposed models in this issue are resolved basing on quadratic programming. Unfortunately, the real state of financial markets makes these problems too complex. Metaheuristics are stochastic methods which aim to solve a large panel of NPhard problems without intervention of users. These methods are inspired from analogies with other fields such as physics, genetics, or ethologic. Already various Metaheuristics approaches have been proposed to solve asset allocation and portfolio optimization problems. In a first time, we survey some approaches on the topic, by categorizing them, describing results and involved techniques. Second part of this paper aims providing a good guide to the application of Metaheuristics to portfolio optimization and asset allocation problems.
Keywords: Portfolio, Asset allocation, Metaheuristics, Mono-objective problems, Multi-objective problems.

1. INTRODUCTION

Among classical research problems in financial theory and operations research we find this of portfolio optimization or asset allocation. The financial institutions (like Banks, insurance companies, fund management firms…) deal always with the problem of managing their funds and how to allocate them optimally by choosing the optimal portfolio in the market. The pioneer in this research area is Markowitz (1952) while proposing the mean-variance portfolio model based on quadratic optimization problem subject to linear constraints. Several theoretical advances have tried making extension to this model and solving it, referring to mathematical modeling methods. For more detailed information on these advances one can refers to the following investigations: Nishimura (1990), Figueroa-Lopez (2005) and Bolshakova et al., (2009). However, this model suffers of several drawbacks limiting its suitability with real situation. Indeed, the developed extensions and modifications of this model (by involving some transaction costs, complex constraints or using alternative objective functions) make the model more and more complex and computationally infeasible. So, there are several classical optimization methods to solve such problem, applicable when some mathematical conditions are met. Such as, linear programming effectively treats the case where the objective function and constraints are expressed as a linear function of decision variables. In addition, the case where the objective function and constraints are nonlinear, non-linear programming is applicable. Unfortunately, the situations encountered in practice usually include one or more complications and as a consequence, these classical methods become obsolete. For example, the objective function could be non-homogeneous, or can't be expressed analytically according to the parameters, or the problem may require consideration of two or more conflicting objectives (multi-objective optimization).

The emergence of a new class of optimization methods, called Metaheuristics, marks a great revolution in the optimization field. These methods are applicable to all types of combinatorial problems, and they can also be adapted to continuous problems. They allow researchers to find a good solution usually with a reasonable computing time, but without guaranteeing the optimality of obtained solution. Metaheuristic methods are advantageously used for solving large problems. These methods can be divided into two classes. First, the specific algorithms characterized by the use of knowledge domain for a given problem. Second, the general algorithms that can be used for a wide variety problems. Accordingly, several research works focused on the portfolio optimization problem have been oriented to apply these meta-heuristics providing practical solutions and overcoming the complexity of this problem.

This paper aims to find a background of Metaheuristics optimization for both mono-objective problems and multi-objective problems and their applications in the portfolio optimization issue. The remainder of this investigation is divided into three sections. Section 2 explores the principal techniques of mono-objective metaheuristics and their applications in portfolio optimization area. Section 3 investigates the principal approaches of multi-objective metaheuristics and their applications in portfolio optimization issue. Finally, section 4 concludes.

2. MONO-OBJECTIVE METAHEURISTICS AND OPTIMAL PORTFOLIO CHOISE

The development of metaheuristic algorithms in addition to the high performance of computing infrastructure provides a wide range of solutions to complex problems. This paragraph highlights some applications of these mono-objective Metaheuristics on portfolio choice and asset allocation issues.

2.1. Survey on the mono-objective applications in the problems of portfolio choices and assets allocation

Our investigation follows a chronological approach (Noting that one can find other research works linked with this research area but they are not included in this work).

Among the first papers interested by the application of mono-objective Metaheuristics on portfolio optimization problem we explore this of Eddelbuttel (1996). This paper deals the index-tracking problem, i.e., how reproducing the behavior of a benchmark index in a target portfolio. The goal of this study is minimizing the variance of return differences between the benchmark index and tracking portfolio. Nevertheless, empirically, this expected return differences is defined in advance and the portfolio is chosen referring to a fewer stocks. This problem's resolution is computationally hard for this reason the author is resorted to use a hybrid Genetic Algorithm. In a first step of each generation, assets included in the portfolio are selected by the genetic algorithm. Then, optimal weights of these selected stocks are defined by a quadratic programming solver. This model is empirically applied on "Deutscher Aktien IndeX" (DAX) by using daily closing prices. Chang et al. (2000) aims to resolve the Mean-Variance portfolio optimization problem including cardinality constraints as well as weights constraints. To attain their goals, authors have used three algorithms: Gentic Algorithm, Tabu Search and Simulated Annealing. The result of this study shows that the presence of Cardinality Constraints provides a discontinuous efficient frontier. So, efficient frontier is constructed via a mathematical programming approach and based on all combinations of unconstrained cases. the authors show that some portions of efficient frontier is hidden and the application of proposed methaheuristics show that Genetic Algorithm is the more performing compared to other used algorithms. Chan et al., (2002) investigate a multi-stage portfolio optimization problem while using genetic algorithms. Based on a Simulated Annealing algorithm Crama and Schyns (2003) resolves portfolio optimization issue with a systematic insertion of constraints. Lin et al., (2005) adds transaction costs in the problem of portfolio optimization and use a metaheuristic approach to resolve it. Chen et al., (2006) develops a constrained portfolio optimization problem and solve it by a particle swarm optimization approach. However, Thong (2007) applies an Ant Colony algorithm for a similar problem. Ruiz-Torrubiano and Suarez (2007) focus on Mean-Variance model with cardinality constraints as well as weights constraints on each assets or groups of assets. The authors use a hybrid approach. So, in a first step, they seek the optimal composition of assets in the target portfolio by an evolutionary algorithm. In a second step, they use a quadratic programming solver to define the optimal weight attributed for each chosen asset. In addition to the traditional objectives sought by other applications (decreasing risks and increasing returns) Aranha and Iba (2007) has dealt another objective that minimize transaction costs between two consecutive time periods. For empirical application the authors use monthly historical returns of NIKKEI and NASDAQ indexes and as a metaheuristic they bring into play Genetic Algorithms. Based on stochastic programming, Hochreiter (2008) incorporates uncertainty in its portfolio model. To resolve this optimization problem the author uses the Genetic Algorithm Metaheuristic. Empirical application

Business Excellence and Management
Volume 3 Issue 4 / December 2013

weekly is based on weekly information of fourteen assets listed in Dow Jones Index. Four types of risk are adopted in empirical simulation of this paper, namely: standard deviation, VaR, mean absolute downside semi deviation, expected shortfall. A stochastic programming approach is adopted by Geyer et al., (2009) to optimize a multi-period portfolio. Chang et al., (2009) investigates the portfolio optimization problem from a risk-aversion point of view. In this study four types of risk measures are adopted namely: the variance, semi-variance, mean absolute deviation and variance with skewness. To resolve this problem, authors of this paper use as Metaheuristic the Genetic Algorithm. The used algorithm is characterized by a binary tournament selection, uniform crossover and a replacement process involving the replacement of worst fitter individuals by the offspring chromosomes. Empirical implications of this work show that any enhancement of cardinality implies an enhancement on computation time. In addition, efficient frontiers of lower cardinality dominate those of higher cardinality. So, to achieve an effective portfolio authors suggest a limit to the cardinality to be at the one-third of total assets. Soleimani et al. (2009) has introduces the sector capitalization constraints in the portfolio optimization problem. According to these authors these constraints have for advantages to reduce the overall risk. However, these constraints make the problem more complicated. They use the Genetic Algorithm as a Metaheuristic to resolve the proposed problem. Yu et al. (2010) apply a simulation model to search the optimal asset allocation that maximizes shareholders' utility function for non-life insurance companies. The authors develop a new evolutionary algorithm while taking account of multi-periodic condition in the asset allocation problem. They show that their model is more effective than other algorithms which optimize mono-periodic problems.

The above presented studies are applications of some Metaheuristics on portfolio optimization problems or asset allocation problems. So, any researcher must have a minimum of skill in the Metaheuristics area to succeed the treated optimization problem. The above presented studies are applications of some Metaheuristics on portfolio optimization problems or asset allocation problems. So, any researcher must have a minimum of skills in the Metaheuristics area to succeed the treated optimization problem. The following subsection will be devoted to present some famous algorithms and their main features.

2.2. A survey on some mono-objective Metaheuristics

Meta-heuristics have usually an iterative behavior. The same pattern is repeated during the optimization until a stopping criterion, specified at the beginning, is met. The Meta-heuristics are direct in the sense they do not involve to derive the gradient of the function. Metaheuristics' users require fast and effective methods but in other hand these methods must be simple to be used. Highlighted techniques in this sub-section are arranged by chronological order of their appearance.

2.2.1. Evolutionary Algorithms

Fraser (1957) is the pioneer of evolutionary algorithms. He represents a family of research algorithms inspired from species' biological evolution. An evolutionary algorithm evolves gradually, by successive iterations or generations, the population composition with maintaining its constant size. The goal is the overall improvement of individuals' performance through generations. In each generation one applies a series of operators (selection operator, crossover operator and mutation operator) to population individuals that generate a new population. Each operator uses one or more population individuals, called parents, to generate new candidates, called offsprings. A complete list of all existing methods to define these operators is available in Eiben and Smith (2003). Evolutionary algorithm contains two principal approaches: Evolutionary Strategy (ES) and Genetic Algorithm (GA).

Evolutionary Strategy (ES)

This approach was originally proposed by Rechenberg (1965), it is the first genuine Metaheuristic and the first evolutionary algorithm. In its basic version the algorithm manipulates iteratively a set of real variables vectors, using mutation and selection operators. The mutation step is typically performed by adding a random value, drawn within a normal distribution. The selection is made by a deterministic choice of the best individuals, according to the value of the fitness function.

Evolution Strategies use a set of μ parents to produce λ offsprings. To produce each offspring, ρ parents are recombined. Then the produced offsprings are mutated. Selection step could be applied either only for offsprings either for parents and offsprings together. In the first case the algorithm is noted $(\mu,\lambda)-ES$ and the second algorithm is noted $(\mu+\lambda)-ES$ Schoenauer and Michèle (1987). The new actual methods use crossover operator to avoid being trapped in local optima.

Genetic Algorithm (GA)

Genetic algorithms are stochastic search techniques and theoretical foundations were established by Holland, (1975). They are inspired from Darwin theory: the natural evolution of living species. There are two mechanisms allowing evolve living species: natural selection and reproduction. Natural selection favors the most adapted population individuals to their environment. The selection is followed by reproduction, performed by crossovers and mutations within individuals' genes. Thus, two parents intersect and transmit some of their genetic heritage to their offspring. In addition some individuals' genes could mutate during the reproductive phase. The combination of these two mechanisms leads to a more adapted population to its environment. In their canonical version, Genetic Algorithms suffer most often of slow convergence or premature problems.

2.2.2. Simulated Annealing Algorithm

This probabilistic Metaheuristic is inspired from physical process of annealing the crystalline materials. This process consists at heating a material at high temperature and then it must be slowly cooling to enhance its crystals' size. Atoms of heated material have a lofty energy that causes them to change positions and they can perform large random movements in the material. The slow cooling reduces atoms' energy and their movement capacity. The different cooling transitory states make it possible to obtain homogeneous materials with good quality. To implement this process by an optimization method, random movements of each point will be associated with a probability of a dependent variable representing the temperature of the material. The link between this algorithm and optimization problems has been proposed for the first time by Pincus (1970), but Kirkpatrick et al. (1983) and Cerny (1985), in a separate research works, are the pioneer of the developed form of simulated annealing algorithm. The simulated annealing algorithm becomes fast popular, owing to its easy adaptation to various problems and its efficiency. However, the principal disadvantage of this algorithm is the large parameters number (initial temperature, the temperature decrease rule, the temperature stages' duration, etc...), that makes it quite empirical. Another drawback is the slowness of this method.

2.2.3. Taboo Search

Glover (1986) introduces Taboo search algorithm. This Metaheuristic is a mathematical optimization method used to solve combinatorial problems. This algorithm improves local search performance by adding to the research process a memory describing visited solutions. From a given position, Taboo search algorithm explores the neighborhood of this position and chooses a new one that optimizes the objective function. This search procedure is iteratively repeated until fixed criterions are satisfied. Thus every potential solution is marked as Taboo and added to the memory, also called Taboo list. This stored solutions list couldn't be visited in the next iterations. The main advantage of the Taboo search algorithm is having less parameter than simulated annealing algorithm. However, the algorithm isn't always efficient, it is often appropriate to add him other intensification and/or diversification processes that involve introduction of new parameters Glover and Laguna (1997).

2.2.4. Algorithms based on swarm intelligence

Collective intelligence refers to communities' cognitive abilities resulting from multiple interactions between community members also called agents. From a simple behavior, agents could perform complex tasks owing a fundamental mechanism known synergy. Under particular conditions, created synergy through collaboration between individuals emerges some opportunities of representation, creation and learning better than isolated individuals. The collective intelligence forms are various according to community types and members that met. These forms of intelligence can be seen principally in social insects (ants, bees...) and animals in movement (migrating birds, fish schools). Based on these phenomenons several algorithms have been created like ant colonies and particle swarm.

Ant Colony Algorithms (ACA)

Ant colony algorithms are born from a simple observation. Insects, particularly ants, solve naturally complex problems. The principal factor that facilitates this behavior is that ants communicate with each other indirectly owing secretions of some chemical substances called pheromones. This indirect communication type is called stigmergy. According Goss et al. (1989) if an obstacle is introduced in ants path, they will all tend, after a research phase, to follow the shortest way between nest and obstacle. They are more attracted to the area where the pheromone substance rate is highest. First algorithms inspired from this analogy were proposed by Colorni et al. (1992) and Dorigo et al. (1996) to resolve problem of business traveler. In these algorithms each solution is considered as an ant moving in the search space. Ants mark better solutions and take account of previous markings to optimize their research. Ant colony algorithms use an implicit probability distribution to perform the transition between iterations.

These algorithms have been extended to resolve several discrete and continuous optimization problems, (Dorigo and Blum (2005); Siarry et al. (2006)). Ant colony algorithms have several advantages such as high intrinsic parallelism, robustness (a colony can maintain an effective search even if some of its individuals are defective) or decentralized (the ants do not obey a centralized authority).

Particle Swarm Optimization (PSO)

Particle Swarm Optimization is a Metaheuristic proposed by Kennedy and Eberhart (1995). This method inspired from animals movement swarms. The most used example is the behavior of fish school (Wilson (1975) and Reynolds (1987)). Indeed, these animals are characterized by a relatively complex and

Business Excellence and Management
Volume 3 Issue 4 / December 2013

Business Excellence and Management
Volume 3 Issue 4 / December 2013

dynamic movement. However, each one has a limited intelligence and local knowledge focused only on its position in the swarm. So, each individual has information only about the position and the speed of its nearest neighbors. As a consequence, each individual's own movement is influenced by, both, its own memory and local information of nearest neighbors. Simple rules, such as "go in same speed as others", "moving in the same direction" or "stay close neighbors" are among key behaviors that maintain cohesion of the swarm and allow the implementation of complex and adaptive collectives behaviors. Local interactions between different particles create the global intelligence behavior of the swarm. Therefore, system performance as whole is greater than the cumulative performance of its different parts. Potential solutions of this algorithm are presented by particles dispersed over the search space to seek the global optimum. Particle's movement is influenced by three components: First, the particle follows the current movement direction (Physical component). Second, the particle moves towards the best place by which it has already past (Cognitive component). Third, the particle relies on congener's experience, and thus move towards the best already reached area by its neighbors (Social component).

Like other Metaheuristics, PSO has several control settings such as: swarm size, maximum size of particles neighborhood, maximum velocity.... The parameter setting is a long and difficult process. As each parameter have a strong influence on the algorithm behavior it is important providing for each problem the suitable set of parameters.

3. MULTI-OBJECTIVE METAHEURISTICS AND OPTIMAL PORTFOLIU CHOISE

All Metaheuristics were originally conceived for solving mono-objective problems. Mono-objective optimization algorithms are based on minimizing or maximizing a single objective function which does not reflect the real system that must be optimized. On the other hand, the multi-objective optimization allows optimization of complex problems characterized by the presence of more than one objective function. Compared to mono objective optimization, multi-objective optimization problems are harder to be solved because they haven't a single solution but a set of solutions.

3.1. Survey on the mono-objective applications in the problems of portfolio choices and assets allocation

Among the first papers interested by the application of multi-objective Metaheuristics on portfolio optimization problem we explore this of Doerner et al., (2001). The paper investigates a multi-objective portfolio selection problem by using as meta-heuristic the Multi-objective Ant Colony Algorithm. In the same year another paper of Lin et al., (2001) investigates the multi-objective portfolio selection problem. This study has linked to the problem other considerations like fixed transaction costs and linear constraints on invested capital. To solve this constrained problem authors use as a multi-objective Metaheuristic the “Non dominated Sorting Genetic Algorithm” (NSGA-II). The adoption of an integer encoding instead of real valued encoding, has invited authors to make radical transformations on the genetic operators used by the NSGA-II. The main idea of this paper is to introduce the better variance as well as the better expected return into the population providing a reduction of range-dependency between the non-dominated solutions. Fieldsend et al., (2004) investigates the mean-variance model of portfolio optimization problem with the presence of cardinality constraints. By cons, of other researches in this paper the cardinality isn't specified in advance. But, the cardinality of portfolios is defined as a supplementary objective function that must be minimized. This choice is explained by the probability of finding two equivalents portfolio one characterized by a lower cardinality and the author with a higher cardinality. Theoretically, portfolio with higher cardinality suffers usually of a considerable group of assets zero-weight. The proposed Metaheuristic to resolve this problem is the “Multiple Objective Evolutionary Algorithms” (MOEA). Streichert et al., (2004) uses a “Non dominated Sorting Genetic Algorithm” (NSGA-I) Metaheuristic for the resolution of a mean-variance model of portfolio optimization problem. The used algorithm is characterized by binary encoding and real-value encoding. This characteristic has required radical transformations on the genetic operators of used Metaheuristic. Subbu et al., (2005) treats the portfolio optimization problem while adopting a hybrid evolutionary multi-objective optimization process. This process based on linear programming and evolutionary computation and relies especially on “Pareto Sorting Evolutionary Algorithm” (PSEA). The model proposed in this paper takes account of different aspects of portfolio risks especially linked with asset-liability management. So the PSEA is initialized by a Random Linear Programming algorithm. The main advantage of this last cited algorithm is that providing initial solutions which are likely to meet the proposed constraints in the problem. So, this algorithm can give extreme limits sampled from the search space. Tsao and Liu (2006) applies a modified version of NSGA-II to a Mean-VaR portfolio framework and consider only a budget constraint in the model. The non-convexity of the VaR function makes it so complex and requires a lot of times for its simulation. In this case the mathematical approaches can’t be suitable adopted techniques. Authors propose adding a threshold to initial generation which randomly generated. Vassiliadis and Dounias (2008) investigates the problem of constrained portfolio optimization problem while applying a bee colony optimization algorithm. However, Mishra et al., (2009) opts for a multi-objective particle swarm optimization approach to resolve the said problem. In the other hand, Branke et al., (2009) adds a variety of non-convex constraints to this problem and resolves it based on an “Envelope-based Multi-Objective Evolutionary Algorithm” (EMOEA). These constraints are inspired from a specific rule in the German investment law. This rule stipulates that: First, the total holdings exceeding five percent are less than fourteen percent of the net asset value of the fund. Second, the share of each asset is not superior than ten percent of the net asset value of the fund. Third, asset shares of the same issuer are no more than five percent of the net asset value of the fund. More recently, Ardia et al., (2010) and Krink and Paterlini, (2011) investigate multi-objective portfolio optimization problem with realistic constraints and resolve the problem based on a differential evolution based stochastic-search heuristic methods.

3.2. A survey on some multi-objective Metaheuristics

The optimum concept in multi-objective problems is different from that of mono-objective problems. So we don't seek a sole global optimum, but an area of solution that gives the better compromise between objectives (Pareto (1896)).

3.2.1. Evolutionary Algorithms

Evolutionary algorithms are widely used to solve multi-objective problems. A comparative study of evolutionary algorithms for multi-objective optimization is available in Zitzler et al. (1999). These methods are based on the Pareto approach and can be classified into two key families of algorithms: the non-elitist and elitist.

Non-Elitist approach

This approach contains three principal techniques. The first technique called "Multiple objective genetic algorithms" (MOGA) proposed by Fonseca and Fleming (1995). The first step of this method aims to rank each individual according to the number of individuals that dominate it. Thus, if an individual is dominated by J other individuals, the rank attributed to this individual is equal to J+1. In a second step, authors apply a scaling function on the rank value in order to assess the fitness of each individual. This function is usually linear. The use of ranking selection tends to distribute the population around the same optimum. But this represents a drawback for a decision-maker because this method provides only one solution. To prevent this tendency, Fonseca and Fleming use a sharing function to distribute the population on the entire Pareto frontier. This method gives high quality of solutions and its implementation is easy. But its performance is dependent on the parameters setting of the sharing function.

The second technique named "Non dominated Sorting Genetic Algorithm" (NSGA) proposed by Srivinas and Deb (1993). In this algorithm the fitness calculation is done by dividing the population into several groups while taking account of individual dominance degree. The algorithm of the rating function is done in four steps. First, one seeks non-dominated individuals in the entire population. These latter constitute the initial Pareto frontier. Second, a factitious fitness value will be assigned to each selected individual. This value is supposed to give an equal reproductive opportunity to all individuals. But it is necessary to apply a sharing function on this value to maintain the population diversity. Third, the initial selected individuals are removed from population. Finally, one restarts this procedure to determine the second Pareto frontier. This method is less efficient in computational time than the MOGA method. But the use of sharing in the state space and sorting solutions in different frontiers appears more appropriate to maintain population diversification and more efficient allocation of solutions on the Pareto frontier. In addition, this method is applicable to problems with any number of objectives. In other hand, it is interesting the use a population sort heuristic to distribute the population on the Pareto frontier. But this procedure slows down the convergence of the algorithm. In addition, this drawback is accentuated by the use of the method of selection by a stochastic remnant.

Horn and Nafpliotis (1993) are the pioneers of the third algorithm called “Niched Pareto Genetic Algorithm" (NPGA) based on the Pareto dominance notion. This method aims to select randomly two individuals and compare them to a sub-population randomly chosen and have for size t. If a sole individual dominates the sub-population, then it is positioned in the next population. In other cases a sharing function is applied to select the individual. The t parameter can exert a variable pressure on the population and thus increases or decreases the algorithm’s convergence. The sub-population size represents a new parameter that must be fixed by the user in addition to the sharing parameters. This approach is faster than previous approaches and has high solutions quality because the sharing is applied only on a portion of the population.

Elitist Approach

The presented approaches in the previous part are considered as non-elitist. First, they don't keep Pareto-optimal individuals found over time. Second, they maintain hardly diversity on the Pareto frontier. Finally, the solutions convergence towards the Pareto frontier is slow.

To resolve the above difficulties some new techniques have been developed. Zitzler and Thiele (1998) propose a new multi-objective optimization called “Strength Pareto Evolutionary Algorithm” (SPEA). This algorithm is based on Pareto concept to compare solutions. So, a set of optimal-Pareto solutions is kept in an external population called archive. According to this method, the fitness of each individual is calculated in relation to solutions stored in the archive. The transition from a generation to another begins with the update of the archive. All non-dominated individuals are stored into archive and dominated individuals are removed. If the number of individuals in the archive exceeds a given number, one applies a clustering technique to reduce it. Then the fitness of each individual is updated before selection. Finally, modification genetic operators are applied. This method distributes solutions effectively on the Pareto frontier. The rating technique allows sampling individuals in the space. However, the main drawback of this method is that rating depends of external population size chosen by user.

“Pareto Archived Evolution Strategy” (PAES) was originally developed as a local search method in an off-line routing information problem. Knowles and Corne (1999) show that this algorithm provides better results than research methods based on population. This method isn’t based on a population and it requires only one individual to achieve the solutions. It uses a population of determined size to temporarily store the optimal-Pareto solutions, The used algorithm is very simple and inspired from an evolution strategy (1 +1) Rechenberg (1973) and It uses a technique of crowding based on a division into hyper-cubes of the objective space. This method's implementation is relatively simple. Also, not being based on a genetic algorithm, thus it avoids the setting of all its parameters. But its effectiveness depends of the new parameter choice allowing the discretization of goals space.

Knowles et al., (2000) proposes the “Pareto Envelope based Selection Algorithm” (PESA). It is approximately based on the crowd principle developed in PAES and defines a parameter called squeeze_factor representing the space congestion assesses. PAES is based on an evolution strategy while PESA is a method based on genetic algorithms. It defines two parameters on the populations’ size: internal population size and external population size or archive. The Squeeze_factor parameter is equal to the number of individuals existing in the same hypercube. It’s used as fitness of the individuals belonging to this zone. This parameter is used to select and to update the archive while in PAES the measure of congestion is only used to update the archive. The main difference compared to PAES is that the selection is based on the measure of the objective space congestion. This allows a good distribution of individuals in the state space, but it increases the dependence of the method efficiency compared to the discretization space factor.

Deb (2000) develops a new version of NSGA called (NSGA-II) that solves some criticisms of the first method such as complexity, non-elitism and the use of sharing. The complexity of the algorithm NSGA is mainly due to the creating process of different borders. To reduce the computational complexity of NSGA, Deb proposes a modification of the sorting procedure of population in several borders. The previews version of NSGA uses sharing which a complex method and requires the setting of one or more parameter(s). In NSGA II, sharing function is replaced by a crowding function. He attributes two characteristics to each individual: The first is the non-domination rank of the individual. This feature depends on the boundary to which the individual belongs. And the second is the crowding distance of the individual and used to estimate the population density around it. To avoid the non-elitism criticism, the author uses in this method a tournament selection and modifies the transition process between two generations. If two solutions are selected to participate in the tournament, the lowest rank solution between them will be retained. But in the case of two similar ranks, it is best to use the point situated in a depopulated area having an important distance value. Finally, this new version of NSGA reduces the algorithm complexity, creates a more elitist method and deletes settings sharing.

Corne (2001) has develops also a new version of PESA named PESA-II. It is a new selection technique based on the use of hyper-cubes in the objective space. Instead of making a selection according to the fitness of individuals as in PESA, this method makes a selection of hyper-cubes occupied by at least one individual. After, one randomly chooses the individual from the selected hypercube. This method is more efficient to spread the solutions over the Pareto frontier. This is due to its ability to choose individuals located in desert areas with a probability higher than the classic tournament. PESA-II allows evolving positively the selection, so it favors the less congested space areas.

3.2.2. Simulated Annealing Algorithm

The simulated annealing method in multi-objective optimization was first addressed using an aggregation perspective (Serafini (1992) and Friesz (1993)). Then, two methods became most popular:

Multiple Objective Simulated Annealing (MOSA)

This method proposed by Ulungu et al. (1999) and it uses the simulated annealing characteristics to find non-dominated solutions.

Pareto Archived Simulated Annealing (PASA)

This method proposed by Engrand (1997) and it uses an aggregate function of all objective function connected to an archived system of non dominated solutions.

3.2.3. Multi-objective algorithms based on swarm intelligence

Ant Colony Algorithms

In single-objective optimization, ant colonies algorithms are very popular in solving combinatorial problems. However, few studies exist in the case of multi-objective. Doerner et al. (2006) proposes an algorithm called P-ACO dedicated to solving the portfolios allocation problem. This algorithm shows good results, compared to NSGA and simulated annealing. The OCF algorithm of Gagné et al., (2004) is based on the same principle as P-ACO. On each iteration, ants change their objective function to optimize. At the end of the iteration, the ant with the best performance updates the pheromone trail.

Business Excellence and Management
Volume 3 Issue 4 / December 2013

Particle Swarm Optimization Algorithm

Ray and Liew (2002) proposes an algorithm using the Pareto dominance, which combines techniques from PSO with evolutionary algorithms. The authors use a density operator of the neighborhood to promote diversity in the swarm. Coello et al., (2002, 2004) develop an algorithm based on the use of an external archive. The update of the archive is done according to a geographical criterion. The search space is divided into hyper-cubes which we give notes, which are according to the non-dominated solutions number located in each of them. Leaders are chosen from the selected hyper-cube using a roulette selection operator based on their notes. A mutation operator is also used.

An hybrid approach PSO-evolutionary algorithm is proposed in (Srinivasan and Seow (2003)). The goal here is to apply the operators of evolutionary algorithms to make the mechanisms of the PSO more efficient. A selection operator is also used to ensure the convergence of non-dominated solutions. The authors don't use an external archive; the population at the last iteration constitutes the final solutions of the problem. Bartz-Beielstein et al., (2003) introduce an elitist strategy to PSO. Different operators of selection and elimination are designed to find the combination which produces the best approximation of the Pareto front. The disposal methods are based on the contribution of particles to the swarm's diversity. In contrast, selection operators are based on the objective functions values. Li, (2003) makes an adaptation of the main mechanisms of the NSGA-II to the multi-objective PSO. Leaders are selected from elements of the archive. We use two selection methods: a niching strategy and a neighborhood density strategy. Similarly, in (Raquel and Naval (2005)), the space distance (crowding distance), already used in NSGA-II, serves as a criterion to maintain the archive diversity. The leaders' selection is also from elements of the archive.

4. CONCLUSIONS

The funds Management problem is very broad and contains several ways to be investigated. A famous sub-problem in this issue is the assets combination can also be named asset allocation or portfolio optimization problem. The seminal paper of Markowitz (1952) is the basis of most advances occurred in this research area. The challenging nature and growing complexities of markets impose some financial and statistical formulations that are vital from various asset management perspectives. Taking account of this complexity mathematical techniques become unable to provide solutions for developed models in this area which a high number of constraints and objectives. However, the theoretical advances that appear in Metaheuristic approaches have provided a revolution and an on-going work in the portfolio optimization field.

Metaheuristics are stochastic methods which aim solving a large panel of problems without intervention of users. These methods are inspired from analogies with other fields such as physics, genetics, or ethologic. Metaheuristics have quickly had a great success thanks to their simplicity of use and their high modularity. They are easily adaptable in order to obtain the best possible performance for a limited time period.

In this article, we have surveyed some investigations linked to the portfolio optimization problem that use Metaheuristics. This investigation is divided into two parts. The first one is allowed to the applications of mono-objective Metaheuristics to portfolio optimization problem. However, the second part focused on application studies of multi-objective Metaheuristics to this problem. In addition to a recapitulation of some famous Metaheuristics approaches is attached to each part.

This research work may not be able to do inclusive justice in exploring the literature to as comprehensive and exhaustive it could be, but it believes to have enclosed the broader spectrum of available works.

REFERENCES

Aranha, C. and Iba, H. (2007), Modelling Cost into a Genetic Algorithm-Based Portfolio Optimization System by Seeding and Objective Sharing, *Evolutionary Computation*, CEC 2007, IEEE Congress 2007, IEEE, 196–203.

Ardia, D., Boudt, K., Carl, P., Mullen, K. M. and Peterson, B. (2010), *Differential Evolution (Deoptim) for Non-Convex Portfolio Optimization*, MPRA Paper 22135, University Library of Munich, Germany.

Bolshakova, I., Girlich, E. and Kovalev, M. (2009), *Portfolio Optimization Problems: A Survey*, Preprint: Faculty of Mathematics, Otto-von-Guericke University Magdeburg.

Branke, J., Scheckenbach, B., Stein, M., Deb, K. and Schmeck, H. (2009), Portfolio Optimization with an Envelope-Based Multi-Objective Evolutionary Algorithm, *European Journal of Operational Research* 199 (3), 684–693.

Cerny, V. (1985), Thermodynamical Approach to the Travelling Salesman Problem, *Journal of Optimization Theory and Applications*, 45(1), 41-51.

Chan, M.C., Wong, C.C., Cheung, B.K.S. and Tang, G.Y.N. (2002), Genetic Algorithms in Multi-Stage Portfolio Optimization System, in *Proceedings of the Eighth International Conference of the Society for Computational Economics*, Computing in Economics and Finance.

Chang, T., Meade, N., Beasley, J., and Sharaiha, Y., (2000), Heuristics for Cardinality Constrained Portfolio Optimization, *Computers and Operations Research* 27(13), 1271–1302.

Chang, T., Yang, S. and Chang, K. (2009), Portfolio Optimization Problems in Different Risk Measures Using Genetic Algorithm, *Expert Systems with Applications* 36(7), 10529–10537.

Chen, W., Zhang, R-T., Cai, Y-M. and Xu, F-S. (2006), Particle Swarm Optimization for Constrained Portfolio Selection Problems, in *International Conference on Machine Learning and Cybernetics*. 2425-2429.

Coello Coello, C.A. and Salazar Lechuga, M., (2002) MOPSO : A Proposal for Multiple Objective Particle Swarm Optimization, in *Proceedings of the 2002 IEEE Congress on Evolutionary Computation*, 2, IEEE Press, 1051-1056.

Coello Coello, C.A., Toscano Pulido, G. and Salazar Lechuga, M. (2004), Handling Multiple Objectives with Particle Swarm Optimization, *IEEE Transactions on Evolutionary Computation*, 8(3), 256-279.

Coello Coello, C.A., Veldhuizen, D.A. and Lamont, G.B. (2002), *Evolutionary Algorithms for Solving Multi-Objective Problems*, Kluwer Academic Publishers.

Colorni, A., Dorigo, M. and Maniezzo V., (1992), Distributed Optimization by Ant Colonies, in *Proceedings of the 1rst European Conference on Artificial Life*, Elsevier Publishing, 134-142.

Corne, D.W. (2001), PESA II: Region-based Selection in Evolutionary Muliobjective Optimization, in *Proceedings of the Genetic and Evolutionary Computation*, Conference GECCO'2001, San Francisco, California, 283-290.

Crama, Y. and Schyns, M. (2003), Simulated Annealing for Complex Portfolio Selection Problems, *European Journal of Operational Research*, 150(3), 546-571.

Deb, K. (2000), A Fast Elitist Non-Dominated Sorting Genetic Algorithm for Multiobjective Optimization: NSGA II, Parallel problem Solving form Nature – PPSN VI, *Springer Lecture Notes in Computer Science*, 849-858.

Doerner et al. (2006), Pareto Ant Colony Optimization: A Metaheuristic Approach to Multiobjective Project Portofolio Selection, *Annals of Operations Research*, 131, 79-99.

Doerner, K., Gutjahr, W.J., Hartl, R.F. and Strauss, C. (2001), Ant Colony Optimization in Multiobjective Portfolio Selection, in *4th Metaheuristics International Conference*, 243-248.

Dorigo, M. and Blum. C. (2005), Ant Colony Optimization Theory: A Survey, *Theoretical Computer Science*, 344, 243-278.

Dorigo, M., Maniezzo, V. and Colorni, A. (1996), Ant System: Optimization by a Colony of Cooperating Agents, *IEEE Transactions on Man*. Cyber., Part B, 26(1), 29-41.

Eddelbuttel, D. (1996), A Hybrid Genetic Algorithm for Passive Management, in *Working Paper presented at the Second conference on computing in economics and Finance, Society of computational economics.*

Eiben, A.E. and Smith, J.E. (2003), *Introduction to Evolutionary Computing*, Natural Computing Series, Springer.

Engrand P. (1997), A Multi-Objective Optimization Approach Based on Simulated Annealing and its Application to Nuclear Fuel Management, in *Proceedings of the 5th International Conference on Nuclear Engineering*, American Society Of Mechanical Engineers, 416-423.

Fieldsend, J., Everson, R. and Singh, S. (2003), Using Unconstrained Elite Archives for Multiobjective Optimization, *Evolutionary Computation*, IEEE Transactions 7(3), 305–323.

Figueroa-Lopez, J.E., (2005), A Selected Survey of Portfolio Optimization Problems

Fonseca, C.M. and Flemming, P.J. (1995), An Overview of Evolutionary Algorithms in Multiobjective Optimization, *Evolutionary Computation*, 3(1), 205-209.

Fraser, A.S. (1957), Simulation of Genetic Systems by Automatic Digital Computers, *Australian Journal of Biological Sciences*, 10, 484-491.

Friesz, T.L. (1993), The Multiobjective Equilibrium Network Design Problem Revisited: A Simulated Annealing Approach, *European Journal of Operational Research*, 65, 44-57.

Gagné, C., Gravel, M., and Price, W.L. (2004), Optimisation multiobjectif à l'aide d'un algorithme de colonies de fourmis, *Inf. Syst. and Operational Research*, 42(1), 23-42.

Geyer, A., Hanke, M. and Weissensteiner, A. (2009), A Stochastic Programming Approach for Multi-Period Portfolio Optimization, *Computational Management Science*, 6(2), 187-208.

Glover, F. (1986), Future Paths for Integer Programming and Links to Artificial Intelligence, *Computers and Operations Research*, 13, 533-549.

Glover, F. and Laguna, M. (1997), *Tabu Search*, Kluwer Academic Publishers.

Goss, S., Aron, S., Deneubourg, J.L. and Pasteels, J.M. (1989), Self-Organized Shortcuts in the Argentine Ant, *Naturwissenschaften*, 76, 579-581.

Hochreiter, R. (2008), Evolutionary Stochastic Portfolio Optimization, *Natural Computing in Computational Finance*, 67–87.

Holland, J. (1975), *Adaptation in Natural and Artificial Systems*, University of Michigan Press, Ann Harbor.

Horn. J. and Nafpliotis, N. (1993), Multiobjective Optimization using the Niched Pareto Genetic Algorithm, *Illigal* TR. n° 93005.

Kennedy, J. and Eberhart, R.C. (1995), Particle Swarm Optimization, in *Proceedings of the IEEE International Conference On Neural Networks*, IEEE Press, 1942-1948.

Kirkpatrick, S., Gelatt, S. and Vecchi, M. (1983), Optimization by Simulated Annealing, *Science*, 220(4598), 671-680.

Knowles, J.D. and Corne, D.W. (1999), The Pareto Archived Evolution Strategy: A New Baseline Algorithm for Multiobjective Optimisation, *Congress on Evolutionary Computation*, Washington, 98-105.

Knowles, J.D., Corne, D.W. and Oates, M.J. (2000), The Pareto-Envelope based Selection Algorithm for Multiobjective Optimization, in *Proceedings of the Sixth International Conference on Parallel Problem Solving from Nature (PPSN VI)*, Berlin, 839-848.

Krink, T. and Paterlini, S. (2011), Multiobjective Optimization Using Differential Evolution for Real-World Portfolio Optimization, *Computational Management Science*, 8(1), 157-179.

Li, X. (2003), A Non-Dominated Sorting Particle Swarm Optimizer for Multiobjective Optimization, in *Proceedings of the 2003 Genetic and Evolutionary Conference*, LNCS 2723, Springer, 37-48.

Lin, D., Li, X. and Li, M. (2005), A Genetic Algorithm for Solving Portfolio Optimization Problems with Transaction Costs and Minimum Transaction Lots, in *Lipo Wang, Ke Chen, and Yew Ong, editors, Advances in Natural Computation*, volume 3612 of *Lecture Notes in Computer Science*, 441-441. Springer Berlin / Heidelberg.

Lin, D., Wang, S. and Yan, H. (2001), A Multiobjective Genetic Algorithm for Portfolio Selection Problem, in *Proceedings of ICOTA* 2001, Hong Kong.

Markowitz, H. (1952), Portfolio selection, *The Journal of Finance,* 7(1), 77–91.

Mishra, S.K., Panda, G. and Meher, S. (2009), Multi-objective Particle Swarm Optimization Approach to Portfolio Optimization, in *World Congress on Nature and Biologically Inspired Computing*, 1612-1615.

Nishimura, K. (1990), On Mathematical Models of Portfolio Selection Problem, *Management Review*, 26(1), 369-391.

Pareto, V. (1986), *Cours d'économie politique*, vol. 1 et 2, F. Rouge, Lausanne.

Pincus, M. (1970), A Monte Carlo Method for the Approximate Solution of Certain Types of Constrained Optimization Problems, *Operational Research*, 18, 1225-1228.

Raquel, C.R. and Naval, J.P.C. (2005), An Effective Use of Crowding Distance in Multiobjective Particle Swarm Optimization, in *Proceedings of the 2005 Genetic and Evolutionary Computation Conference*, ACM Press, 257-264.

Ray, T. and Liew, K.M. (2002), A Swarm Metaphor for Multiobjective Design Optimization, *Engineering Optimization*, 34(2), 141-153.

Rechenberg, I. (1965), *Cybernetic Solution Path of an Experimental Problem*, Royal Aircarft Establishment Library Translation.

Rechenberg, I. (1973), *Evolutionsstrategie: Optimierung technischer Systeme nach Prinzipien der biologischen Evolution*, Stuttgart.

Reynolds, C.W. (1987), Flocks, Herds and Schools: A Distributed Behavioral Model, *Computer Graphics*, 21(4), 25-34.

Ruiz-Torrubiano, R. and Suarez, A. (2007), Use of Heuristic Rules in Evolutionary Methods for the Selection of Optimal Investment Portfolios, in *Evolutionary Computation*, CEC 2007, IEEE Congress 2007, IEEE, 212–219.

Schoenauer, M. and Michèle, S. (1987), Contrôle d'un algorithme génétique, *Revue d'intelligence artificielle*, 10(2-3), 389-428.

Serafini, P. (1992), Simulated Annealing for Multiple Objective Optimization Problems, in *Proceedings of the Tenth International Conference on Multiple Criteria Decision Making*, Taipei, 1, 87-96.

Siarry, P., Dréo, J., Pétrowski, A. and Taillard, E. (2006), *Métaheuristiques pour l'optimisation difficile*, Eyrolles.

Soleimani, H., Golmakani, H. and Salimi, M. (2009), Markowitz-based Portfolio Selection with Minimum Transaction Lots, Cardinality Constraints and regarding Sector Capitalization Using Genetic Algorithm, *Expert Systems with Applications* 36(3), 5058–5063.

Srinivasan, D. and Seow, T.H. (2003), Particle Swarm Inspired Evolutionary Algorithm for Multiobjective Optimization Problem, in *Proceedings of the 2003 Congress on Evolutionary Computation*, IEEE Press, 3, 2292-2297.

Srivinas, N. and Deb, K. (1993), *Multiobjective Optimization using Nondominated Sorting in Genetic Algorithms, Technical Report*, Departement of Mechanical Engineering, Institute of Technology, India.

Streichert, F., Ulmer, H. and Zell, A. (2004), Comparing Discrete and Continuous Genotypes on the Constrained Portfolio Selection Problem, in *Genetic and Evolutionary Computation–GECCO*, Springer, 1239–1250.

Subbu, R., Bonissone, P.P., Eklund, N.H.W., Bollapragada, S. and Chalermkraivuth, K.C. (2005), Multiobjective Financial Portfolio Design: A Hybrid Evolutionary Approach, in *Congress on Evolutionary Computation*, 1722–1729.

Thong, V. (2007), *Constrained Markowitz Portfolio Selection Using Ant Colony Optimization*, Erasmus Universiteit.

Tsao, C. and Liu, C. (2006), *Incorporating Value-at-risk in Portfolio Selection: An Evolutionary Approach*, Tech. rep., Working Paper.

Ulungu, E.L., Teghem, J., Fortemps, P. and Tuyttens, D. (1999), MOSA Method: A Tool for Solving Multiobjective Combinatorial Optimization Problems, *Journal of Multicriteria Decision Analysis*, 20, 221-236.

Vassiliadis, V. and Dounias, G. (2008), Nature Inspired Intelligence for the Constrained Portfolio Optimization Problem, in *John Darzentas, George Vouros, Spyros Vosinakis, and Argyris Arnellos, editors, Artificial Intelligence: Theories, Models and Applications*, 5138 of Lecture Notes in Computer Science, Springer Berlin / Heidelberg, 431-436.

Wilson, E.O. (1975), *Sociobiology: The new synthesis*, Belknap Press.

Yu, T-Y., Tsai, C. and Huang, H.-T. (2010), Applying Simulation Optimization to the Asset Allocation of a Property–Casualty Insurer, *European Journal of Operational Research*, 207, 499-507.

Zitzler, E., Deb K. and Thiele, L. (1999), *Comparison of Multiobjective Evolutionary Algorithms: Empirical Results*, Tech. Report n° 70, Swiss Federal Institute of Technology (ETH), Zurich.

Zitzler, E. and Thiele, L. (1998), *An Evolutionary Algorithm for Multiobjective Optimization: The Strength Pareto Approach*, TIKReport n° 43.