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Non-static cylindrically symmetric Maxwell's field in Bimetric relativity.

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ABSTRACT

Non-Static Cylindrically symmetric inhomogeneous model is studied in the context of Rosen's bimetric theory of relativity with the matter Maxwell's field and it is interesting to note that the model is free from singularity at t = 0.

Keywords: Static, Cylindrically symmetric, inhomogeneous, Maxwell's field, bimetric relativity.

INTRODUCTION

To remove the singularity which appears in general theory of relativity, Rosen's N [1,2] proposed a new theory known as bimetric theory of relativity within the frame work of general theory of relativity. In this theory there exist two matric tensors at each point of space time, g_{ij} which describes gravitation and background metric γ_{ij} which enters into the field equations and interact with g_{ij} but not interact directly with the matter. At each space time point there are two line elements –

$$ds^2 = g_{ij}\,dx^i dx^j$$

and

$$d\sigma^2 = \gamma_{ij}\,dx^i dx^j$$

where ds is interval between two neighboring elements and can be measured by a clock and a measuring rod and the interval dσ is an abstract and it is not directly measurable.

Krori K.D., Barua J. [3] has studied Magnetic energy and cylindrically symmetric distribution of matter in equilibrium in general relativity.

Leteliar [4] has studied spherical symmetric, plane symmetric and some particular cases of cylindrical symmetric space-time and obtained general solutions of Einstein field equations for the matter cosmic cloud string. Baysal H. et.al.[5] also studied cylindrically symmetric inhomogeneous string universe in general theory of relativity and physical and geometrical properties of this model have been studied. Mahurpawar and Deo [6] observed that cosmic string with cylindrical symmetric cosmological model does not exist in bimetric theory of relativity. Deo [7] has studied cylindrically symmetric universe with the matter cosmic string and domain walls. Deo et.al. [8] studied cylindrically symmetric inhomogeneous Maxwell's field coupled with perfect fluid in bimetric relativity and vacuum solutions are obtained. Deo and Qureshi [9] have studied cylindrically symmetric space time with bulk viscous fluid in Rosen's bimetric theory of relativity and observed that bulk viscous fluid does not exist in this theory. P.K. Sahoo, and B. Mishra [10] have studied Cylindrically symmetric cosmic strings coupled with Maxwell's fields in bimetric relativity.

In this paper we have studied non-static cylindrically symmetric Maxwell's field in Bimetric relativity and observed that the resulting space time is free from singularity at time t = 0.

Field equation in Bimetric theory of relativity:

To remove the singularity which appears in general theory of relativity, in 1940 Rosen has proposed the new theory known as Bimetric theory of relativity within the frame work of general theory of relativity.
He has obtained the field equation from variation principle as

$$K_i^j = N_i^j - \frac{1}{2}N g_i^j = -8\pi k T_i^j \qquad (2.1)$$

Where,

$$N_i^j = \frac{1}{2}\gamma^{\alpha\beta}[g^{hj}g_{hi|\alpha}]_{|\beta} \qquad (2.2)$$

$N = N_\alpha^\alpha$, $k = \sqrt{\frac{g}{\gamma}}$, $g =$ det. (g_{ij}) and $\gamma = det.(\gamma_{ij})$ (2.3)

And a vertical bar (|) denotes the covariant differentiation with respect to γ_{ij}.

Cylindrically symmetric space time with Maxwell's field:

Consider cylindrical symmetric line element of the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2dy^2 + C^2dz^2 \qquad (3.1)$$

Where A, B & C are functions of t only
For corresponding equation (3.1) consider the line element for background metric γ_{ij} as

$$d\sigma^2 = dx^2 + dy^2 + dz^2 - dt^2 \qquad (3.2)$$

Since, γ_{ij} is the Lorentz metric i.e. (1,1,1,-1) , therefore γ- covariant derivative becomes the ordinary partial derivative.
And T_i^j – the energy momentum tensor for Maxwell's field is given by

$$T_i^j = E_i^j = F_{ir}F^{jr} - \frac{1}{4}F_{ab}F^{ab}g_i^j \qquad (3.3)$$

Where E_i^j is the electromagnetic energy tensor and F_{ir} is the electromagnetic field tensor.

It is assume that the commoving system contains a magnetic field along the axis of symmetry. Only the components $F_{13} = -F_{31}$ of the electromagnetic field tensor F_{ir} are different from zero. And Maxwell's field equation

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \qquad (3.4)$$

Give rise to

$$F_{13} = -F_{31} = H \text{ (Constant)} \qquad (3.5)$$

Using equation (2.1)-(2.3) and (3.1)-(3.5) we get

$$K_1^1 = \frac{1}{2}\left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right) + \frac{1}{2}\left(\frac{\ddot{C}}{C} - \frac{\dot{C}^2}{C^2}\right) = -4\pi K \frac{H^2}{A^2C^2}$$
(3.6)

$$K_2^2 = -\frac{1}{2}\left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right) + \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right) + \frac{1}{2}\left(\frac{\ddot{C}}{C} - \frac{\dot{C}^2}{C^2}\right) = 4\pi K \frac{H^2}{A^2C^2}$$
(3.7)

$$K_3^3 = \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}\right) + \frac{1}{2}\left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right) - \frac{1}{2}\left(\frac{\ddot{C}}{C} - \frac{\dot{C}^2}{C^2}\right) = -4\pi K \frac{H^2}{A^2C^2}$$
(3.8)

$$K_4^4 = \frac{1}{2}\left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right) + \frac{1}{2}\left(\frac{\ddot{C}}{C} - \frac{\dot{C}^2}{C^2}\right) = 4\pi K \frac{H^2}{A^2C^2}$$
(3.9)

Using equation (3.7) and (3.8) we get

$$\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} = 0$$

i.e. A=exp (C_1t) (3.10)

Using equation (3.6) and (3.9) we get

$$\left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right) + \left(\frac{\ddot{C}}{C} - \frac{\dot{C}^2}{C^2}\right) = 0$$

$$i.e.\ \frac{d}{dt}\left(\frac{\dot{B}}{B}\right) = -\frac{d}{dt}\left(\frac{\dot{C}}{C}\right) = K$$

After integration we get

B = exp($C_2t^2 + m_1$t) (3.11)

C = exp [-($C_3t^2 + m_2t$)] (3.12)

Where C_1, C_2, C_3, m_1, m_2 are constants of integration. Substituting values of A, B and C, equation (3.1) we have

$$ds^2 = e^{2\,C_1 t}(dx^2 - dt^2) + e^{2\,(C_2t^2 + m_1t)}\,dy^2 + e^{-2(C_3t^2 + m_2t)}\,dz^2$$
(3.14)

It is interesting to note that the equation (3.14) is free from singularity at t = 0.

Conflicts of interest: The authors stated that no conflicts of interest.

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