IRJSE *Int. Res. J. of Science & Engineering*, 2020; Special Issue A7: 305-315
ISSN: 2322-0015
SJIF Impact Factor 6.68
RESEARCH ARTICLE **OPEN ACCESS**

Two-Dimensional Transient Thermoelastic Problem of Semi-Infinite Solid Circular Cylinder

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Manuscript Details
Available online on http://www.irjse.in
ISSN: 2322-0015

Cite this article as:
Dange WK. Two-Dimensional Transient Thermoelastic Problem of Semi- Infinite Solid Circular Cylinder, *Int. Res. Journal of Science & Engineering,* February, 2020, Special Issue A7 : 306-315.

ABSTRACT

In this paper an attempt has been made to study thermoelastic problem of a semi- infinite solid circular cylinder occupying the space D: $0 \le r \le a$, $0 \le z < \infty$, where *a* radius of semi- infinite solid circular cylinder .The lower face $z = 0$ and outer curved surface of semi-infinite solid circular cylinder are maintained at zero temperature .Initially the temperature of semi-infinite solid circular cylinder is known . One apply integral transform techniques to find the thermoelastic solution. The results are obtained as series of Bessel functions. Numerical calculations are carried out for semi- infinite solid circular cylinder of aluminium metal and illustrated graphically.

Key words: thermoelastic problem, semi- infinite solid circular cylinder, thermoelastic solution, integral transform.

INTRODUCTION

As a result of the increased usage of industrial and construction materials the interest in isotropic thermal stress problems has grown considerably. However there are only a few studies concerned with the two-dimensional steady state thermal stress. Nowacki [1] has determined steady-state thermal stresses in a thick circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and circular edge. Wankhede [5] has determined the quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. However, there aren't many investigations on transient state.

Roy Choudhuri [4] has succeeded in determining the quasi-static thermal stresses in a circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and the fixed circular edge thermally insulated. Warsha K. Dange, has studied [6] two-dimensional transient thermoelastic problem for a hollow cylinder with internal heat generation and [7] two-dimensional transient thermoelastic problem of a annular disc due to radiation . In all aforementioned investigations, they have not however considered any thermoelastic problem with The lower face $z=0$ and outer curved surface of semi-infinite solid circular cylinder are maintained at zero temperature .Initially the temperature of semi-infinite solid circular cylinder is known . This paper is concerned with transient thermoelastic problem of a semi- infinite solid circular cylinder occupying the space D: 0 ≤ r ≤ a , 0 ≤ z < ∞,

Statement of the problem

Consider the semi-infinite solid circular cylinder whose axis is coincident with z-axis, defined by : 0 ≤ r ≤ a and 0 ≤ z < ∞, where *a* radius of solid circular cylinder .The lower face $z=0$ and outer curved surface of semi-infinite solid circular cylinder are maintained at zero temperature .Initially the temperature of semi-infinite solid circular cylinder is known . Under these boundary and initial conditions the temperature, Goodier's thermoelastic displacement potential, the displacement components in the radial and axial directions , dilatation , Michell's function , component of the stresses in semi-infinite solid circular cylinder are required to be determined.

The equation for $T(r,z,t)$, the temperature in cylindrical coordinates is:

$$\kappa\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]=\frac{\partial T}{\partial t} \tag{1}$$

Subject to the initial and boundary conditions

$$M_t(T,1,0,0)=\begin{cases} g(r)e^{\omega z} & at\ \ 0\le z\le 1 \\ \\ 0 & at\ \ z>1\ ,\ 0\le r\le a \end{cases} \tag{2}$$

$$M_r(T,1,0,a)=0,\ \text{for all}\ \ 0\le z<\infty,\quad t>0 \tag{3}$$

$$M_z(T,1,0,0)=0,\ \text{for all}\ \ 0\le r\le a\ ,\ t>0 \tag{4}$$

The most general expression for these conditions can be given by

$$M_v(f,k,\bar{\bar{k}},\$)=(\bar{k}f+\bar{\bar{k}}\hat{f})_{v=\$}$$

Where the prime (^) denotes differentiation with respect to v and k is thermal diffusivity of the solid circular cylinder. $\omega > 0$ is the constant .

The Navier's equations without the body forces for axisymmetric two-dimensional thermoelastic problem can be expressed as

$$\left.\begin{aligned} &\nabla^2 u_r-\frac{u_r}{r^2}+\frac{1}{1-2\upsilon}\frac{\partial e}{\partial r}-\frac{2(1+\upsilon)}{1-2\upsilon}\alpha_t\frac{\partial T}{\partial r}=0 \\ &\nabla^2 u_z-\frac{1}{1-2\upsilon}\frac{\partial e}{\partial z}-\frac{2(1+\upsilon)}{1-2\upsilon}\alpha_t\frac{\partial T}{\partial z}=0 \end{aligned}\right\} \tag{5}$$

where u_r and u_z are the displacement components in the radial and axial directions respectively and the dilatation *e* as

$$e=\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z} \tag{6}$$

The displacement function in the cylindrical coordinate system are represented by the Goodier’s thermoelastic displacement potential ϕ and Michell’s function M as

$$u_r = \frac{\partial\phi}{\partial r} - \frac{\partial^2 M}{\partial r\partial z}, \tag{7}$$

$$u_z = \frac{\partial\phi}{\partial z} + 2(1-\upsilon)\nabla^2 M - \frac{\partial^2 M}{\partial^2 z} \tag{8}$$

in which Goodier’s thermoelastic potential must satisfy the equation

$$\nabla^2\phi = \left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t T \tag{9}$$

and the Michell’s function M must satisfy the equation

$$\nabla^2(\nabla^2 M) = 0 \tag{10}$$

Where

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}$$

The component of the stresses are represented by the use of the potential ϕ and Michell’s function M as

$$\sigma_{rr} = 2G\left\{\left(\frac{\partial^2\phi}{\partial r^2} - \nabla^2\phi\right) + \frac{\partial}{\partial z}\left(\upsilon\nabla^2 M - \frac{\partial^2 M}{\partial r^2}\right)\right\}, \tag{11}$$

$$\sigma_{\theta\theta} = 2G\left\{\left(\frac{1}{r}\frac{\partial\phi}{\partial r} - \nabla^2\phi\right) + \frac{\partial}{\partial z}\left(\upsilon\nabla^2 M - \frac{1}{r}\frac{\partial M}{\partial r}\right)\right\}, \tag{12}$$

$$\sigma_{zz} = 2G\left\{\left(\frac{\partial^2\phi}{\partial r^2} - \nabla^2\phi\right) + \frac{\partial}{\partial z}\left((2-\upsilon)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right\}, \tag{13}$$

and

$$\sigma_{rz} = 2G\left\{\frac{\partial^2\phi}{\partial r\partial z} + \frac{\partial}{\partial r}\left((1-\upsilon)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right\} \tag{14}$$

where G and υ are the shear modulus and Poisson’s ratio respectively.
The equations (1) to (14) constitute the mathematical formulation of the problem under consideration.

Solution of the problem

Determination Temperature Function $T(r,z,t)$**:**

By applying finite Hankel transform over variable r to the equations (1) ,(2) and (4) and using (3) to reduce differential equation in Hankel transform domain and then applying Fourier transform over variable over variable z and making use of respective inversion over the heat conduction equation one obtains the expression for temperature distribution in semi-infinite solid circular cylinder as

$$T(r,z,t) = \frac{4}{\pi a^2}\sum_{n,m=1}^{\infty}\frac{J_0(\mu_n r)}{[J_0'(\mu_n a)]^2}\int_0^{\infty}\frac{\bar{g}(\mu_n)}{\beta_m^2+\omega^2}\left([\omega\sin\beta_m - \beta_m\cos\beta_m]e^{\omega} + \beta_m^2\right)d\beta_m$$

$$\left[\exp[-k(\mu_n^2+\beta_m^2)t\right]\sin\beta_m z \tag{15}$$

Where μ_n are the positive roots of the transcendental equation $J_0(\mu_n a) = 0$

Thermoelastic solution :

Referring to the fundamental equation (1) and its solution (15) for the heat conduction problem, the solution for the displacement function are represented by the Goodier's thermoelastic displacement potential ϕ governed by equation (9) are represented by

$$\phi(r,z,t) = -\left(\frac{1+\upsilon}{1-\upsilon}\right)\frac{4\alpha_t}{\pi a^2}\sum_{n,m=1}^{\infty}\frac{J_0(\mu_n r)}{(\mu_n^2+\beta_m^2)[J_0'(\mu_n a)]^2}\int_0^{\infty}\frac{\bar{g}(\mu_n)}{\beta_m^2+\omega^2}\left([\omega\sin\beta_m-\beta_m\cos\beta_m]e^{\omega}+\beta_m^2\right)d\beta_m$$

$$\left[\exp[-k(\mu_n^2+\beta_m^2)t\right]\sin\beta_m z \qquad (16)$$

Similarly, the solution for Michell's function M are assumed so as to satisfy the governed condition of equation (9) as

$$M(r,z,t) = -\left(\frac{1+\upsilon}{1-\upsilon}\right)\frac{4\alpha_t}{\pi a^2}\sum_{n,m=1}^{\infty}\frac{J_0(\mu_n r)}{(\mu_n^2+\beta_m^2)[J_0'(\mu_n a)]^2}\int_0^{\infty}\frac{\bar{g}(\mu_n)}{\beta_m^2+\omega^2}\left([\omega\sin\beta_m-\beta_m\cos\beta_m]e^{\omega}+\beta_m^2\right)d\beta_m$$

$$\left[\exp[-k(\mu_n^2+\beta_m^2)t\right]\left[\sinh(\mu_n z)+z\cosh(\mu_n z)\right] \qquad (17)$$

In this manner two displacement functions in the cylindrical coordinate system ϕ and M are fully formulated. Now, in order to obtain the displacement components, we substitute the values of thermoelastic displacement potential ϕ and Michell's function M in equations (7) and (8), one obtains

$$u_r = -\left(\frac{1+\upsilon}{1-\upsilon}\right)\frac{4\alpha_t}{\pi a^2}\sum_{n,m=1}^{\infty}\frac{\mu_n\, J_0'(\mu_n r)}{(\mu_n^2+\beta_m^2)[J_0'(\mu_n a)]^2}\int_1^{\infty}\frac{\bar{g}(\mu_n)}{\beta_m^2+\omega^2}\left([\omega\sin\beta_m-\beta_m\cos\beta_m]e^{\omega}+\beta_m^2\right)d\beta_m$$
$$\left[\exp[-k(\mu_n^2+\beta_m^2)t\right]\left[\sin(\beta_m z)-(\mu_n+1)\cosh(\mu_n z)-\mu_n z\sinh(\mu_n z)\right] \qquad (18)$$

$$u_z = -\left(\frac{1+\upsilon}{1-\upsilon}\right)\frac{4\alpha_t}{\pi a^2}\sum_{n,m=1}^{\infty}\frac{J_0(\mu_n r)}{(\mu_n^2+\beta_m^2)[J_0'(\mu_n a)]^2}\int_1^{\infty}\frac{\bar{g}(\mu_n)}{\beta_m^2+\omega^2}\left([\omega\sin\beta_m-\beta_m\cos\beta_m]e^{\omega}+\beta_m^2\right)d\beta_m$$
$$\left[\exp[-k(\mu_n^2+\beta_m^2)t\right]\left[\beta_m\cos(\beta_m z)+[2(1-2\upsilon)+\mu_n]\mu_n\sinh(\mu_n z)+z\cosh(\mu_n z)\right] \qquad (19)$$

Thus, making use of the two displacement components the dilation is established as

$$e = -\left(\frac{1+\upsilon}{1-\upsilon}\right)\frac{4\alpha_t}{\pi a^2}\sum_{n,m=1}^{\infty}\frac{J_0(\mu_n r)}{(\mu_n^2+\beta_m^2)[J_0'(\mu_n a)]^2}\int_1^{\infty}\frac{\bar{g}(\mu_n)}{\beta_m^2+\omega^2}\left([\omega\sin\beta_m-\beta_m\cos\beta_m]e^{\omega}+\beta_m^2\right)d\beta_m$$
$$\left[\exp[-k(\mu_n^2+\beta_m^2)t\right]\left[(\mu_n^2-\beta_m^2)\sin(\beta_m z)+\mu_n(z-\mu_n^2)\sinh(\mu_n z)+(1-2\upsilon)\mu_n^2\cosh(\mu_n z)\right] \qquad (20)$$

Then, the stress components can be evaluated by substituting the values of thermoelastic displacement potential ϕ from equation (16) and Michell's function M from equation (17) in equations (11), (12), (13) and (14), one obtains

$$\sigma_{rr} = -\left(\frac{1+\upsilon}{1-\upsilon}\right)\frac{8\alpha_t G}{\pi a^2}\sum_{n,m=1}^{\infty}\frac{1}{(\mu_n^2+\beta_m^2)[J_0'(\mu_n a)]^2}\int_1^{\infty}\frac{\bar{g}(\mu_n)}{\beta_m^2+\omega^2}\left([\omega\sin\beta_m-\beta_m\cos\beta_m]e^{\omega}+\beta_m^2\right)d\beta_m$$
$$\left\{J_0(\mu_n r)\left[(\mu_n^2+\beta_m^2)\sin(\beta_m z)+2\mu_n^2\cosh(\mu_n z)\right]+J''_0(\mu_n r)\left[\mu_n^2\sin(\beta_m z)-\mu_n^2(\mu_n+1)\cosh(\mu_n z)-z\mu_n^3\sinh h(\mu_n z)\right]\right\}$$
$$\left[\exp[-k(\mu_n^2+\beta_m^2)t\right] \qquad (21)$$

$$\sigma_{\theta\theta} = -\left(\frac{1+\upsilon}{1-\upsilon}\right)\frac{8\alpha_t G}{\pi a^2}\sum_{n,m=1}^{\infty}\frac{1}{(\mu_n^2+\beta_m^2)[J_0'(\mu_n a)]^2}\int_1^{\infty}\frac{\bar{g}(\mu_n)}{\beta_m{}^2+\omega^2}\left([\omega\sin\beta_m-\beta_m\cos\beta_m]e^{\omega}+\beta_m{}^2\right)d\beta_m$$
$$\left\{\frac{J_0(\mu_n r)}{r}\left[\sin(\beta_m z)-\sinh h(\mu_n z)-z\cosh(\mu_n z)\right]+J_0(\mu_n r)\left[(\mu_n^2+\beta_m^2)\sin(\beta_m z)+2\upsilon\mu_n^2\cosh(\mu_n z)\right]\right\}$$
$$\left[\exp[-k(\mu_n{}^2+\beta_m{}^2)t\right] \tag{22}$$

$$\sigma_{zz} = -\left(\frac{1+\upsilon}{1-\upsilon}\right)\frac{8\alpha_t G}{\pi a^2}\sum_{n,m=1}^{\infty}\frac{1}{(\mu_n^2+\beta_m^2)[J_0'(\mu_n a)]^2}\int_1^{\infty}\frac{\bar{g}(\mu_n)}{\beta_m{}^2+\omega^2}\left([\omega\sin\beta_m-\beta_m\cos\beta_m]e^{\omega}+\beta_m{}^2\right)d\beta_m$$
$$\left\{J_0''(\mu_n r)\left[\mu_n^2\sin(\beta_m z)\right]+J_0(\mu_n r)\left[(\mu_n^2+\beta_m^2)\sin(\beta_m z)+(1-2\upsilon)\mu_n^2\cosh(\mu_n z)-\mu_n^3(\cosh(\mu_n z)+z\sinh(\mu_n z))\right]\right\}$$
$$\left[\exp[-k(\mu_n{}^2+\beta_m{}^2)t\right] \tag{23}$$

$$\sigma_{rz} = -\left(\frac{1+\upsilon}{1-\upsilon}\right)\frac{8\alpha_t G}{\pi a^2}\sum_{n,m=1}^{\infty}\frac{\mu_n J_0(\mu_n r)}{(\mu_n^2+\beta_m^2)[J_0'(\mu_n a)]^2}\int_1^{\infty}\frac{\bar{g}(\mu_n)}{\beta_m{}^2+\omega^2}\left([\omega\sin\beta_m-\beta_m\cos\beta_m]e^{\omega}+\beta_m{}^2\right)d\beta_m$$
$$\left[\beta_m\cos(\beta_m z)-\mu_n^2(\sinh(\mu_n z)+z\cosh(\mu_n z))\right]\left[\exp[-k(\mu_n{}^2+\beta_m{}^2)t\right] \tag{24}$$

Numerical Results, Discussion and Remarks:

To interpret the numerical computations, we consider material properties of Aluminium metal, which can be commonly used in both wrought and cast forms. The low density of aluminium results in its extensive use in the aerospace industry.

Poisson ratio, υ	0.281
Thermal expansion coefficient, α_t (cm/cm-^{0}C)	25.5×10^{-6}
Thermal diffusivity, κ (cm^2/sec)	0.86
Inner radius, a (cm)	1

Table 1: Material properties and parameters used in this study

In the foregoing analysis are performed by setting the radiation constants, r_0 = 0.7cm, and set $\omega = 1$

$g(r) = \delta(r - r_0)$

Equations (15) becomes

$$T(r,z,t) = \frac{4}{\pi a^2}\sum_{n,m=1}^{\infty}\frac{r_0 J_0(\mu_n r_0)J_0(\mu_n r)}{[J_0'(\mu_n a)]^2}\int_0^{\infty}\frac{1}{\beta_m{}^2+1}\left([\sin\beta_m-\beta_m\cos\beta_m]e+\beta_m{}^2\right)d\beta_m$$
$$\left[\exp[-k(\mu_n{}^2+\beta_m{}^2)t\right]\sin(\beta_m z) \tag{25}$$

The derived numerical results from equation (15) to (24) have been illustrated graphically in figures 1 to 9.

Figure 1 represents graph of temperature $T(r,z,t)$ versus r for different values of t. It is observed that unknown temperature $T(r,z,t)$ develops tensile stress within semi-infinite solid circular cylinder in the circular region $r = 0.5 to r = 0.7$ and develops compressive stress in circular region

$r = 0.3 to r = 0.5$ & $r = 0.7 to r = 0.9$ It is also observed that $T(r,z,t)$ goes on decreasing to the point $r = 0.3$ then goes on increasing from to $r = 0.9$ of the semi-infinite solid circular cylinder.

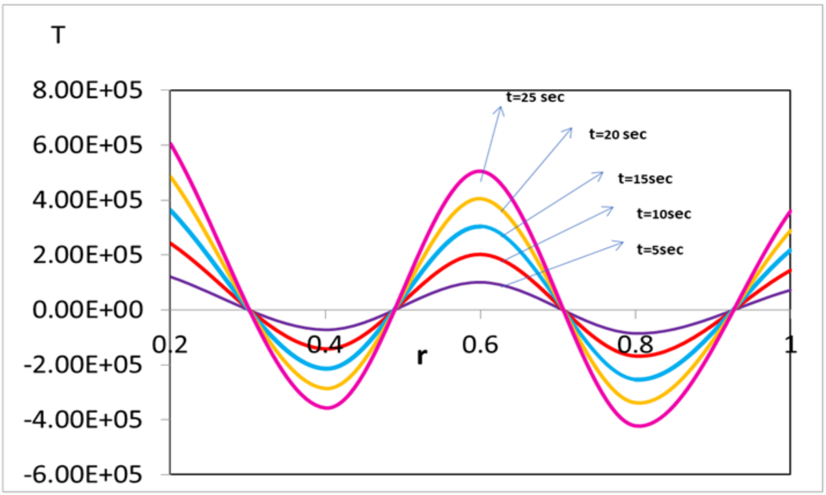

Fig 1 Graph of $T(r,z,t)$ versus r for different values of t

Figure 2 represents graph of $\phi(r,z,t)$ versus r for different values of t. It is observed that $\phi(r,z,t)$ develops tensile stress within semi-infinite solid circular cylinder in the circular region $r = 0.4 to r = 0.7$ and develops compressive stress in circular region $r = 0.7 to r = 0.9$. It is also observed that $\phi(r,z,t)$ goes on increasing to $r = 0.4$ & from $r = 0.9$ of the semi-infinite solid circular cylinder.

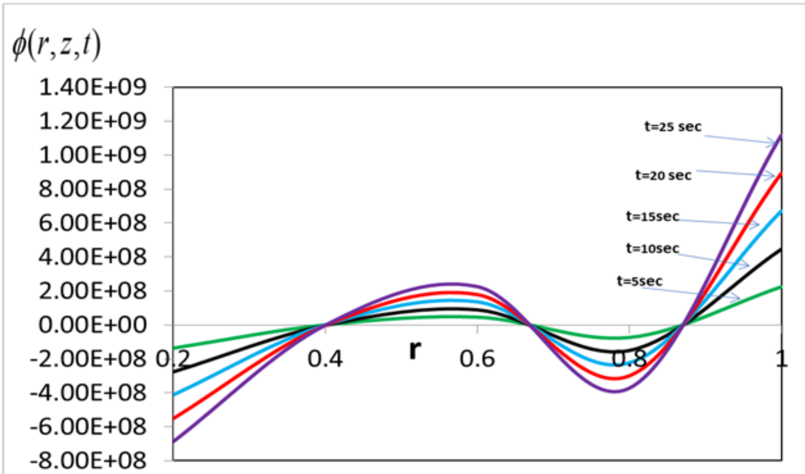

Fig 2 : Graph of $\phi(r,z,t)$ versus r for different values of t

Figure 3 represents graph of $M(r,z,t)$ versus r for different values of t. It is observed that $M(r,z,t)$ develops tensile stress within semi-infinite solid circular cylinder in the circular region $r = 0.4 to r = 0.8$. It is also observed that $M(r,z,t)$ goes on increasing to $r = 0.4$ & decreasing from $r = 0.83$ of the semi-infinite solid circular cylinder.

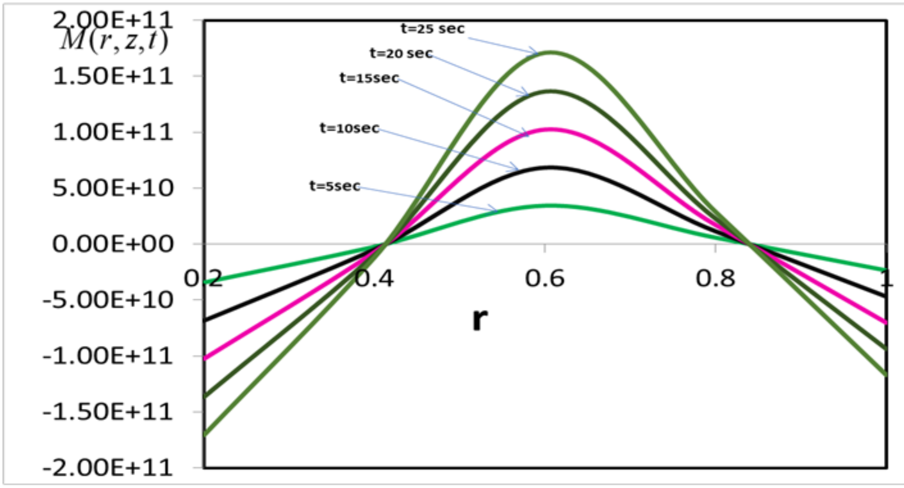

Fig 3 : Graph of $M(r,z,t)$ versus r for different values of t

Figure 4 represents graph of u_r versus r for different values of t. It is observed that unknown u_r develops tensile stress within semi-infinite solid circular cylinder in the circular region $r = 0.4 to r = 0.8$. It is also observed that u_r goes on increasing to $r = 0.4$ & from $r = 0.8$ of the semi-infinite solid circular cylinder.

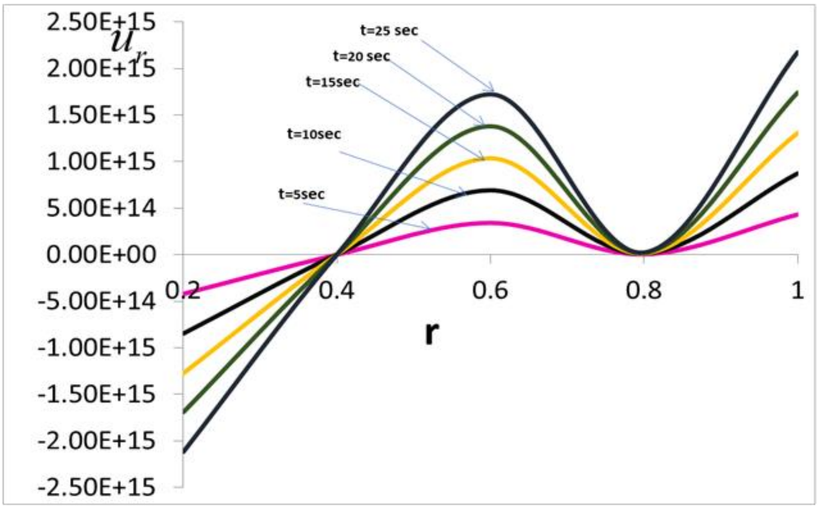

Fig 4 : Graph of u_r versus r for different values of t

Figure 5 represents graph of u_z versus r for different values of t. It is observed that u_z develops compressive stress within semi-infinite solid circular cylinder in the circular region $r = 0.6 to r = 0.8$. It is also observed that u_z goes on increasing to $r = 0.4$ then & from $r = 0.4$ to $r = 0.6$ value of u_z is approximately equal to zero of the semi-infinite solid circular cylinder.

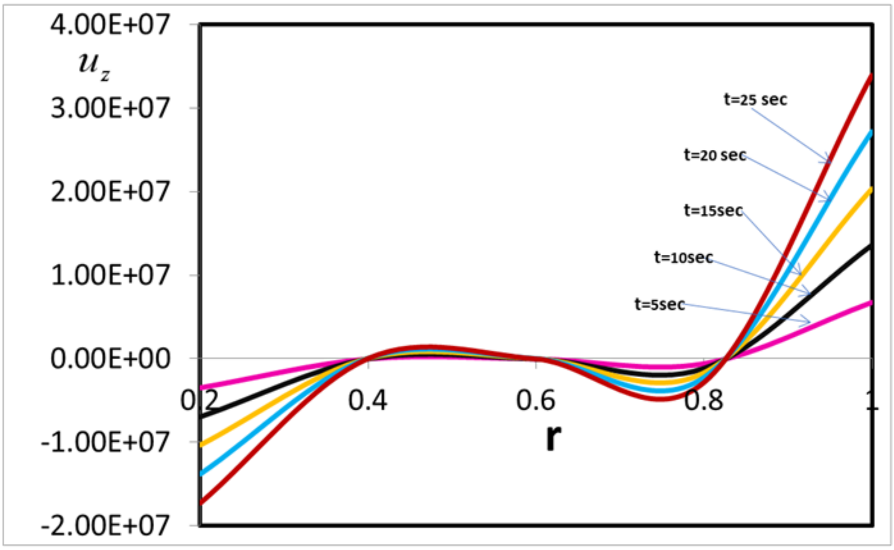

Fig 5 : Graph of u_z versus r for different values of t

Figure 6 represents graph of σ_{rr} versus r for different values of t. It is observed that σ_{rr} develops compressive stress within semi-infinite solid circular cylinder in the circular region $r = 0.4 to r = 0.8$. It is also observed that σ_{rr} goes on increasing to $r = 0.3$ & then from $r = 0.8$ σ_{rr} goes on increasing of the semi-infinite solid circular cylinder.

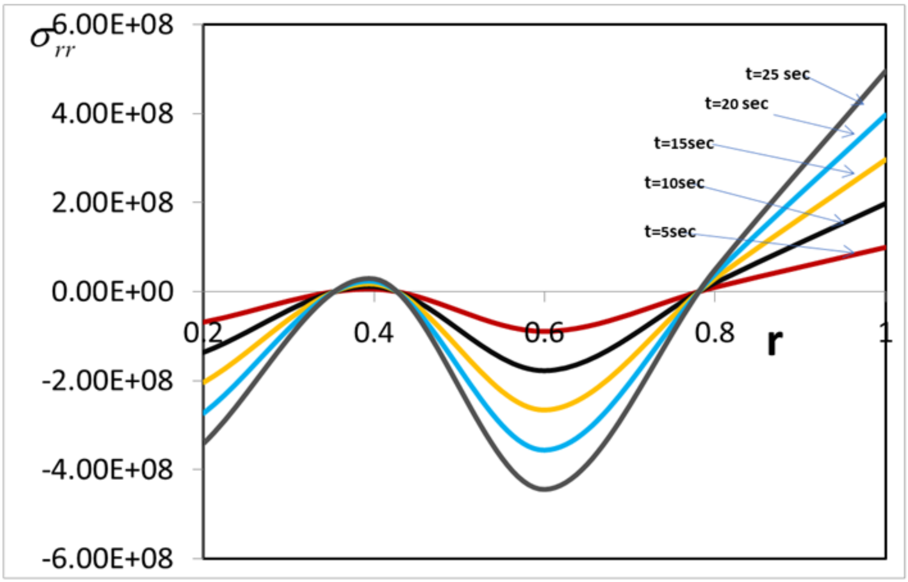

Fig 6 : Graph of σ_{rr} versus r for different values of t

Figure 7 represents graph of $\sigma_{\theta\theta}$ versus r for different values of t. It is observed that $\sigma_{\theta\theta}$ develops tensile stress within semi-infinite solid circular cylinder in the circular region $r = 0.3 to r = 0.6$ and $r = 0.7 to r = 0.9$. It is also observed that σ_{rr} goes on increasing to $r = 0.3$ & then from $r = 0.8$ σ_{rr} goes on decreasing in the semi-infinite solid circular cylinder.

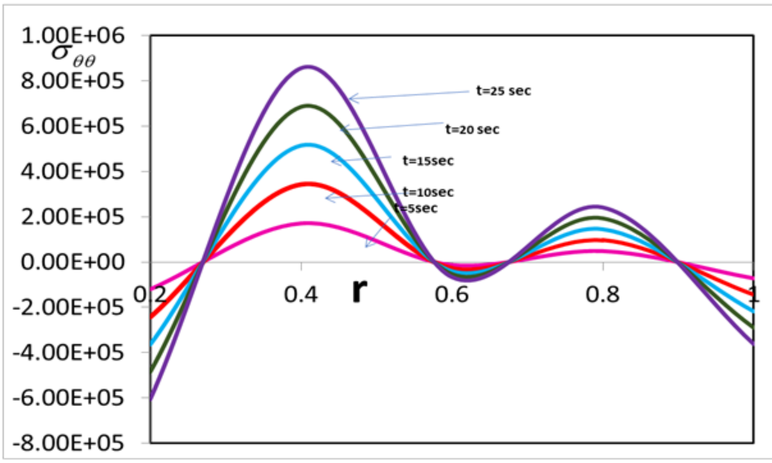

Fig 7 : Graph of $\sigma_{\theta\theta}$ versus r for different values of t

Figure 8 represents graph of σ_{zz} versus r for different values of t . It is observed that σ_{zz} develops compressive stress within semi-infinite solid circular cylinder in the circular region approximately $r = 0.44 to r = 0.8$. It is also observed that σ_{zz} goes on increasing to $r = 0.3$ value of σ_{zz} is approximately equal to zero from $r = 0.3 to r = 0.44$ & $r = 0.8 to r = 0.9$ & then σ_{zz} goes on decreasing in the semi-infinite solid circular cylinder.

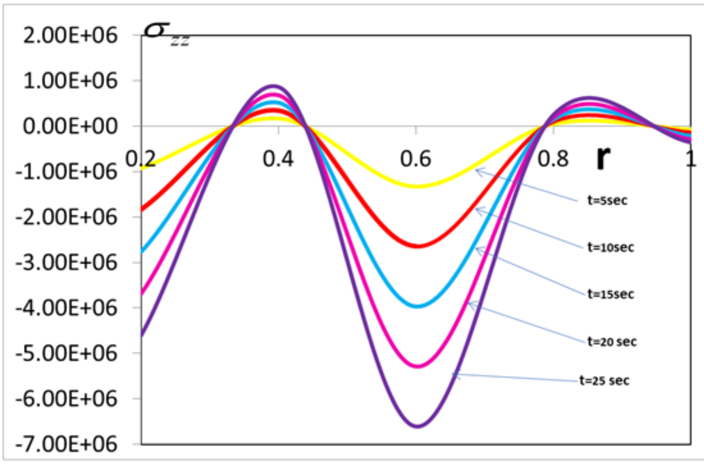

Fig 8 : Graph of σ_{zz} versus r for different values of t

Figure 9 represents graph of σ_{rz} versus r for different values of t . It is observed that σ_{rz} develops tensile stress within semi-infinite solid circular cylinder in the circular region.It is also observed that σ_{rz} goes on increasing to $r = 0.4$ & then from $r = 0.6$ σ_{rz} goes on decreasing in the semi-infinite solid circular cylinder.

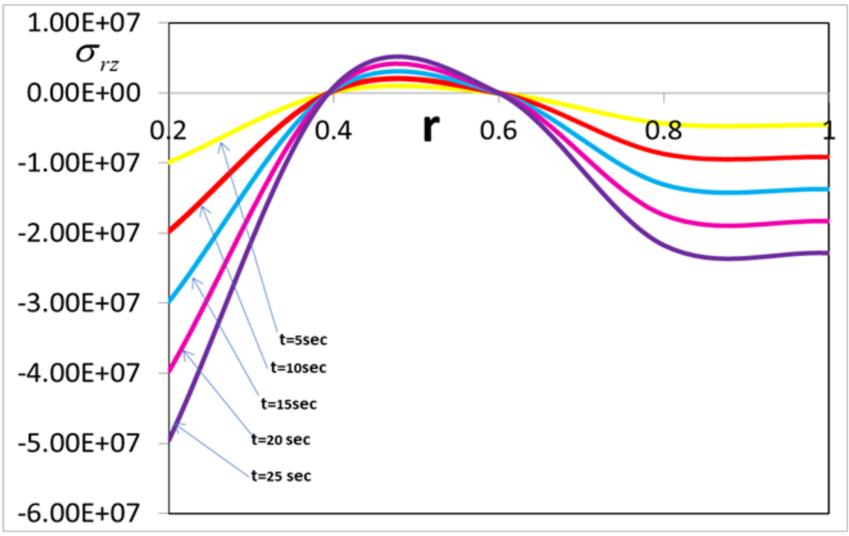

Fig 9 : Graph of σ_{rz} versus r for different values of t

CONCLUSION

In this study one treated the two-dimensional thermoelastic problem of the semi-infinite solid circular cylinder. Under given initial and boundary conditions the temperature , Goodier's thermoelastic displacement potential, the displacement components in the radial and axial directions , Michell's function , component of the stresses in semi-infinite solid circular cylinder have been determined with the help of Hankel transform and Fourier transform techniques.

The results are obtained as series of Bessel's functions in the form of infinite series. Moreover, by assigning suitable values to the parameters and functions in the equation of temperature expression of special interest can be derived for any particular case. and illustrated graphically.

Conflicts of interest: The authors stated that no conflicts of interest.

REFERENCES

1. Kulkarni, V. S. and Deshmukh, K. C.: Quasi-static transient thermal stresses in a thick annular disc, Sādhanā, 32 (2007) 561–575.
2. Nowacki, W.: The state of stress in a thick circular plate due to temperature field, Bull. Sci. Acad. Polon Sci. Tech., 5 (1957), 227.
3. Noda, N; Hetnarski, R. B.and Tanigawa, Y.: Thermal stresses, Second ed. Taylor and Francis, New York (2003), 260.
4. Roy Choudhary, S. K.: A note on quasi-static thermal deflection of a thin clamped circular plate due to ramp-type heating of a concentric circular region of the upper face, J. of the Franklin Institute, 206 (1973), 213-219.
5. Wankhede, P. C.: On the quasi-static thermal stresses in a circular plate, Indian J. Pure Appl. Math., Vol. 13 (8) (1982), 1273-1277.
6. Warsha K. Dange, "Two-Dimensional Transient Thermoelastic Problem For A Hollow Cylinder With Internal Heat Generation" International Journal Of ScienceAndResearch (IJSR), Volume 4, Issue 1,January 2015.
7. Warsha K. Dange, "Two-Dimensional Transient Thermoelastic Problem Of A Annular Disc Due To Radiation "International Journal Of Mathematics Trends And Technology- Volume17,Issue1-January 2015.