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Parametric Analysis of Gravity Vortex Turbines as a Low Cost Renewable Energy Alternative from Low Head Hydraulic Resources

Deniz ÜNSALAN[1], **Kunsel IZET-ÜNSALAN**[2], **K. Turgut GÜRSEL**[3]

1, 3 Dokuz Eylül University Institute of Marine Sciences and Technology, İzmir, Turkey
2 Ovidius University, Faculty of Mechanical, Industrial and Maritime Engineering, Constanta, Romania
deniz.unsalan@deu.edu.tr

Abstract. The long existent worldwide trend for large scale hydroelectric power plants, relying on dams are now under severe criticism for the large areas their reservoirs occupy, which are often fertile agricultural areas and sometimes flood cultural heritage sites. However, there are also environment-friendly alternatives for hydroelectric power production, which are capable to obtain energy from small scale streams with relatively low heads. Such smaller scale sources with low cost facilities can be used for electric production by alternative schemes that use small streams, irrigation canals and divertions from rivers, tidal pools, overtopping wave energy converters and urban wastewater. One of the recent types of such plants are the gravity vortex turbines that use the naturally occurring "sink vortex" draining such water. They are highly efficient and able to obtain energy from sources with flow rates as low as 1 m3/s and heads as low as 0.80 m. Such water sources are abundant in most of the rural areas and it is possible to obtain either an important part or the total need of the energy requirement of the nearby communities with such systems. Gravity vortex turbines have low costs due to their simple structure and are easy to maintain. They can also be implemented for overtopping wave energy and tidal energy systems, as well as recovery units of pumped energy storage schemes.

The purpose of this paper is to propose relations for the design and parametric analysis to size the relevant parts of the plant- the pool and the turbine. Potential flow is assumed throughout the analysis. Attempts to obtain optimized relations between the relative sizes and rotational speeds for the pool, water source, turbine are made and inputs for preliminary design are obtained.

Keywords: Gravity vortex, potential flow, low head hydraulic turbine, renewable energy

1. Introduction

Hydroelectric power accounts for about 15.8 % of Worldwide electric power generation and 63 % of electrical power from the renewable sources as of June 2019. (Anonymous, 2020). However, they are under severe criticism due to the areas they flood, which are often fertile areas and cultural heritage sites, in addition to their negative environmental impacts. These concerns have led people to concentrate upon other "renewable" energy sources, small scale hydroelectric power production being one of them. Such plants do not require large reservoirs or can function with no reservoirs at all. The energy source for such plants are mostly streams that are scattered over the land, and therefore shall answer mostly to the needs of rural communities. They can provide energy for residents, local small-scale industries and can also feed the nationwide grid when surplus power is generated. A summary and feasibility study of such small scale power plants is provided by Narrain (2017).

The purpose of this study is to provide information about one of such small scale hydroelectric power plant concepts and to propose basic design equations for their sizing: Gravity vortex turbines, or Zotlöterer turbines, after their inventor. Gravity vortex turbines are compact, simple in construction and do not require high pressure heads and therefore eliminate the need for dams. They are suitable for pressure heads at the order of one meter and for flow rates at the order of 1 m3/s. Apart from natural streams, they can be used to extract power from irrigation canals, recycled water, tidal height differences, wave energy systems of overtopping type, and from diverted flow of rivers. They are also suitable for the energy recovery units of pumped-energy storage systems (Anonymous from Wikipedia, 2020).

Gravity vortex turbines are based on the formation of gravity-driven sink vortices or "bathtub" vortices. As the result of conservation of angular momentum, any vortex that is already present in the fluid mass (Coriolis-acceleration induced vortices) or vortices introduced externally result in high tangential velocities as they approach to the "sink". This high-velocity stream can be used in a way similar to the fluid jet from the nozzles of a radial-flow turbine. Therefore, one needs:

- Introduction of high velocity flow into a circular "pool", to produce vorticity,
- A sink in the center of the pool,
- A radial-flow turbine to extract energy from the fluid flow.

A photograph of a gravity vortex turbine is shown in Figure 1 (Anonymous, 2020).

Figure 1. A gravity vortex turbine (anonymous from Wikipedia, last accessed in March 2020)

The analysis below is based on two-dimensional, inviscid flow assumption, a shallow cylindrical pool with a rectangular water entrance over the height of the pool, and water entering in tangential direction.

2. Analysis

The modelling of a vortex with a sink is represented as the superposition of a vorticity and a sink flow. The potential function Φ (r,φ) and stream function ψ (r,φ), for a vortex-sink flow is represented in polar coordinates (r, φ) as:

$$\Phi(r,\varphi) = \frac{\Gamma}{2\pi}\varphi - \frac{q}{2\pi}\ln r \tag{1}$$

$$\Psi(r,\varphi)=\frac{\Gamma}{2\pi}\ln r-\frac{q}{2\pi}\varphi \tag{2}$$

where Γ and q are the vortex strength and sink strength respectively (Prandtl and Tietjens, 2011). Tangential and radial components of velocity can be derived from Equations (19 and (2) respectively:

$$v_\theta=\frac{1}{r}\frac{d\Phi}{d\varphi}=\frac{\Gamma}{2\pi r} \tag{3}$$

$$v_r=\frac{d\Phi}{dr}=-\frac{q}{2\pi r} \tag{4}$$

The vortex is created by a tangentially introduced water stream with velocity V_0 into the cylindrical pool with radius R_0. However, the water stream shall have a finite width, b, and therefore not all particles shall attain the same vorticity due to changes in the tangential component of the velocity, V_0 (Figure 2). Any fluid particle with distance y from the external wall shall have a tangential and radial component, v_r and v_θ. The relevant circulation, as a function of radial distance y from the external wall, Γ(y) shall be:

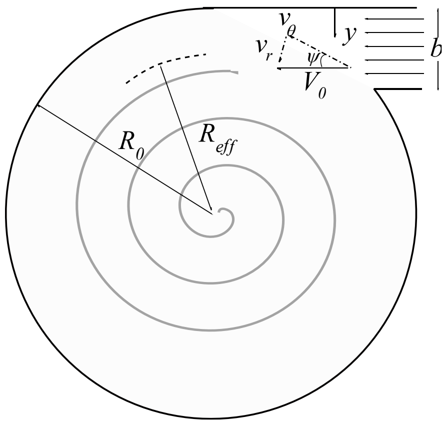

Figure 2. The vortex and the pool

$$\Gamma(y)=\oint v_\theta(y)\cdot ds=\oint v_\theta(y)R_0\cdot d\theta \tag{5}$$

However, the water shall enter the pool through a rectangular entrance port with a width b, and therefore not all water particles shall contribute equal to the vorticity within the pool.

From Figure 3, $v_\theta(y)=V_0\cos\psi$, $R(y)=R_0-y/\cos\psi$ and $\cos\psi=1-y/R_0$;

$$\Gamma(y)=\int_0^{2\pi}V_0\cos\psi R_0\cdot d\theta=2\pi V_0R_0\left(1-\frac{y}{R_0}\right) \tag{6}$$

Taking the average value of circulation over the inlet width b as the effective circulation:

$$\Gamma_{eff}=\frac{1}{b}\int_0^b\Gamma(y)\cdot dy=2\pi V_0R_0\left(1-\frac{b}{2R_0}\right) \tag{7}$$

Similarly, the sink strength q can be calculated by the flow rate Q, assuming the radial velocity component v_r is invariant with depth at the core of the vortex:

$$Q=-\int_0^H\int_0^{2\pi}v_r\cdot r\,d\theta\,dz=\frac{q}{2\pi}\int_0^H\int_0^{2\pi}d\theta\,dz=qH \tag{8}$$

or, for a rectangular pool entrance of width b and height H,

$$q=\frac{Q}{H}=bV_0 \tag{9}$$

A particle in the rotating mass of water shall keep its angular momentum per unit mass, ωr^2 as it approaches to the sink at the center of the pool. The tangential velocity at any radial coordinate r shall then be:

$$v_\theta(r) = V_0 \cos\psi \cdot \frac{R_0}{r} = \frac{V_0 R_0}{r}\left(1 - \frac{y}{R_0}\right) \tag{10}$$

The effective value of tangential velocity at a radial ordinate r of the entering water mass can be found by averaging Equation (13)

$$v_\theta(r) \simeq \frac{V_0 R_0}{r}\left(1 - \frac{b}{2R_0}\right) = \frac{V_0 R_{eff}}{r} \quad ; \quad where\ \ R_{eff} = R_0\left(1 - \frac{b}{2R_0}\right) \tag{11}$$

Assuming that the water entering the turbine blade is perfectly deflected downward, the torque created at the turbine blade can be found from the Euler turbine equation (Figure 3):

$$T = \rho Q\left(v_\theta - \Omega r_t\right) r_t = \rho Q\left(V_0 R_{eff} - \Omega r_t^2\right) \tag{12}$$

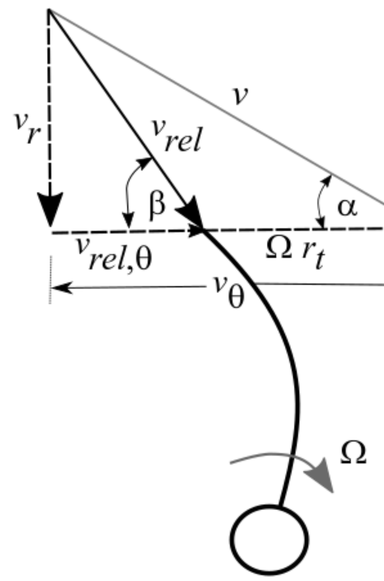

Figure 3. Vector diagram of velocities entering the turbine blade

where ρ is the mass density of the water, Ω is the angular velocity of the turbine, and r_t is the external radius of the turbine rotor. The power obtained, P can be found to be:

$$P = T\,\Omega = \rho Q\left(V_0 R_{eff} - \Omega r_t^2\right)\Omega \tag{13}$$

The power obtained from the turbine is therefore a second degree polynomial in terms of angular velocity of shaft Ω, with the optimum value:

$$\Omega_{opt} = \frac{V_0 R_{eff}}{2r_t^2} = \frac{v_\theta}{2r_t} = \frac{1}{2}\omega_t \tag{14}$$

where ω_t is the angular velocity of water at the turbine entrance. Hence, it is understood that for optimum efficiency, the turbine should rotate at half the angular velocity of the surrounding water mass. The optimum power from the turbine shall therefore be:

$$P_{out} = \frac{1}{4}\rho Q\left(\frac{V_0 R_{eff}}{r_t}\right)^2 \tag{15}$$

The efficiency of the turbine system can be written as:

$$\eta = \frac{P_{out}}{\dot{E}_{in}} = \frac{\frac{1}{4}\rho Q\left(\frac{V_0 R_{eff}}{r_t}\right)^2}{\frac{1}{2}\rho Q V_0^2} = \frac{1}{2}\left(\frac{R_{eff}}{r_t}\right)^2 \tag{16}$$

This equation implies that for an ideal energy conversion, i.e., η = 1.0, $r_t = \frac{R_{eff}}{\sqrt{2}}$. However, assuming that for an ideal operation, the radial component of kinetic energy is totally converted to mechanical work, the energy lost in the turbine can be thought to be due to the radial and vertical components of the flow at the immediately after the turbine entrance:

$$\eta = \frac{\dot{E}_{in} - \frac{1}{2}\rho Q v_{r,t}^2}{\dot{E}_{in}} = 1 - \left[\left(\frac{v_{r,t}}{V_0}\right)^2 + \left(\frac{v_{z,t}}{V_0}\right)^2\right] \tag{17}$$

From Equations (4) and (11), and in terms of the entrance port cross sectional area, assuming a rectangular entrance port,

$$v_{r,t} = \frac{Q}{2\pi r_t H} = \frac{bV_0}{2\pi r_t} \tag{18}$$

In order to calculate the vertical component of velocity within the vortex, v_z, the shape of the free surface of the vorticial flow has to be calculated. From the law of Bernoulli, written for two distinct points 1 and 2 on the same streamline:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2 \tag{19}$$

If both points have the same atmospheric pressure, and point 1 is taken as the reference point at a stagnant location ($v_1 = 0$, $z_1 = 0$), and point 2 can be any point (r, z)

$$z = -\frac{v_2^2}{2g} = -\frac{1}{2g}\left(v_r^2 + v_\theta^2\right) = -\frac{q^2 + \Gamma^2}{8\pi^2 g r^2} \tag{20}$$

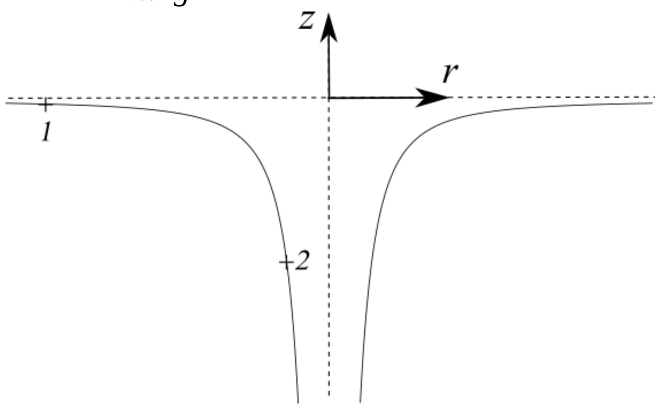

Figure 4. Gravity vortex

$$v_z = \frac{dz}{dt} = \frac{dz}{dr}\cdot\frac{dr}{dt} = v_r \frac{dz}{dr} \tag{21}$$

$$z = -\frac{v^2}{2g} \cong -\frac{v_\theta^2 + v_r^2}{2g} = -\frac{1}{2g} v_r^2 \left[1 + \left(\frac{v_\theta}{v_r}\right)^2\right] \tag{22}$$

From equations (9) and (11), and using chain rule,

$$\frac{dz}{dr} = \frac{dz}{dv_r} \cdot \frac{dv_r}{dr} = -\frac{v_r}{g}\left[1+\left(\frac{2\pi R_{eq}}{b}\right)^2\right] \cdot \frac{d}{dr}\left(\frac{bV_0}{2\pi r}\right) \tag{23}$$

$$v_z = \frac{v_r^3}{gr}\left[1+\left(\frac{2\pi R_{eq}}{b}\right)^2\right] \tag{24}$$

$$\eta = 1-\left(\frac{v_r}{V_0}\right)^2 \left\{1+\left(\frac{v_r^2}{gr}\right)^2\left[1+\left(\frac{2\pi R_{eq}}{b}\right)^2\right]\right\} \tag{25}$$

or,

$$\eta = 1-\left(\frac{v_r}{V_0}\right)^2 \left\{1+Fn_r^{\ 4}\left[1+\left(\frac{2\pi R_{eq}}{b}\right)^2\right]\right\} \tag{26}$$

Fn_r is the Froude number based on radial velocity and turbine radius:

$$Fn_r = \frac{v_r}{\sqrt{gr_t}} \tag{27}$$

In more convenient nondimensional terms $\rho = \frac{r_t}{R_0}$, $b' = \frac{b}{R_0}$ and $Fn_{R0} = \frac{V_0}{\sqrt{gR_0}}$, equation (26) can be written as:

$$\eta = 1-\left(\frac{b'}{2\pi\rho}\right)^2 \left\{1+\frac{b'^4}{16\pi^4} Fn_{R0}\rho^{-6}\left[1+4\pi^2\left(\frac{1}{b'}-1\right)\right]\right\} \tag{28}$$

A plot of turbine efficiency for various radii (for Fn_{R0} =1.5, b' = 0.15) can be seen in Figure 5.

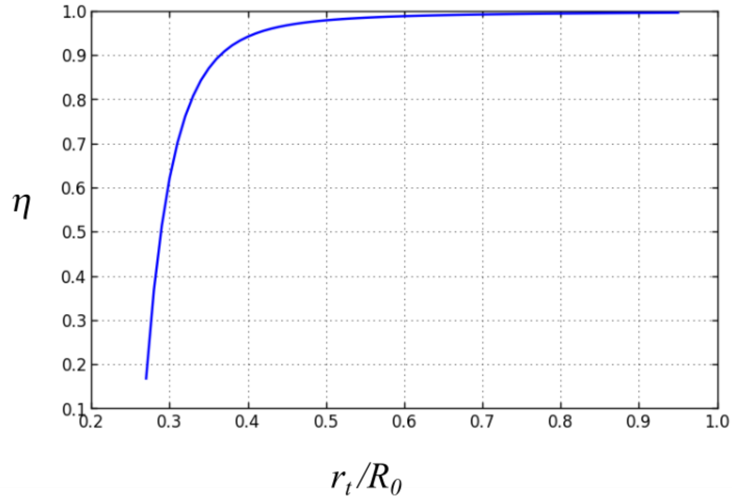

Figure 5. The efficiency of the turbine for particles on free surface

As can be observed, the efficiency of the turbine is approximately equal to unity for $r_t/R_0 > 0.7$, which agrees with equation (16). Therefore, the following relation is proposed that the turbine diameter:

$$r_t = \frac{R_0}{\sqrt{2}}\left(1 - \frac{b}{2R_0}\right) \tag{29}$$

Hence, the optimum angular velocity of the water entering the turbine:

$$\Omega_{opt} = \frac{V_0 R_{eff}}{2 \cdot r_t^2} = \frac{0.707 V_0}{R_0\left(1 - 0.5\dfrac{b}{R_0}\right)} \tag{30}$$

The angle of entrance to turbine blades, from Figure 4:

$$\tan\beta = \frac{v_{r,t}}{v_{\theta,t} - \Omega r_t} = 2\frac{v_{r,t}}{v_{\theta,t}} = \frac{b}{\pi R_{eff}} = \frac{2b'}{\pi(2 - b')} \tag{31}$$

The size of vortex gravity turbine can be determined by the cavitation and ventilation considerations at the turbine blades. From equation (2), the equation for the path of a fluid particle can be written as:

$$\frac{r}{R} = e^{-\frac{q}{\Gamma}\varphi} = e^{-\frac{b'}{2\pi}\varphi} \tag{32}$$

where the angle φ is arbitrarily taken to be zero when $r = R_0$. Taking the consideration that the jet emanating from the entrance should not interact with itself, and for the perfect utilization of the turbine perimeter, for a particle at the rim of the pool the angle φ can be taken equal to 2π. Hence, from Equation (33), the relation between pool radius and entrance width can be obtained:

$$b = 0.347 R_0 \tag{33}$$

If this criterion is accepted, then the optimum diameter of the turbine shall be $r_t = 0.584\ R_0$, optimum speed of rotation shall be $\Omega_{opt} = 1.711\left(\frac{V_0}{R_0}\right)$ rad/s and the optimum blade angle shall be $\beta_{opt} = 7.6^o$.

3. A hypothetical gravity vortex power plant and conclusions

In order to illustrate the feasibility of the concept, a typical plant, with 2 meters of water head, or 6.26 m/s inlet velocity was selected. The plant shall have a pool of diameter 4.0 m with pool depth 1.0 m. According to the criteria adopted in the paper, this shall correspond to a port opening of 0.69 m and an inlet flow rate of 4.4 m^3/s. The power available is 86 kW and with an actual efficiency (including the eddy losses and secondary flows) of 90%, a 87 kW power output can be anticipated. Effective radius of the pool shall be 1.7 m, turbine diameter 1.17 m and the shaft shall rotate at about 51.1 rpm, suggesting the use of a gearbox to obtain a convenient generator shaft speed.

The cost of this plant shall be relatively low compared to other similar hydro-power plants, simple in construction and hence easy to maintain.

For further research into this study, it is recommended that the flow inside the pool should be simulated by computational fluid dynamics (CFD) techniques and pilot plants should be constructed and operated.

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