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Homogeneous Bianchi Type III Bulk Viscous Model In Presence of G and Λ In Scalar Tensor Theory of Gravitation

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ABSTRACT

In the present paper, we investigate Homogenious Bianchi Type-III bulk viscous fluid cosmological model with variable gravitational and cosmological constant 'Λ' in the framework of Seaz Ballester scalar tensor theory of gravitation. In order to find exact solutions of the Einstein's field equations, we assume i) the expansion scalar 'θ' is proportional to shear scalar 'σ', which leads to $C = B^n$, where B and C are functions of time only ii) the coefficient of bulk viscosity is a power function of the energy density and iii) the cosmic fluid obeys the barotropic equation of state. The nature of the model is discussed in the presence of variable gravitational and cosmological constant. Some physical and kinematical aspects of the model are also discussed.

Keywords: Bianchi Type III Cosmology, Bulk viscosity, Variable G and Λ.

INTRODUCTION

Einstein's general theory of relativity has been successful in describing gravitational phenomena. It has also served as a basis for models of the universe. However since Einstein first published his theory of gravitation, there have been many criticisms of general relativity because of the lack of certain desirable features in the theory. For example Einstein himself pointed out that general relativity does not account satisfactorily for inertial properties of matter, i.e. Mach's principle is not substantiated by general relativity. So in recent years there has been lot of interest in several alternative theories of gravitation.

The most important among them are scalar tensor theories of gravitation formulated by Brans and Dicke(1961), Nordtvedt (1970) and Saez and Ballester (1985). All version of the scalar tensor theories are based on the introduction of a scalar field ϕ into the formulation of general relativity, this scalar field together with the metric tensor field then forms a scalar tensor field representing the gravitational field.

In Saez-Ballester theory the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of weak fields and suggest a possible way to solve missing matter problem in non-flat FRW cosmologies.

The Saez Ballester (1985) field equations are

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -8\pi T_{ij}, \tag{1}$$

$$2\phi^n\phi^i_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{2}$$

Where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is the Einstein tensor , T_{ij} is the stress energy tensor of the matter, ω and n are constant , comma (,) and semicolon (;) denotes partial and co-variant differentiation respectively.

Also energy conservation equation

$$T^{ij}_{;j} = 0 \tag{3}$$

Is the consequence of field equations (1) and (2).

A detailed discussion of Saez-Ballester cosmological models is contained in the work of Saez (1985), Sing and Agrawal (1991) , Shri Ram and Tiwari (1998), Reddy and Venkateswara Rao (2001). Recentaly Adhav et al. (2007) have studied Axially symmetric non-static domain walls in scalar-tensor theories formulated by Brans and Dick (1961) and Saez-Ballester (1985).

Bulk viscosity is supposed to play a very important role in the early evolution of the universe. There are many circumstances during the evolution of the universe in which bulk viscosity could arise.The bulk viscosity coeffiecient determines the magnitude of the viscous stress relative to the expansion. Ribeiro and Sanyal (1987) studied Bianchi type VI model containing the viscous fluid in the presence of an axial magnetic field. Also several aspects of viscous fluid cosmological model in early universe have been extensively investigated by many authors Raj Bali and Dave S. (2001), Adhav et al. (2009), M.K.Verma and Shri Ram (2011), Kandalkar et al (2012).

The cosmological constant Λ and the gravitational constant G are two parameters present in the Einstein's Field equations. The Newtonian constant G plays the role of coupling constant between geometry and matter in Einstein's field equations.There have been numerous modification of general relativityin which G varies with time in order to achive possible unification of gravitation and elementary partical physics or to incorporate Mach's principle in general relativity. The Λ term have been interpreted in terms of Higg's scalar field Wagoner (1970). Linde (1974) proposed that the Λ term is a function of temperature and related it to the process of broken system. The cosmological constant problem related to the existence of Λ have been discussed in the literature.A number of authors e.g. Kalligas et al. (1992), Arbab (1997), Abdussattar and Vishwakarma (1997),proposed linking of variations of G and Λ within the framework of general relativity. Verma et al.(2011) investigate bianchi type-VI bulk viscous fluid models with variable gravitationa and cosmological constant. Recently. Deo et al.(2015) discussed bianchi type-III cosmological model electromagnetic field with cosmic string in general theory of relativity.

In this paper, we investigated Bianchi Type III bulk viscous fluid cosmological model with variable G and Λ in Seaz Ballester theory of gravitation. The paper is organized as follows. We present the metric and Einstein's field equation for viscous fluid with time dependent G and Λ We deals with solution of the field equations and we obtain solution of the field equation under the assumption that 1) the expansion scalar 'θ' is proportional to shear scalar 'σ 2) the coefficient of bulk viscosity is a power function of the energy density and 3) the cosmic fluid obeys the barotropic equation of state. The nature of the model is discussed in the presence of variable gravitational and cosmological constant.The physical and kinematical properties of the model have also been discuss

2.The metric and field equation

We consider the spatially homogeneous and anisotropic Bianchi type-III metric in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{-2ax}dy^2 + C^2(t)dz^2 \tag{4}$$

Where a is nonzero constant and A, B, C are functions of the proper time t

The energy momentum-tensor for a bulk viscous fluid distribution is given by

$$T_i^j = (\rho + \bar{p})v_i v^j + \bar{p} g_i^j \tag{5}$$

where

$$\bar{p} = p - \xi\, v^j_{;i} \tag{6}$$

Here $\rho, p, \bar{p}$ and ξ are the energy densityof matter, thermodynamic pressure, effective pressure and bulk viscosity coefficient respectively and v_i is the flow vector satisfying the relations

$$g_{ij}v^i v^j = -1$$

we choose the co ordinates to be commoving, so that

$$v^1 = 0 = v^2 = v^3, v^4 = 1 \tag{7}$$

The semicolon stands for the covariant differentiation.

The field equations (1), (2) and (3) for the metric (4) with the help of (5) and (7) can be written as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{\omega}{2}\phi^n {\phi_4}^2 = -8\pi G\,\bar{p} + \Lambda \tag{8}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} + \frac{\omega}{2}\phi^n {\phi_4}^2 = -8\pi G\,\bar{p} + \Lambda \tag{9}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{a^2}{A^2} + \frac{\omega}{2}\phi^n {\phi_4}^2 = -8\pi G\,\bar{p} + \Lambda \tag{10}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{a^2}{A^2} - \frac{\omega}{2}\phi^n {\phi_4}^2 = 8\pi G\,\rho + \Lambda \tag{11}$$

$$a\left(\frac{B_4}{B} - \frac{A_4}{A}\right) = 0 \tag{12}$$

and

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{n}{2}\left(\frac{{\phi_4}^2}{\phi}\right) = 0 \tag{13}$$

where suffix 4 at the symbols A, B, C and ϕ denotes ordinary differentiation with respective to t . An additional equation for timr changes of G and Λ is obtained by the divergence of Einstein tensor,

i.e. $\left(R_i^j - \frac{1}{2} R g_i^j\right)_{;j}$ which leads to $\left(8\pi G T_i^j - \Lambda g_i^j\right)_{;j} = 0$ which gives

$$8\pi G_4 \rho + \Lambda_4 + 8\pi G\left[\rho_4 + (\rho + \bar{p})\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\right] \tag{14}$$

The conservation of energy equation (14), after using equation (6), split into two equation

$$\rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \tag{15}$$

and

$$8\pi G_4 \rho + \Lambda_4 = 8\pi G \xi \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)^2 \tag{16}$$

The average scale factor R for the metric (4) is defined by

$$R^3 = ABCe^{-ax} \tag{17}$$

The volume scale factor V is given by

$$V = R^3 = ABCe^{-ax} \tag{18}$$

The generalized mean Hubble parameter H is given by

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \tag{19}$$

Where $H_1 = \frac{A_4}{A}$, $H_2 = \frac{B_4}{B}$, $H_3 = \frac{C_4}{C}$

The expansion scalar θ and shear scalar σ are given by

$$\theta = v^i_{;i} = \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) \tag{20}$$

and

$$\sigma^2 = \frac{1}{3}\left[\left(\frac{A_4}{A}\right)^2 + \left(\frac{B_4}{B}\right)^2 + \left(\frac{C_4}{C}\right)^2 - \frac{A_4 B_4}{AB} - \frac{B_4 C_4}{BC} - \frac{A_4 C_4}{AC}\right] \tag{21}$$

The important observational quantity in cosmology is the deceleration parameter q which is defined as

$$q = -\frac{RR_{44}}{R_4^2} \tag{22}$$

The sign of q indicates whether is model inflates or not. The positive sign corresponds to the standard decelerating model whereas the negative sign indicates inflation.

3. Solution of the field equations:

Equation (8) – (13) are six independent equations in seven unknowns A, B, C, ρ, p, ξ and ϕ .for the complete determinacy of the system, we need extra conditions. We consider the equation (12), yielding

$$A = kB \tag{23}$$

As we wish to consider space-time with Bianchi type-III, we have $A = B$ by taking $k = 1$ without loss of generality equation (23) yields,

$$A = B \tag{24}$$

Using equation (24) the field equations (8)-(13) becomes

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{\omega}{2}\phi^n {\phi_4}^2 = -8\pi G\, \bar{p} + \Lambda \tag{25}$$

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \left(\frac{a}{B}\right)^2 + \frac{\omega}{2}\phi^n {\phi_4}^2 = -8\pi G\, \bar{p} + \Lambda \tag{26}$$

$$\left(\frac{B_4}{B}\right)^2 + 2\frac{B_4 C_4}{BC} - \frac{a^2}{B^2} - \frac{\omega}{2}\phi^n {\phi_4}^2 = -8\pi G\, \rho + \Lambda \tag{27}$$

and

$$\phi_{44} + \phi_4 \left(2\frac{B_4}{B} + \frac{C_4}{C} \right) + \frac{n}{2}\left(\frac{{\phi_4}^2}{\phi} \right) = 0 \tag{28}$$

Solving equations (25) and (26), yield

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{B_4}{B}\left(\frac{B_4}{B} - \frac{C_4}{C} \right) - \left(\frac{a}{B} \right)^2 = 0 \tag{29}$$

Firstly we assume that the expansion is proportional to the shear which is physical condition. This condition leads to

$$C = B^n \tag{30}$$

where n is real number.

equation (29) together with (30) leads to

$$\frac{B_{44}}{B} + (1+n)\left(\frac{B_4}{B} \right)^2 - \frac{1}{1-n}\left(\frac{a}{B} \right)^2 = 0 \tag{31}$$

which can be rewritten as

$$\frac{d}{dB}(f^2) + \frac{2(1+n)}{B}(f^2) = \frac{2}{1-n}\left(\frac{a}{B} \right)^2 \tag{32}$$

where

$$B_4 = f(B) \tag{33}$$

From (32) we obtain

$$\left(\frac{dB}{dt} \right)^2 = \frac{a^2}{(1-n)^2} + \frac{k_1}{B^{2(1+n)}} \tag{34}$$

where k_1 is the constant of integration. After a suitable transformation of co ordinates, the metric (4) reduces to the form

$$ds^2 = -\left(\frac{a^2}{(1-n)^2} + \frac{k_1}{B^{2(1+n)}} \right)^{-1} dT^2 + T^2 dx^2 + T^2 e^{-2ax} dy^2 + T^{2n} dz^2 \tag{35}$$

where $B = T$

furthermore, to obtain the expression for Saez-Ballester scalar field ϕ, we rewrite the equation (28) as

$$\frac{\phi_{44}}{\phi_4} + (2+n)\frac{B_4}{B} + \frac{n}{2}\frac{\phi_4}{\phi} = 0 \tag{36}$$

after simplifying, we obtain

$$B^{(n+2)} \phi^{\frac{n}{2}} d\phi = \varphi_0 dt \tag{37}$$

We now substitute the value of B, we obtained

$$\phi^{\frac{n}{2}} d\phi = \frac{\varphi_0}{T^{n+2}} dt \tag{38}$$

Integrating, we obtain

$$\phi^{\frac{n+2}{2}} = -\varphi_0 \frac{(n+2)}{2(n+1)} \left(\frac{a^2}{(1-n)^2 T^{2(n+1)}} + \frac{k_1}{T^{4(1+n)}} \right)^{\frac{1}{2}} + \psi_0 \tag{39}$$

where ψ_0 is integrating constant.

It is clear that, given $\xi(t)$, we can find the physical and kinematical parameters associated with metric (35). The effect of bulk viscosity is to produce a change in the cosmic fluid and therefore exhibits essential change on character of the solution. In most of the investigations, the bulk viscosity is assumed to be a simple power function of the energy density(1995, 1972)

$$\xi(t) = \xi_0 \rho^{\alpha} \tag{40}$$

where ξ_0 and α (>1) are constant. For small density α may even be equal to unity [35]. The case $\alpha = 1$ corresponds to a radiative fluid (1972) Near a big-bang, v $0 \le \alpha \le 1/2$ is more appropriate assumption to obtain realistic models (1976).

For the specification of ξ, we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho \tag{41}$$

where $\gamma(0 \le \gamma \le 1)$ is constant.

From equation (15) and (41), we obtain

$$\rho' = \frac{-c(n+2)(1+\gamma)}{T}\rho \tag{42}$$

Where a dash denotes differentiation with respect to T.

Integrating of equation (42), yields

on using (37) in (34), we obtain

$$\rho = \frac{c}{T^{(n+2)(1+\gamma)}} \tag{43}$$

Where c is integrating constant. Diff. equation (42) we obtain

$$\rho' = \frac{-c(n+2)(1+\gamma)}{T^{(n+3)+(n+2)\gamma}} \tag{44}$$

Also using equation (39), from equation (27), we find

$$8\pi G\rho + \Lambda = (1+2n)\left(\frac{a^2}{(1-n)^2 T^2} + \frac{k_1}{T^{4(1+n)}}\right) - \frac{\alpha^2}{T^2} - \frac{\omega}{2}\varphi_0^2\left(\frac{a^2}{(1-n)^2 T^{(3n+4)}} + \frac{k_1}{T^{(6+5n)}}\right) \tag{45}$$

Which on differentiation leads to

$$8\pi G'\rho + 8\pi G\rho' + \Lambda' = \omega\varphi_0^2\left(\frac{(3n+4)a^4}{(1-n)^2 T^{3(2n+3)}} + \frac{4(4n+5)a^2 k_1}{(1-n)^2 T^{(8n+11)}} + \frac{4(5n+6)k_1^2}{T^{(13+10n)}}\right)$$
$$-\frac{4(2n^2+3n+1)}{T^{(5+4n)}} - \frac{2n(n+2)\alpha^2}{T^3} \tag{46}$$

Now using (15), (40) and (44) in equation (46), we get

$$G = \left\{\omega\varphi_0^2\left(\frac{(3n+4)a^4}{(1-n)^2 T^{2(2n+3)}} + \frac{4(4n+5)a^2 k_1}{(1-n)^2 T^{(8n+11)}} + \frac{4(5n+6)k_1^2}{T^{(13+10n)}}\right)\right.$$
$$\left. - \frac{4(2n^2+3n+1)}{T^{(5+4n)}} - \frac{2n(n+2)\alpha^2}{T^3}\right\} \times \left[\frac{8\pi\xi_0 c^{\alpha}(n+2)^2}{T^{\alpha(n+2)(1+\gamma)}}\sqrt{\frac{\alpha^2}{(1-n^2)T^4} + \frac{k_1}{T^{8(1+n)}}}\right.$$
$$\left. - \frac{8\pi(n+2)(1+\gamma)}{T^{(n+3)+(n+2)\gamma}}\right]^{-1} \tag{47}$$

Equation (43) and (47) in (45), we get

$$\Lambda=(1+2n)\left(\frac{a^2}{(1-n)^2T^2}+\frac{k_1}{T^{4(1+n)}}\right)-\frac{\alpha^2}{T^2}-\frac{\omega}{2}\varphi_0^2\left(\frac{a^2}{(1-n)^2T^{(3n+4)}}+\frac{k_1}{T^{(6+5n)}}\right)$$
$$-\left\{\omega\varphi_0^2\left(\frac{(3n+4)a^4}{(1-n)^2T^{2(2n+3)}}+\frac{4(4n+5)a^2k_1}{(1-n)^2T^{(8n+11)}}+\frac{4(5n+6)k_1^2}{T^{(13+10n)}}\right)\right.$$
$$\left.-\frac{4(2n^2+3n+1)}{T^{(5+4n)}}-\frac{2n(n+2)\alpha^2}{T^3}\right\}\times\left[\frac{\xi_0c^\alpha(n+2)^2}{T^{\alpha(n+2)(1+\gamma)}}\sqrt{\frac{\alpha^2}{(1-n^2)T^4}+\frac{k_1}{T^{8(1+n)}}}\right.$$
$$\left.-\frac{(n+2)(1+\gamma)}{T^{(n+3)+(n+2)\gamma}}\right]^{-1}\frac{c}{T^{(n+2)(1+\gamma)}} \tag{48}$$

From equation (40) and (43), we obtain

$$\xi(t)=\xi_0\frac{c^\alpha}{T^{\alpha(n+2)(1+\gamma)}} \tag{49}$$

5. Some physical and Kinematical Properties.

In this section we discuss some physical and kinematical properties of the velocity vector v^i of the metric (29), the spatial volume (V), the scalar expansion (θ), the shear scalar (σ) and deceleration parameter (q) of the fluid are given by

$$V=\sqrt{-g}=nT^3e^{-ax} \tag{50}$$

$$\theta=(n+2)\sqrt{\frac{\alpha^2}{(1-n^2)T^2}+\frac{k_1}{T^{4(1+n)}}} \tag{51}$$

$$\sigma^2=(2-4n+4n^2)\left(\frac{\alpha^2}{(1-n^2)T^2}+\frac{k_1}{T^{4(1+n)}}\right) \tag{52}$$ and,

$$q=\frac{2+3\alpha-2m}{1+2m} \tag{53}$$

The Hubble parameter is given by

$$H=\frac{(n+2)}{3}\sqrt{\frac{\alpha^2}{(1-n^2)T^2}+\frac{k_1}{T^{4(1+n)}}} \tag{54}$$

The spatial volume of the model given by (49) shows the anisotropic expansion of the universe with time. For the model expansion scalar θ, and shear scalar σ tends to zero as $T\to\infty$. The position value of deceleration parameter indicates the model decelerates in the standard way.

CONCLUSION

In this paper, we investigated Bianchi Type III bulk viscous fluid cosmological model with variable G and Λ in Seaz Ballester theory of gravitation. To get a determinate solution of the field equations, we have assumed the relation between metric potential and shear viscosity is proportional to the scale expansion. We observe that the spatial volume is zero at $T=0$. At this epoch the energy density ρ, expansion θ, shear scalar σ and the bulk viscosity coefficient ξ are all infinite. Therefore the model (35) starts involving with a big-bang at $T=0$. For large T energy density becomes zero, the rate of expansion in the model shows down tending to zero as $T\to\infty$. The cosmological constant term Λ is infinite at the beginning of the model and decreases at late time. The gravitational constant G is zero initially tends to infinity as $T \rightarrow \infty$ These are supported by recent result from the observations of the typen La Supernova explosion (SN la).

Conflicts of interest: The authors stated that no conflicts of interest.

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