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Analysis of Various Performance Measures of Finite Source Multiple-Server Queuing Model by using Binomial Distribution and Uniform Distribution.

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ABSTRACT

Queuing Model of Finite Source having Multiple-Server is discussed here in order to analyze the various performance measures. The Calling Population in this model is limited. The model is working under Queue Discipline GD. The numbers of arriving customers are fixed. When the system is busy in serving the customers that are already in the system, the arrival rate of next additional customer gets reduced which minimizes the further congestion. Since the Calling Population is limited, the arrival rate of customers and service rate of server follows Binomial Distribution, whereas the inter-arrival time and service time follows Uniform Distribution. For the model, formula to find the expected waiting time of a customer in the queue is derived. This formula and Little's formula are then used to analyze the other performance measures of the Queuing Model. The calculated results obtained from the new derived formulae are compared with the results of existing available formulae to interpret the conclusion.

Keywords calling population, utilization factor, Multiple-Server, mean arrival rate, mean service rate, inter-arrival time, service time

INTRODUCTION

The application of Queuing theory plays an important in Queuing Model to reduced the waiting time of customer [9].

The Queuing Model discussed here is of Finite Source having Multiple-Server to study its various performance measures. The arriving customers in this model are finite in numbers and it works under Queue Discipline GD. The numbers of arriving customers are fixed. When the system is busy in serving the customers that are already in the system, the arrival rate of next coming customer gets reduced which minimizes the further congestion. The most common application of this model is machine repairing or servicing by the serviceman. The servicemen appointed for this work are considered as servers and machines are considered as customers. Many of the researchers worked on these types of model [1], [2], [6], [7].

Since the Calling Population is limited, the arrival rate of customers and service rate of server follows Binomial Distribution, whereas the inter-arrival time and service time follows Uniform Distribution [10], [11], [12]. Formula to find the expected waiting time of a customer in the queue is derived by using the Binomial Distribution. By using this formula and Little's Formula the other respective formulae are derived and studied to analyze the different aspects of the Queuing Model with the help of an example. The results obtained are compared with the results of existing available formulae to interpret the conclusion.

METHODOLOGY

Let the calling population be limited to R . α be the mean arrival rate of the customers and β be the mean service rate of server. The system is having n number of multiple-server and model works under the General Discipline.

Utilization factor of server $\rho = \alpha / n\beta$ (1)

PROBABILITY OF x CUSTOMERS ARRIVED/SERVED IN TIME t: [10], [12]

If suppose x number of customers be there in the system then the remaining customers arriving left in the system are $(R - x)$. Thus the further arrival rate of customers will be $\alpha(R - x)$. The arrival rate will be zero for system having greater than or equal to R number of customers.

$\therefore$ By Binomial Distribution, Probability of x customers arrived in time t is given by

P (x customers arrived in time t)

$$= {}^{R}C_{x}\left(\frac{\alpha(R-x)t}{R}\right)^{x}\left(1-\frac{\alpha(R-x)t}{R}\right)^{R-x}, \quad t < \frac{R}{\alpha(R-x)} \qquad (2)$$

This equation assures the time interval for the arrival of R number of customers is $0 \le t < \frac{R}{\alpha(R-x)}$.

The system is having n number of multiple-server then the mean service rate of server is classified as:

Case-1: Mean service rate $= x\beta$ for $(x < n)$

Case-2: Mean service rate $= n\beta$ for $(n \le x \le R)$

Case-1: For $(x < n)$

Probability of x customers where $(x < n)$ served by the servers in time t is given by

P (x customers served in time t)

$$= {}^{R}C_{x}\left(\frac{x\beta t}{R}\right)^{x}\left(1-\frac{x\beta t}{R}\right)^{R-x}, \quad t < \frac{R}{x\beta} \text{ and } (x < n) \qquad (3)$$

This equation gives an assurance that the time interval to serve the number of customers which are less than the number of servers is $0 \le t < \frac{R}{x\beta}$.

Case-2: For $(n \le x \le R)$

Probability of x customers where $(n \le x \le R)$ served by the n number of servers in time t is given by

P (x customers served in time t)

$$= {}^{R}C_{x}\left(\frac{n\beta t}{R}\right)^{x}\left(1-\frac{n\beta t}{R}\right)^{R-x}, \quad t < \frac{R}{n\beta} \text{ and } (n \le x \le R) \qquad (4)$$

This equation gives an assurance that the time interval to serve $(n \le x \le R)$ customers by n number of servers is $0 \le t < \frac{R}{n\beta}$.

ARRIVAL AND SERVICE TIME DISTRIBUTION: [10]

Arrival time distribution:

Mean inter-arrival time of customers is $\left(1/\alpha\right)$ and mean of Uniform Distribution is $\left((a+b)/2\right)$

$$\therefore \frac{1}{\alpha} = \frac{a+b}{2} \Rightarrow \alpha = \frac{2}{a+b} \Rightarrow b = \frac{2}{\alpha} - a$$

Here time starts from $t = 0$ which gives $a = 0 \Rightarrow b = \frac{2}{\alpha}$

Probability density function,

$$f(t) = \frac{1}{\left(\frac{2}{\alpha} - 0\right)} = \frac{\alpha}{2} \ , \ 0 \le t \le \frac{2}{\alpha}$$

and Cumulative distribution function,

$$F(t) = \frac{1}{b-a}[t-a] = \frac{1}{\left(\frac{2}{\alpha} - 0\right)}[t-0] = \frac{\alpha t}{2} \quad , 0 \le t \le \frac{2}{\alpha}$$

Which gives the Probability of another customer arriving in the next coming time t, if a customer is already arrived.

Also it gives an assurance of 100% arrival of another customer in the next coming time $\left(2/\alpha\right)$

Service Time Distribution:

Mean service time required to serve a customer $= 1/\beta$

Probability density function,

$$f(t) = \frac{1}{\left(\frac{2}{\beta} - 0\right)} = \frac{\beta}{2} \ , \ 0 \le t \le \frac{2}{\beta}$$

and Cumulative distribution function,

$$F(t) = \frac{\beta\, t}{2} \ , \ 0 \le t \le \frac{2}{\beta}$$

Which gives the Probability of another customer served in the next coming time t, if a customer already served.

Also it gives an assurance of 100% served another customer in the next coming time $\left(2/\beta\right)$.

PROBABILITY OF x CUSTOMERS IN THE SYSTEM BY USING THE SYSTEM OF DIFFERENTIAL DIFFERENCE EQUATIONS FOR MULTIPLE-SERVER FINITE SOURCE QUEUING MODEL:

System will have Probability of containing x number of customers at time $(t + \omega t)$,

$P_x(t+\omega t) = P_x(t)$\{ Prob (zero arrival in ωt & zero departure in ωt) \} + $P_{x+1}(t)$\{ Prob (zero arrival in ωt & one departure in ωt)\} + $P_{x-1}(t)$\{ Prob (one arrival in ωt & zero departure in ωt)\} [8]

(where ωt is very small)

Case-1: For $(x < n)$

Using equations (2) & (3) for small time ωt, yields

$$P_x(t+\omega t) = P_x(t)\left\{\left(1-\frac{\alpha(R-x)\omega t}{R}\right)^R\left(1-\frac{x\beta\omega t}{R}\right)^R\right\}$$
$$+P_{x+1}(t)\left\{\left(1-\frac{\alpha(R-x-1)\omega t}{R}\right)^R R\left(\frac{(x+1)\beta\omega t}{R}\right)\left(1-\frac{(x+1)\beta\omega t}{R}\right)^{R-1}\right\}$$
$$+P_{x-1}(t)\left\{R\left(\frac{\alpha(R-x+1)\omega t}{R}\right)\left(1-\frac{\alpha(R-x+1)\omega t}{R}\right)^{R-1}\left(1-\frac{(x-1)\beta\omega t}{R}\right)^R\right\}$$

On simplifying we get,

$$\therefore P_x'(t) = \alpha(R-x+1)P_{x-1}(t) + (x+1)\beta\, P_{x+1}(t) - [\alpha(R-x) + x\beta]P_x(t)$$
(5)[10]

Case-2: For $(n \le x \le R)$

Using equations (2) & (4) for small time ωt, yields

$$P_x(t+\omega t) = P_x(t)\left\{\left(1-\frac{\alpha(R-x)\omega t}{R}\right)^R \left(1-\frac{n\beta\omega t}{R}\right)^R\right\}$$
$$+P_{x+1}(t)\left\{\left(1-\frac{\alpha(R-x-1)\omega t}{R}\right)^R R\left(\frac{n\beta\omega t}{R}\right)\left(1-\frac{n\beta\omega t}{R}\right)^{R-1}\right\}$$
$$+P_{x-1}(t)\left\{R\left(\frac{\alpha(R-x+1)\omega t}{R}\right)\left(1-\frac{\alpha(R-x+1)\omega t}{R}\right)^{R-1}\left(1-\frac{n\beta\omega t}{R}\right)^R\right\}$$

On simplifying we get,

$$\therefore P_x'(t) = \alpha(R-x+1)P_{x-1}(t) + n\beta\, P_{x+1}(t) - [\alpha(R-x) + n\beta]P_x(t)$$
(6)[10]

In the steady state condition, system is independent of time. Hence under the steady state condition we get, the Probability of x customers in the system,

$$\therefore P_x = \frac{R!}{x!(R-x)!}\left(\frac{\alpha}{\beta}\right)^x P_0 \quad , \ (x<n) \qquad (7)[10]$$

$$= \frac{R!}{(R-x)!\,n!\,n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x P_0 \ , \quad (n \le x \le R) \ (8)[10]$$
$$= 0 \qquad , \ (x > R)$$

Where (no customer for service) server stand idle,

$$P_0 = \left[\sum_{x=0}^{n-1}\frac{R!}{x!(R-x)!}\left(\frac{\alpha}{\beta}\right)^x + \sum_{x=n}^{R}\frac{R!}{(R-x)!\,n!\,n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x\right]^{-1}$$
(9)[10]

WAITING TIME DISTRIBUTION OF A CUSTOMER IN THE QUEUE FOR MULTIPLE-SERVER FINITE SOURCE QUEUING MODEL:

In Case 2, number of arriving customers are either equal or more than the number of servers and hence next arriving customer must wait in the queue for service. So waiting time of a customer in the queue comes in existence only in case-2. In the steady state condition, the waiting time distribution of each customer is identical and a continuous random variable. Let T be the time required by the servers to serve all the customers who are already in the system. Let $F_T(t)$ be the Probability distribution function of T then,

$F_T(0) =$ {Probability of no customer} + {Probability of customers $(x<n)$ }

$$= 1 - \sum_{x=n}^{R} \frac{R!}{(R-x)!\,n!\,n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x P_0 \qquad (10)$$

If a customer is arriving for service and there are already $(x \ge n)$ customers present in the system then the arriving customer will get service after the service completion of all the customers in the system. As the Calling Population is finite and limited to R, arriving customer is allow to join the system only if $n \le x \le (R-1)$ customers are there in the system and getting service after $n \le x \le (R-1)$ are served.

Let $f_x(t)$ be the Probability function of $n \le x \le (R-1)$ customers

then, $f_x(t) = \sum_{x=n}^{R-1} P_x$ [Prob { $(x-n)$ customers got service at time t }

$\times$ Prob { n customers are under service during time ωt }][3], [8] (where ωt is very small)

Using equations (4) & (8), yields

$$f_x(t) = \sum_{x=n}^{R-1}\frac{R!}{(R-x)!\,n!\,n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x P_0\left[{}^{R-1}C_{x-n}\left(\frac{n\beta t}{R-1}\right)^{x-n}\left(1-\frac{n\beta t}{R-1}\right)^{(R-1)-(x-n)}\right] n\beta\omega t$$
(11)

and from equations (10) & (11), we get

$$\therefore F_T(t) = P(T \le t) = F_T(0) +$$
$$\int_0^t \sum_{x=n}^{R-1}\frac{R!}{(R-x)!\,n!\,n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x P_0\left[{}^{R-1}C_{x-n}\left(\frac{n\beta t}{R-1}\right)^{x-n}\left(1-\frac{n\beta t}{R-1}\right)^{(R-1)-(x-n)}\right] n\beta dt$$

By diff w. r. to t we get Probability density function of waiting time distribution is

$F_T'(t)$

$$=0+\sum_{x=n}^{R-1}\frac{R!}{(R-x)!n!n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x P_0\left[{}^{R-1}C_{x-n}\left(\frac{n\beta t}{R-1}\right)^{x-n}\left(1-\frac{n\beta t}{R-1}\right)^{(R-1)-(x-n)}\right]n\beta$$

($\because$ from equation (10), $F_T(0)$ is independent of t)

$$=\sum_{x=n}^{R-1}\frac{R!}{(R-x)!n!n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x P_0\left[{}^{R-1}C_{x-n}\left(\frac{n\beta t}{R-1}\right)^{x-n}\left(1-\frac{n\beta t}{R-1}\right)^{(R-1)-(x-n)}\right]n\beta$$

(12)

Now, time interval to serve $(R-1)$ number of customers is $0 \le t < \dfrac{R-1}{n\beta}$

$\therefore$ Expected waiting time of a customer in the queue is given by

$$T_q = \int_0^{R-1/n\beta} t \times F_T'(t)\,dt$$

and using equation (12), we get

$$T_q = \int_0^{R-1/n\beta} t \times \sum_{x=n}^{R-1}\frac{R!}{(R-x)!n!n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x P_0\left[{}^{R-1}C_{x-n}\left(\frac{n\beta t}{R-1}\right)^{x-n}\left(1-\frac{n\beta t}{R-1}\right)^{(R-1)-(x-n)}\right]n\beta dt$$

$$=\frac{R!}{(n-1)!}P_0\beta \times\left[\sum_{x=n}^{R-1}\frac{1}{(R-x)!n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x {}^{R-1}C_{x-n}\left(\frac{n\beta}{R-1}\right)^{x-n}\int_0^{R-1/n\beta} t^{x-n+1}\left(1-\frac{n\beta t}{R-1}\right)^{(R-1)-(x-n)} dt\right]$$

To solve the integration using substitution

$$\frac{n\beta t}{R-1}=y_1 \Rightarrow t=\left(\frac{R-1}{n\beta}\right)y_1 \quad \Rightarrow dt=\left(\frac{R-1}{n\beta}\right)dy_1 ,$$

$0 \le y_1 \le 1$

$$\therefore T_q=\frac{R!}{(n-1)!}P_0\beta \times \left[\sum_{x=n}^{R-1}\frac{1}{(R-x)!n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x\left(\frac{n\beta}{R-1}\right)^{x-n}\frac{(R-1)!}{(x-n)!(R-1-x+n)!}\int_0^1\left(\frac{R-1}{n\beta}y_1\right)^{x-n+1}(1-y_1)^{(R-1)-(x-n)}\left(\frac{R-1}{n\beta}\right)dy_1\right]$$

$$=\frac{R!}{(n-1)!}P_0\beta\left[\sum_{x=n}^{R-1}\frac{1}{(R-x)!n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x\left(\frac{R-1}{n\beta}\right)^2\frac{(R-1)!}{(x-n)!(R-1-x+n)!}\int_0^1 y_1^{x-n+1}(1-y_1)^{(R-1)-(x-n)}dy_1\right]$$

$$=\frac{R!}{(n-1)!}P_0\beta\left[\sum_{x=n}^{R-1}\frac{1}{(R-x)!n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x\left(\frac{R-1}{n\beta}\right)^2\frac{(R-1)!}{(x-n)!(R-1-x+n)!}\beta'(x-n+2,R-x+n)\right]$$

$$=\frac{R!}{(n-1)!}P_0\beta\left[\sum_{x=n}^{R-1}\frac{1}{(R-x)!n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x\left(\frac{R-1}{n\beta}\right)^2\frac{(R-1)!}{(x-n)!(R-1-x+n)!}\frac{\Gamma(x-n+2)\Gamma(R-x+n)}{\Gamma(R+2)}\right]$$

$$=\frac{R!}{(n-1)!}P_0\beta\left[\sum_{x=n}^{R-1}\frac{1}{(R-x)!n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x\left(\frac{R-1}{n\beta}\right)^2\frac{(R-1)!}{(x-n)!(R-1-x+n)!}\frac{(x-n+1)!(R-1-x+n)!}{(R+1)!}\right]$$

$$=\frac{(R-1)!(R-1)^2P_0}{n^2(n-1)!\beta(R+1)}\left[\sum_{x=n}^{R-1}\frac{(x-n+1)}{(R-x)!n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x\right]$$

By using substitution $x=n+r$, we obtain

The expected waiting time of a customer in the queue,

$$T_q=\frac{(R-1)!(R-1)^2P_0}{n^2(n-1)!\beta(R+1)}\left[\sum_{r=0}^{R-n-1}\frac{(r+1)}{(R-n-r)!n^r}\left(\frac{\alpha}{\beta}\right)^{n+r}\right] \quad (13)$$

EXAMPLE FOR ANALYSIS AND COMPARISON:

For analysis and comparison of Finite Source Queuing Model consider an example having data of five customers given in following Table-1.

Arriving customers be limited to $R=5$ and for Multiple-Server Model number of severs $n=2$ (cph. customers per hour).

Calculations: [3], [4], [5], [8]

Arriving customers be limited to $R=5$, and for Multiple-Server take $n=2$

1. Utilization factor of server, (from equation (1))

$$\rho = \frac{\alpha}{n\beta} = \frac{0.1834}{2(0.3505)} = 0.2616$$

2. Server stands idle, (from equation (9))

$$P_0 = \left[\sum_{x=0}^{n-1} \frac{R!}{x!(R-x)!}\left(\frac{\alpha}{\beta}\right)^x + \sum_{x=n}^{R} \frac{R!}{(R-x)!n!n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x\right]^{-1}$$

$$= \left[\sum_{x=0}^{1} \frac{5!}{x!(5-x)!}(0.5233)^x + \sum_{x=2}^{5} \frac{5!}{(5-x)!2!2^{x-2}}(0.5233)^x\right]^{-1} = 0.1008$$

3. Expected waiting time of a customer in the queue, (from equation (13))

$$T_q = \frac{(R-1)!(R-1)^2 P_0}{n^2(n-1)!\beta(R+1)}$$

$$\left[\sum_{r=0}^{R-n-1} \frac{(r+1)}{(R-n-r)!n^r}\left(\frac{\alpha}{\beta}\right)^{n+r}\right]$$

$$= \frac{4! \times 4^2 \times 0.1008}{2^2(2-1)! \times 0.3505 \times 6}$$

$$\left[\sum_{r=0}^{2} \frac{(r+1)}{(3-r)!2^r}(0.5233)^{2+r}\right] = 0.7983 hrs.$$

4. Expected waiting time of a customer in the system,

T_s = (Expected waiting time in the queue) + (service time)

$$= T_q + \frac{1}{\beta} = 0.7983 + \frac{1}{0.3505} = 3.6514 hrs.$$

5. Expected number of customers in the queue, (Using Little's formula [3])

$$N_q = \alpha' T_q \quad \text{Where} \quad \alpha' = \alpha \sum_{x=0}^{R} (R-x)P_x$$

$$\therefore N_q = 0.5381 \times 0.7983 = 0.4296 customer$$

6. Expected number of customers in the system, (Using Little's formula [3])

$$N_s = \alpha' T_s \quad \text{Where} \quad \alpha' = \alpha \sum_{x=0}^{R} (R-x)P_x$$

$$\therefore N_s = 0.5381 \times 3.6514 = 1.9648 customers$$

7. Expected waiting time of a customer in the queue for busy system,

$$T_b = \frac{T_q}{(\text{Prob.of system being engaged})} = \frac{T_q}{1-P_0}$$

$$= \frac{0.7983}{1-0.1008} = 0.8878 hrs.$$

8. Expected number of customers served per busy period,

$$N_b = \frac{N_s}{(\text{Prob.of system being engaged})} = \frac{N_s}{1-P_0}$$

$$= \frac{1.9648}{1-0.1008} = 2.1851 customers$$

Table-1: Data of Five Customers used for Multiple-Server Finite Source Queuing Model.

Customer for Service in Sr. No.	**Inter-arrival Time Between Two Successive Customers in hrs.** (a)	**Arrival Rate of Customers** $\left(1/a\right)$	**Service Time in hrs.** (b)	**Service Rate** $\left(1/b\right)$	α **Mean Value of** $\left(1/a\right)$	β **Mean Value of** $\left(1/b\right)$
1	6	0.1667	3	0.3333	0.1834 cph.	0.3505 cph.
2	5	0.2	2.5	0.4		
3	5	0.2	3.5	0.2857		
4	6	0.1667	3	0.3333		

5			2.5	0.4		

Table-2: Calculated Results, Existing Results and % Decrease in the value of Queuing Parameters of the Multiple-Server Finite Source Queuing Model: (Time measured in hrs.)

Sr. No.	Queuing Parameters	Multiple-Server		
		Calculated Results	**Existing Results[4], [5], [8]**	**% Decrease**
1	T_q	0.7983	0.9896	19.33
2	T_s	3.6514	3.8422	4.97
3	N_q	0.4296	0.5325	19.32
4	N_s	1.9648	2.0677	4.98
5	T_b	0.8878	1.1005	19.33
6	N_b	2.1851	2.2995	4.97

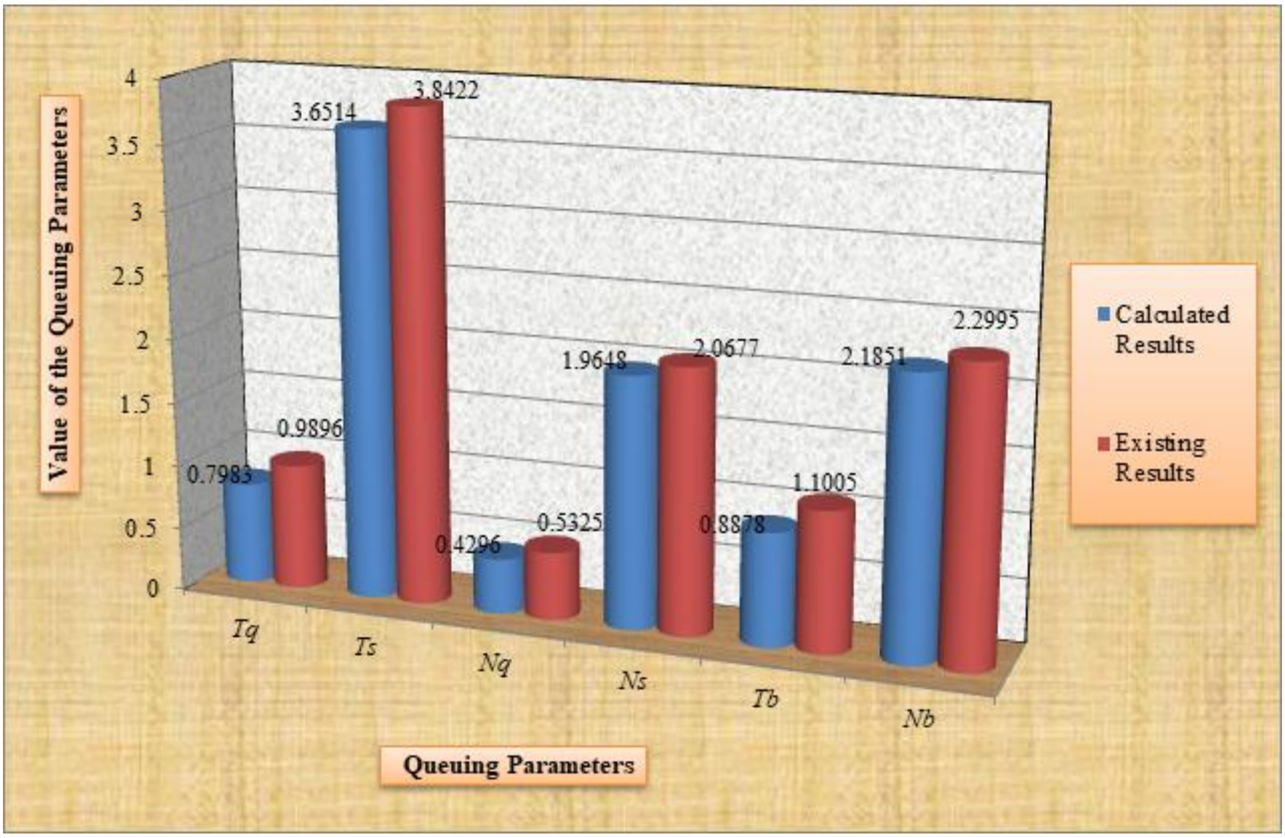

Figure -1 Graph Showing Calculated Results and Existing Results of Queuing Parameters of Multiple-Server (Two Servers) Finite Source Queuing Model

CONCLUSION

The application of Binomial Distribution to the Finite Source Multiple-Server Queuing Model gives an assurance of time interval to arrive and serve the particular number of customers. The application of Binomial Distribution shows a big change of decrease in the results of approximately 5% to 19% as compared to the existing results of Poisson Distribution. Waiting time of a customer in the queue, number of customers in the queue and waiting time in the queue for busy system shows approximately

19% reduction. Whereas waiting time of a customer in the system, number of customers in the system and served per busy period reduce approximately 5%. From the Table-2 and the graph as shown in Fig.-1 of calculated results and existing results indicates that the application of Binomial Distribution may improve the result analysis.

Conflicts of interest: The authors stated that no conflicts of interest.

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