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**SECTION 4. Computer science, computer
engineering and automation.**

INVESTIGATION OF THE ACCURACY AND SPEED OF THE ALGORITHMS OF STOCHASTIC OPTIMIZATION FUNCTIONS ON A TWO-DIMENSIONAL SPACE

Abstract: The purpose of this paper is to analyze the accuracy of calculations and the amount of time spent on finding optimal values for functions of several variables using optimization algorithms based on several methods of stochastic search. To conduct research, the staff of the Department of General Mechanics of the Lipetsk State Technical University created software that implements algorithms for searching for extreme values for functions of several variables. The functional purpose of the software is to find the minimum of a given function, represented as a string of characters. Optimization is performed on a specific and fixed search area, which is a hyperparallelepiped. Each separate program uses its own method of algorithmic optimization. In the development of programs, optimization algorithms based on the Monte Carlo method, an annealing simulation method, an interval analysis method, and a genetic algorithm were used. The results of a computational experiment for three different functions of two variables are presented in the article, a comparative analysis of the closeness of the results to values obtained analytically is carried out. The obtained data allowed us to draw conclusions about the advantages and disadvantages of each of the algorithms. Based on the results of computational experiments, the regularities between the time costs of algorithms and their numerical parameters are determined.

Key words: stochastic optimization, annealing simulation method, Monte Carlo method, genetic algorithm, interval analysis method, convergence rate, extremum.

Language: English

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Introduction

To solve optimization problems, numerous stochastic search methods based on Markov random processes have been developed [1, 2, 3, 4, 5, 6]. In particular, one of the most effective methods for optimizing a multifactorial function is meta-heuristic: genetic algorithms, tabu search, simulated annealing, maximum ant system (max-min ant system), ant colony optimization method, optimization of particle swarm optimization, differential evolution method, and others [6, 7, 8, 9, 10]. The advantage of meta-heuristic methods based on stochastic algorithms is their ability to solve complex problems in the absence of knowledge about search space; moreover, a good implementation of the meta-heuristic method can ensure finding a solution close to optimal within

a reasonable time or number of iterations. In many ways, this is why such methods allow us to find optimal solutions for problems that are difficult to solve from the point of view of direct analytical research. Very simplistic metaheuristics can be considered as algorithms realizing a direct random search on discrete space of possible optimal or close to optimal solutions of the problem until the set condition is met or a specified number of iterations is reached.

Materials and Methods

Modern researchers metaheuristics are recognized as a powerful and extremely popular class of optimization methods, which with high efficiency allow finding solutions for a wide range of problems



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from various applications [7, 11, 12]. However, often in solving applied problems, researchers have to resort to the search for compromise solutions when constructing schemes of search algorithms, it is necessary to make a choice between the high speed of computation and their accuracy [13].

The purpose of this study is to analyze the accuracy and speed of convergence of optimization algorithms for stochastic optimization that allow searching for optimal values of functions of several variables on sets of dimension 2. In this paper, we give a comparative analysis of the speed of operation and the accuracy of the required values for the following algorithms: the Monte- Carlo, simulated annealing, genetic algorithm and interval analysis algorithm. Optimization is performed on a specific set of functions and on a fixed search area, which is a hyperparal-lelepipid.

To conduct research, a complex of three computer programs was written (a certificate of state registration of computer programs No. 2017613650 on 07.06.2017). The programs are written in C++ in Builder 6.0 and run on Windows 95, 98, XP and the following. Programs run in windowed mode and allow you to enter data manually. The output of the calculation results directly in the windows of the program interface.

The functional purpose of programs is to find the minimum of a given function $f(x)$, represented as a string of symbols, on a given segment. Each separate program uses its own method of algorithmic optimization.

Figures 1, 2, and 3 show visual displays of the interface of the main program modules.

The screenshot shows a software window with two main sections: 'Входные данные' (Input data) and 'Выходные данные' (Output data).
Входные данные:
 - Function: $f(x_1, x_2) = ((x_1)^2 - 100)^2 + ((x_2)^2 - 100)^2$
 - Search area: Hyperparallelepiped with variables X1 and X2, each ranging from -2 to 2.
Выходные данные:
 - Method: Monte-Carlo
 - Iterations: 100000 (100% progress)
 - F min = 0.407057502325165
 - Coordinates: X1 = 9.98324533829768, X2 = 9.97280800805689
 - Time: 0.211841535688851 seconds
 - Method: Method of simulation of annealing
 - Max temperature (T): 1000 (100% progress)
 - Number of cycles (L): 1000
 - Cooling parameter (r): 0.98
 - F min = 0.437567864486749
 - Coordinates: X1 = 9.97843867305511, X2 = 9.97486800744612
 - Time: 1.92513399561112 seconds
 - Buttons: 'Найти минимум методом Монте-Карло', 'Найти минимум методом имитации отжига', 'Выход'

Figure 1 - The working window of the program module realizing the search for the optimal value of a function of several variables using the Monte Carlo algorithm and the method of simulation of annealing

The screenshot shows a software window with three main sections: 'Входные данные' (Input data), 'Параметры алгоритма' (Algorithm parameters), and 'Результаты вычислений' (Calculation results).
Входные данные:
 - Function: $F(x, y, z) = (x + y - 2)^2$
 - Search area: Hyperparallelepiped with variables x, y, and z, each ranging from -2 to 2.
Параметры алгоритма:
 - Number of individuals in population: 10
 - Mutation probability: 0.5
 - Number of intervals of division of hyperparallelepiped: 1000
 - Number of generations: 10
 - Time: 1.74209924233511 c.
Результаты вычислений:
 - F min = 0
 - x = 3.3598523844663, y = 1.67784693327687
 - Button: 'Найти оптимум'

Figure 2 - The working window of the program module realizing the search for the optimal value of a function of several variables using the genetic algorithm

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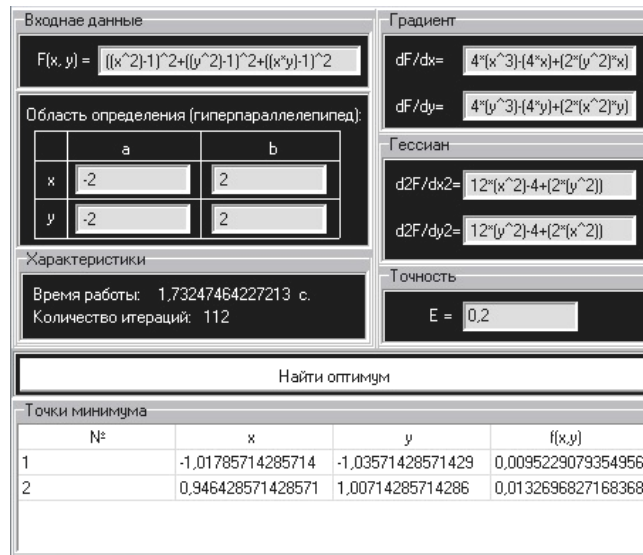


Figure 3 - The working window of the program module that implements the search for the optimal value of a function of several variables using the interval analysis method

Incremental algorithms for finding the minimum (optimal) value of the function, based on the Monte Carlo method, the annealing simulation method, the genetic algorithm and the algorithm of the interval estimation method are presented in [1, 2, 3, 6, 14].

For successful execution of the program, which uses the algorithm of interval estimates, it is required to enter a function, its first and second derivatives for each of the variables, the domain of the function definition. It should be borne in mind that a given function must have the property of twice differentiability on a given interval.

For successful execution of the program, it is required to enter a function, the area of its definition, and also the values of the method parameters.

Example of an input function:

$$\arctg((x1*x2+4)/(x1*x3*x4*x5-3*x1*x4+1)).$$

In programs, there is a limit on the number of variables - the function can not depend on more than nine variables. Programs do not provide for limits on the size of the domain of definition of a function and the magnitude of the values of numerical parameters of computational algorithms. Note that the program run time and the accuracy of the data obtained with

the help of it depend on these numerical parameters (Tables 3, 4).

The study was carried out using three functions:

$$F_1(x, y) = 1 - \cos(x - 1) + \sin(y - 1) - \cos(x + y - 2);$$

$$F_2(x, y) = (x^2 - 1)^2 + (y^2 - 1)^2 + (x - y)^2;$$

$$F_3(x, y) = (x + y - 2)^2.$$

The functions reach a minimum at the point with coordinates (1, 1).

The search area is segments x

$$\in [-2, 2], y \in [-2, 2].$$

Computational experiments were carried out on a computer with an Intel(R) Core(TM) i5-2300 CPU @ 2800GHz.

The results of optimization of the functions $F_1(x, y)$, $F_2(x, y)$, $F_3(x, y)$ for all four algorithms are presented in Table 1.

Compare the running time of the algorithms allow the data presented in Table 2.

To investigate the time dependence of the algorithm based on the Monte Carlo method on the number of iterations, a computational experiment was carried out, and the optimal values of the function $F_2(x, y)$ on the interval $[-2, 2]$ were searched for. The results of the computational experiment are presented in Table 3.

Table 1

The values of the optimums of functions found using computer programs

№	Function	Monte Carlo Method (100000 iterations)	Simulation method for annealing ($T=1000, L=1000, r=0,98$)	The genetic algorithm (number of individuals: 10, probability of mutation: 0.5, number of	Method of interval estimates (accuracy $E = 0.2$)



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		generations: 10)							
		x	y	x	y	x	y	x	y
1	$F_1(x, y)$	0,878	1,050	1,309	0,069	2,035	1,891	1,010	1,544
2		0,954	0,658	1,255	0,902	1,598	0,255	1,287	0,558
3		0,845	1,101	1,025	1,003	1,258	2,337	1,221	0,655
4		0,755	1,180	1,008	0,891	1,298	1,336	0,505	1,012
5		0,806	1,462	0,815	0,981	0,567	1,622	1,456	0,778
6	$F_2(x, y)$	0,998	1,353	1,329	1,202	1,954	1,333	1,469	2,941
7		0,954	0,648	0,997	1,044	1,305	0,557	1,332	1,233
8		0,999	1,636	1,001	1,637	1,556	0,713	1,258	0,245
9		0,998	0,669	0,999	0,841	1,223	2,036	1,058	0,801
10		0,869	1,025	0,999	1,164	1,879	1,825	1,322	0,210
11	$F_3(x, y)$	0,302	1,883	0,859	1,336	1,369	0,001	1,229	1,331
12		0,265	0,244	0,987	0,995	0,546	1,223	1,003	0,932
13		1,626	0,614	1,204	0,753	1,821	2,003	1,975	2,364
14		0,984	1,932	1,006	0,861	1,409	1,337	1,764	1,223
15		0,389	0,867	1,833	0,991	1,336	0,998	1,584	2,369
Average value, $F_{cp}(x, y)$		0,80192	1,0757	1,04858	1,00541	1,38233	1,26075	1,1791	1,0200
$\Delta = F_{cp}(x, y) - 1 $		0,19808	0,0757	0,04858	0,00541	0,38233	0,26075	0,1791	0,0200

Table 2

Time-consuming work of algorithms to find optimal values

№	Function	Время, затраченное на поиск оптимального значения, с			
		Monte Carlo Method (100000 iterations)	Simulation method for annealing ($T=1000, L=1000, r=0,98$)	The genetic algorithm (number of individuals: 10, probability of mutation: 0.5, number of generations: 10)	Method of interval estimates (accuracy $E = 0.2$)
1	$F_1(x, y)$	0,434896893	1,948114753	1,45677189761	1,72342123855
2		0,439852551	2,043567893	1,45123454380	1,78749053456
3		0,438157642	2,456690103	1,39943267833	1,72868920054
4		0,435936690	2,123423539	1,35752345675	1,65681234692
5		0,436751039	1,890678754	1,48765482303	1,64458895915
6	$F_2(x, y)$	0,417397450	1,434553778	1,21234185769	1,50345678515
7		0,400477815	1,567836168	1,32512894783	1,39468463289
8		0,387677813	1,776500083	1,18678007031	1,41387636774
9		0,418835855	1,826473534	1,21569043216	1,43648586996
10		0,400695770	1,612332124	1,36070765403	1,48849803033
11	$F_3(x, y)$	0,164545287	1,014389397	1,19541467957	0,84453576555
12		0,129453465	0,914345922	1,18347550456	0,76541345647
13		0,159634598	1,156043937	0,99657746712	0,67513454644
14		0,102030678	1,200459945	1,14343554670	0,61704606357
15		0,146566570	1,073459369	0,98878347848	0,59423370054
Среднее значение		0,327527	1,602591	1,264064	1,284958

Table 3

The time dependence of the Monte Carlo algorithm on the number of iterations

№	The number of iterations, I	The running time of the algorithm, t , sec.
1	1	0,0003350
2	2	0,0002990
3	5	0,0014324
4	10	0,0025763
5	50	0,0112647

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6	100	0,0224290
7	1000	0,0242715
8	5000	0,0323467
9	10000	0,0408283
10	50000	0,1219972

With the help of linear regression, an analytical dependence was obtained between the time of operation of the algorithm based on the Monte Carlo method and the number of iterations:

$$I = 535500r - 8919,4$$

To study the time dependence of the algorithm on the basis of the method of simulating the fat from

the numerical parameters of the algorithm (the maximum (initial) temperature, the number of cycles, the temperature decrease parameter), a computational experiment was carried out. We searched for the optimal values of the function $F_2(x, y)$ on to the segment $[-2, 2]$. The results of the computational experiment are presented in Table 4.

Table 4

The running time of the algorithm based on the annealing simulation method

№	Maximum temperature, T	Number of cycles for each temperature, L	Parameter of temperature decrease, r	Time of operation of the algorithm, t , sec.
1	1000	10	0,2	0,003357
2	1000	10	0,4	0,00675
3	1000	10	0,7	0,012608
4	1000	10	0,9	0,026091
5	1000	50	0,2	0,004464
6	1000	50	0,4	0,007469
7	1000	50	0,7	0,017348
8	1000	50	0,9	0,04116
9	1000	100	0,2	0,00572
10	1000	100	0,4	0,00979
11	10000	100	0,7	0,02593
12	10000	100	0,9	0,06536
13	10000	500	0,2	0,01758
14	10000	500	0,4	0,03116
15	10000	500	0,7	0,07885
16	10000	500	0,9	0,23834
17	10000	1000	0,2	0,03127
18	10000	1000	0,4	0,05513
19	10000	1000	0,7	0,14127
20	10000	1000	0,9	0,44616

Conclusion

With the help of linear regression, an analytical dependence is obtained that allows us to calculate the

$$t = 0,00025 + 0,00000016T + 0,000294L + 0,012r$$

From the obtained results of computational experiments it is evident that the algorithm has the greatest accuracy when determining extremal values of functions from two variables on the basis of the annealing simulation method and the method of interval estimations. The results obtained using an algorithm based on the Monte Carlo method and the genetic algorithm have significant deviations from the optimal values found analytically. At the same time, the rate of convergence of the algorithm based on the simulation of annealing significantly exceeds

running time of the algorithm on the basis of the annealing simulation method through its numerical parameters:

the time costs of other algorithms. A significant disadvantage of the algorithm based on the method of interval analysis is that for its practical application it is necessary to know the analytical record of the first and second derivatives of the optimized function, which substantially narrows its application domain. Note that for algorithms based on the Monte Carlo method and the annealing simulation method, correction of the design parameters-maximum (initial) temperature, the number of cycles, the temperature reduction parameter, is possible-so that

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the accuracy of the calculation results increases with increasing time for -rate and, conversely, in tasks that do not require high accuracy of the results obtained, to achieve the speed of calculations. Thus, the algorithm based on the method of imitation of fat is

the most acceptable for applied research, since it allows for the variance between the accuracy of calculations and the rate of convergence of the stochastic search.

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