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ON DECOMPOSITION THEOREMS OF MULTIFUNCTIONAL BERGMAN TYPE SPACES IN SOME DOMAINS IN C^n

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We present some extensions of well-known one functional results on atomic decompositions in classical Bergman spaces obtained earlier by various authors in some new multifunctional Bergman type spaces in various domains in higher dimension.

Keywords: Bergman type spaces, analytic functions, decomposition theorems.

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О ТЕОРЕМАХ ДЕКОМПОЗИЦИИ В МУЛЬТИФУНКЦИОНАЛЬНЫХ ПРОСТРАНСТВАХ БЕРГМАНА

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Мы приводим некоторые новые теоремы о декомпозиции аналитических функций из мультифункциональных пространств Бергмана, обобщающие ранее известные результаты подобного типа для обычных пространств Бергмана в различных областях.

Ключевые слова: аналитические функции, пространства Бергмана мультифункциональные пространства, теоремы декомпозиции.

1. Introduction, preliminaries and main results

The intention of this paper to provide complete analogues of our recent results (see [4],[5])on atomic decompositions in new multifunctional Bergman spaces in the unit ball and bounded pseudoconvex domains in some new multifunctional Bergman spaces in tubular domains over symmetric cones and in some related domains.

The problem of atomic decompositions of Bergman and other spaces in one functional case was considered in various domains in one and higher dimension by various authors (see for example [1]-[3],[14-16],[7],[10],[12] and various references there).It is well-known that these theorems have numerous applications in various problems of complex function theory in one and higher dimension. Note that our results in particular are heavily based on certain new theorems from [7] and [6] on onefunctional Bergman spaces in tubular domains over symmetric cones and in Siegel domains of second type(direct generalizations of bounded symmetric domains). As in mentioned cases of the unit ball and bounded strictly pseudoconvex domains (see[4] and [5]) adding a simple new integral condition (which vanish in case of one functional space) we obtain a new atomic decomposition theorem for analytic multifunctional Bergman spaces in these domains. These assertions can be considered at the same time as direct extensions of previously known results concerning one functional Bergman spaces. To formulate our theorems we need to introduce a group of definitions and notations taken from [7], [6]. Further we also note our results partially are also valid in so-called bounded minimal homogeneous domains in $\mathbb{C}^n$, based on recent results of S.Yamaji (see [8-9] and various references there).These can be done using same methods of proof which we present below.The only tool of our rather transparent proof is so-called Forelly-Rudin type estimate which is available in all these domains(see [7],[8],[6],[4],[5]) and a well-known uniform estimate from below of a norm of Bergman space which is also available in various domains in higher dimension. Note more precisely one part of all our assertions below in various domains and spaces on them is a direct simple corollary of an argument related with ordinary induction and an uniform estimate we just mentioned.The other part uses only Holder"s inequality applied twice and the standard Forelly-Rudin estimate. This can be seen after simple very careful analysis of the proof of the unit ball and tube cases below. First we provide some known assertions on atomic decomposition for one functional Bergman spaces , then we provide direct multifunctional generalizations of these assertions. First we discuss the simpler case of the unit ball then pass same arguments to other domains. We consider Bergman spaces on bounded symmetric domains, tubular domains over symmetric cones and then even more general Siegel domains of second type in $\mathbb{C}^n$ and new multifunctional A^p_α Bergman spaces on them. Since proofs of all assertions in various domains are very similar we will omit some details below leaving them to interested reader. Let B_n or B be the unit ball in $\mathbb{C}^n$, let further $H(B_n)$ be the space of all analytic functions in B_n(or B).

We define the Bergman class in the unit ball in a usual way.

$$\left(A^p_\alpha\right)(B_n) = \left\{ f \in H(B_n) : \int_{B_n} |f(z)|^p (1-|z|)^\alpha dv(z) < \infty \right\},$$

$0 < p < \infty, \alpha > -1$, where dv is a normalized Lebeques measure on B_n (we will also use this notation for Lebegues measure in other domains). We formulate a well-known theorem on atomic decomposition of usual Bergman spaces $A^p_\alpha(B_n)$.

We denote positive constants in this paper as usual by $c, c_1, c_2, \ldots$ or by c_α, c_β.

Theorem A. *(see, for example, [1,3,14])*
Let $\alpha > -1$, $f \in H(B_n)$, $0 < p < \infty$, let , $b > b_0$, $b_0 = b_0(p,n,\alpha)$. Let $f \in A^p_\alpha(B_n)$. Then there exist a sequence $\{a_j\}$ in B_n, such that

$$f(z) = \sum_{j=1}^{+\infty} c_j \frac{\left(1-|a_j|^2\right)^{\left(\frac{bp-n-1-\alpha}{p}\right)}}{\left(1-\langle z,a_j\rangle\right)^b}, z \in B_n \qquad (s_1)$$

Where the series converges in the norm topology of $\left(A^p_\alpha\right)(B_n)$ and $\sum_{j=1}^{\infty} |c_j|^p < \infty$, and where $\alpha > \alpha_0$, $\alpha_0 = \alpha_0(p,n,m)$ and the reverse is also true if (s_1) holds then $f \in A^p_\alpha(B_n)$ for same values of parameters.

Let T_Ω be a tubular domain over symmetric cone, let $\{z_j\}_{j=1}^{\infty}$ be a δ-lattice in T_Ω (see [7], [11]), $\delta \in (0,1)$, $\{z_j\} \in T_\Omega$, $H(T_\Omega)$ – be the space of all analytic functions on T_Ω.

Let further

$$A^p_\alpha(T_\Omega) = \left\{ f \in H(T_\Omega) : \int_{T_\Omega} |f(z)|^p \Delta^{\alpha-\frac{n}{r}}(Imz)\, dv(z) < \infty \right\},$$

where $\alpha > \frac{n}{r} - 1, 1 \le p < \infty$ be the Bergman space in T_Ω, (Δ^t) be determinant function in T_Ω.(or we use below sometimes T_Λ)(see [7,11]).

We formulate a known theorem on atomic decomposition of $\left(A^p_\alpha\right)(T_\Omega)$ spaces in T_Ω (see [7,11]).(well-known one functional result which has many applications)

Theorem B. *(see, for example, [7,11])*
Let $\{z_j\}$ be a δ-lattice in T_Ω , $\delta \in (0,1)$, $z_j = x_j + iy_j$, $z_j \in T_\Omega, j \ge 0$. Then

$$\|f\|_{A^p_\nu} \asymp \sum_j |f(z_j)|^p \Delta^{\nu+\frac{n}{r}}(y_j).$$

If $f \in A^p_\nu$, then

$$\sum_{j=1}^{\infty} |\lambda_j|^p \Delta^{\nu+\frac{n}{r}}(y_j) \le c\|f\|^p_{A^p_\nu}. \qquad (s_2)$$

If

$$\sum_{j=1}^{\infty} |\lambda_j|^p \Delta^{\nu+\frac{n}{r}}(y_j) < \infty$$

then the reverse to (s_2) holds, if $f \in \left(A^p_\nu\right)$ then

$$f(z) = \sum_j (\lambda_j)(B_\nu(z,z_j))\left(\Delta^{\nu+\frac{n}{r}}(y_j)\right)$$

where B_ν is Bergman kernel ([7,11]).

In [1] and [5], [10], [14] similar atomic decomposition theorems were obtained (or mentioned) for A^p_α Bergman spaces in bounded symmetric domains, Siegel domains and in bounded pseudoconvex domains with smooth boundary for A^p_α spaces (one functional Bergman spaces).

In [4], [5] these assertions in analytic space were extended to multifunctional $\left(A^p_\alpha\right)$ spaces in the unit ball and in bounded strongly pseudoconvex domains with smooth boundary. This paper provide such results in other type domains in $\mathbb{C}^n$. To formulate these results we need some very basic definitions and lemmas on these domains,namely tube domains, Siegel domains and bounded symmetric domains, and on Bergman A^p_α spaces on them.

Let Ω be a bounded symmetric domain in $\mathbb{C}^n$ (see [13]). Then Ω is uniquely determined by their analytic invariants namely r-rank of Ω, a, b, all of them are positive integers. The Bergman reproducing kernel is

$$K(z,w) = \frac{1}{h(z,w)}, z, w \in \Omega,$$

where $h(z,w)$ is a sum of homogeneous monomials in z and $\bar{w}$,

$$N = a(r-1) + b + 2$$

and the orthogonal projection P of $L^2(dV)$ onto $A^2(dV)$ is given by the well-known formula

$$(Pf)(z) = \int_\Omega \frac{f(w)\, dV(w)}{h(z,w)^N}, f \in L^2(dV), z \in \Omega,$$

where dV is the normalized volume measure in Ω.

Let further

$$\alpha > -1, dV_\alpha(z) = c_\alpha h(z,z)^\alpha dV(z),$$

where c_α is special constant so that $dV_\alpha(z)$ has total mass 1 on Ω.

Let also further

$$A^p_\alpha(\Omega) = \left\{ f \in H(\Omega) : \int_\Omega |f(w)|^p \left(h(w,w)^\alpha\right) dV(w) < \infty \right\},$$

where $\alpha > (-1), 1 < p < \infty$, and where $H(\Omega)$ is a space of all analytic functions on Ω.

The definition of the problem weighted Bergman spaces in classical simplest bounded pseudoconvex domain the unit ball is the following.

Let

$$\int_{B_n} |f_1(z)|^{q_1} \cdots |f_m(z)|^{q_m} \left(1 - |z|^2\right)^{\left(\sum_{k=1}^m \alpha_k\right)} dv(z) < \infty,$$

where

$$\sum_{k=1}^m \alpha_k > -1, q_j \in (0, \infty), j = 1, \cdots, m.$$

Then can we say that there is a atomic decomposition for each $\{f_j\}$ function, $j = 1, ..., m$?

The answer is true when $m = 1$ (see [1, 5, 7]). Our goal is to show that when $q_j = p, j = 1, ..., m$, $p \in (0, \infty)$, the answer is also true, that is each function $f_j, j = 1, \cdots, m$ can be decomposed into atoms, under the following additional new integral condition

$$f_1(w_1) \cdots f_m(w_m) = C_b \int_B \frac{f_1(z) \cdots f_m(z)\, dv_\alpha(z)}{(1 - \langle z, w_1 \rangle)^{\frac{n+1+\alpha}{m}} \cdots (1 - \langle z, w_n \rangle)^{\frac{n+1+\alpha}{m}}}, \qquad (1)$$

where $w_j \in B_n$, $j = 1, \cdots, m$, $\alpha > -1$.

The following is our theorem on atomic decomposition for multifunctional weighted Bergman spaces which completely extends the theorem on atomic decomposition of one functional weighted Bergman spaces in the unit ball from [14],[15].

Theorem 1.
Let $\alpha_k > -1,\ f_k \in H(B_n),\ k = 1,\cdots,m,\ m \in \mathbb{N}$ *and* $1 < p < \infty$ *or* $p = 1$. *Suppose that*

$$b > n + \frac{\max \alpha_k + 1}{p}$$

Let

$$(f_1,\cdots,f_m)_{A^p_\alpha} = \int_B \prod_{k=1}^m |f_k|^p \left(1-|z|^2\right)^{(m-1)(n+1)+\sum_{k=1}^m \alpha_k} dv(z).$$

If for all $z_j \in B,\ j = 1,\cdots,m,$

$$f_1(z_1)\cdots f_m(z_m) = C_b \int_B \frac{f_1(z)\cdots f_m(z)\,dv_\alpha(z)}{\prod_{j=1}^m \left(1-\langle z,z_j\rangle\right)^{(n+1+\alpha)/m}} \tag{2}$$

and $(f_1,\cdots,f_m)_{A^p_\alpha} < \infty$, *then there exists a sequence* (a_j) *in* B *such that every function* f_k *can be represented in a form*

$$f_k(z) = \sum_{j=1}^\infty C_j^{(k)} \frac{\left(1-|a_j|^2\right)^{\frac{(pb-n-1-\alpha_k)}{p}}}{\left(1-\langle z,a_j\rangle\right)^b}, k = 1,...,m, \tag{3}$$

where the series converges in the norm topology of $A^p_{\alpha_k}$ *and* $\sum_{j=1}^\infty \left|C_j^{(k)}\right| < \infty,\ k = 1,...,m,\ b > b_0,\ \alpha > \alpha_0,\ b_0 = b_0(n,p,\alpha_1,...,\alpha_m),\ \alpha_0 = \alpha_0(n,p,\alpha_1...\alpha_m)$.

Conversely if $k = 1,...,m$ *has the form* (3) *then* $(f_1,\cdots,f_m)_{A^p_\alpha} < \infty$.

Simple arguments used in proof of this theorem 1 easily can be passed to various difficult domains in C^n.

We formulate now our new theorem on atomic decompositions in multifunctional Bergman spaces in tubular T_Ω domains over symmetric cones.

Theorem 2. *Let* $\nu_k > \frac{n}{r} - 1,\ k = 1,...,m,\ m \in N,\ m > 1$. *Let* $1 \le p < \infty$. *Let for some big enough* (β_0) *and all* $\beta_j > \beta_0$ *and* $z_j \in T_\Omega,\ f_j \in H(T_\Omega),\ j = 1,...,m.$

$$f_1(z_1)...f_m(z_m) = (\vec{c}_\beta) \int_{T_\Omega} \frac{\left(\prod_{j=1}^m f_j(z)\right)\left(\Delta^{\frac{1}{m}\sum_{j=1}^m \beta_j - \frac{n}{r}}(Imz)\right) dv(z)}{\prod_{j=1}^m \Delta^{\left(\frac{1}{m}\right)\left(\frac{n}{r}+\beta_j\right)}\left(\frac{z_j-\bar{z}}{i}\right)}$$

for some constant $\vec{c}_\beta$.

Let also

$$G(f_1,\ldots,f_m) = \int_{T_\Omega} \prod_{k=1}^m |f_k(z)|^p \left[\Delta^{(m-1)2\frac{n}{r}+\sum_{k=1}^m(\nu_k-\frac{n}{r})}(Imz)\right] dv(z) < \infty$$

then $f_k \in \left(A^p_{\nu_k}\right)(T_\Omega)$ *and hence the conclusions of theorem B for each* f_k *is valid and the reverse is also true if* $f_k \in A^p_{\nu_k}(T_\Lambda)$ *then* $G(f_1,\ldots,f_m) < \infty$.

The integral condition (it vanishes in both theorems for one function $m = 1$ case according to known result,namely since for all functions from Bergman class so called Bergman representation formula with large enough index is valid) as it is easy to note in the unit ball and in tube simply coincide if we put all β_j in our last theorem equal to each other. Proofs in both cases (different β_j and equal β_j) are very similar and to simplify calculations it we will work below only with simpler condition.

We formulate complete analogue of theorem 1 in bounded symmetric domains.We denote by A_0^p Bergman spaces without weight. We define similarly (as we already did for unit ball) Bergman space with appropriaate weight with several functions in this domain.

Theorem 3. *Let $\alpha_j > -1$, $j = 1,...,m$, $m \in N$, $m > 1$. Let $1 \le p < \infty$. Let for some big enough α_0 and all $\alpha_j > \alpha_0$ and $z_j \in \Omega$, $f_j \in H(\Omega)$, $j = 1,...,m$.*

$$\prod_{j=1}^{m} (f_j)(z_j) = (c(\alpha,\dots,\alpha_m)) \int_{\Omega} \frac{\left(\prod_{j=1}^{m} f_j(z)\right)\left(h(z,z)^{\alpha}\right) dV(z)}{\prod_{j=1}^{m} [h(z,w_k)]^{\frac{(N+\alpha)}{m}}}$$

for some constant c.

Then $f_j \in A_o^p(\Omega)$, $j = 1,...,m$ and there exists and constants $c_1, c_2 > 0$ and a sequence $\left(w_i^{(m)}\right)_i \in \Omega$ such that

$$(f_k(z)) = \sum_j \left(\lambda_j^k\right) \left(\frac{h\left(z, w_j^m\right)^{2N}}{h\left(w_j, w_j^m\right)^{N}} \right)^{\frac{1}{p}}$$

$$\sum_j \left(|\lambda_j|^p\right) \le c_1 \|f_k\|_{A_o^p}, z \in \Omega, k = 1,...,m$$

And if $f_i \in A_o^p, i = 1,\dots,m$ then $(f_1,...,f_m)_{A_o^p} < \infty$. so the reverse is valid also ,if each function is From Bergmman class then all group is from multifunctional space.

We will formulate below similar type assertion for more general Siegel domains of second type based on onefunctional known result of D.Bekolle and T.Kagou (see ,for example, [12], [10].)

First some basic facts on these domains. Let D be a as usual homogeneous Siegel domain of type II. Let $d\nu$ denote the Lebesque measure on D and let as usual $H(D)$ be the space of holomorphic functions in D endowed as usual with the topology of uniform convergence on compact subsets of D.

The Bergman projection P of D as usual the orthogonal projection of $L^2(D, d\nu)$ onto its subspace $A^2(D)$ consisting of holomorphic functions. Moreover it is known P is the integral operator defined on $L^2(D, d\nu)$ by the Bergman kernel $B(z, \zeta)$ which for D was computed for example in [8], [6,10].

Let r be a real number, for example. We fix it. Since D is homogeneous the function $\zeta \to B(\zeta, \zeta)$ does not vanish on D, we can set

$$L^{p,r}(D) = L^p\left(D, B^{-r}(\zeta,\zeta)\, d\nu(\zeta)\right), 0 < p < \infty.$$

Let p be an arbitrary positive number. The weighted Bergman space is defined as usual by

$$A^{p,r}(D) = L^{p,r}(D) \bigcap H(D).$$

The so-called weighted Bergman projection P_ε is the orthogonal projection of $L^{2,\varepsilon}(D)$ onto $A^{2,\varepsilon}(D)$. This facts can be found in [12,10]. It is proved [12,10] that there exists a real number $\varepsilon_D < 0$ such that $A^{2,\varepsilon}(D) = \{0\}$ if $\varepsilon \le \varepsilon_D$; and that for $\varepsilon \le \varepsilon_D$, P_ε is the integral operator defined on $L^{2,\varepsilon}(D)$ by the weighted Bergman kernel $c_\varepsilon B^{1+\varepsilon}(\zeta, z)$. In all our work we assume that $\varepsilon \ge \varepsilon_D$.

The “norm” $\|\cdot\|_{p,r}$ of $A^{p,r}(D)$ with $r > \varepsilon_D$ is defined by

$$\|f\|_{p,r} = \left(\int_D |f(z)|^p B^{-r}(z,z)\,d\nu(z)\right)^{\frac{1}{p}}, f \in A^{p,r}(D).$$

We need some assertions (see, for example, [6,10],[12])

Lemma A. *Let $h \in L^\infty(D)$. Take $\rho > \rho_0$ for large fired ρ_0. Then the function*

$$z \to G(z) = \int_D B^{1+\rho}(z,\zeta)\,h(\zeta)\,d\nu(\zeta)$$

satisfies the estimate $\sup\limits_{z\in D}|G(z)|B^{-\rho}(z,z) \le c\|h\|_\infty$ and $G \in H(D)$.

Lemma B. *For each ρ sufficiently large and for each $G \in H(D)$ such that*

$$\left(\sup_{z\in D}\right)|G(z)|\left|B^{-\rho}(z,z)\right| < \infty$$

one has the reproducing formula

$$(G(\zeta)) = (c_\rho)\int_D B^{1+\rho}(\zeta,z)\,(G(z))\left(B^{-\rho}(z,z)\right)d\nu(z), z \in D$$

Lemma C. *Let α and ε be in R^l, $(\zeta,\nu) \in D$. Then we have*

$$\int_D \left|b^{1+\alpha}((\zeta,\nu),(z,u))\right| b^{-\varepsilon}((z,u),(z,u))\,d\nu(z,u) < \infty$$

if $\varepsilon_i > \left(\frac{n_i+2}{2(2d-q)_i}\right)$ and $(\alpha_i-\varepsilon)_i > \frac{n_i}{-2(2d-q)_i}$, $i = 1,\dots,l$.

Lemma D. *(Forelly-Rudin estimate) Let α and ε be in R^l, $(\zeta,\nu) \in D$. Then for*

$$\varepsilon_i > \frac{n_i+2}{2(2d-q)_i}$$

and

$$(\alpha_i-\varepsilon)_i > \frac{n_i}{(-2)(2d-q)_i}, i = 1,\dots,l$$

$$\int_D \left|b^{1+\alpha}((\zeta,\nu),(z,u))\right| b^{-\varepsilon}((z,u),(z,u))\,d\nu(z,u) = c_{\alpha,\varepsilon} b^{\alpha-\varepsilon}((\zeta,\nu),(\zeta,\nu)).$$

Lemma E. *(Bergman representation formula) Let r be a vector of R^l such that $r_i > \left(\frac{n_i+2}{2(2d-q)_i}\right)$ for all $i = 1,\dots,l$ and a p is a real number such that*

$$1 \le p < \min\left\{\frac{n_i - 2(2d-q)_i(1+r_i)}{n_i}\right\}.$$

Then for all $\varepsilon \in R^l$ such that

$$(\varepsilon_i) > \frac{n_i+2}{2(2d-q)_i}\left(\frac{p-1}{p}\right) + \left(\frac{r_i}{p}\right),$$

$$i = 1,\dots,l, P_\varepsilon f = f, f \in A^{p,r}.$$

The known theorem on atomic decomposition of Bergman spaces in Siegel domain is the following.

Theorem C. (see [10],[12]) *Let $D \subset C^N$ be a symmetric Siegel domains of type II,*

$$p \in \left(\frac{2N}{2N+1}, 1\right),$$

$$r \in R^l; r_j > \frac{n_i + 2}{2(2d-q)_i}.$$

Then there are two constants $c = c(p,r)$ and $c_1 = c_1(p,r)$ such that for every $f \in A^{p,r}(D)$ there exists an l^p sequence $\{\lambda_i\}$ such that

$$f(z) = \sum_{i=0}^{\infty} \lambda_i b^{\frac{\alpha}{p}}(z, z_i) b^{\frac{1+r-\alpha}{p}}(z_j, z_j)$$

where $\{z_i\}$ is a lattice in D and the following estimate holds

$$c\|f\|_{p,r}^p \le |\lambda_i|^p \le c_1 \|f\|_{p,r}^p.$$

Remark 1. This theorem as it was shown later is true for $p \in (0,1)$.

Our extension is the following.

Theorem 4. *(On atomic decompositions of Siegel domains of second type for multifunctional Bergman spaces)*

1. Let $r_j > \varepsilon_D$, $j = 1,...,m$, $1 < p < \infty$, $f_k \in H(D)$, $k = 1,...,m$. Let

$$\left(\vec{f}\right) = (f_1,...,f_m)_{A^p_{\vec{r}}} = \int_D \prod_{j=1}^m |f_j(z)|^p B^{-(m-1)-\sum_{j=1}^m r_k}(z,z)\, d\nu(z) < \infty \qquad (A)$$

If for all such r_j, $j = 1,...,m$

$$\prod_{j=1}^m f_j(\zeta_j) = c\int_D \prod_{j=1}^m f_j(z) B^{-\rho}(z,z) \prod_{j=1}^m B^{\frac{1+\rho}{m}}(\zeta_j, z)\, d\nu(z) \qquad (B)$$

Then $(f_j) \in A^p_{r_j}$ and the reverse is also true if $(f_j) \in A^p_{r_j}$, $j = 1,...,m$ then $\left(\vec{f}\right)_{A^p_{\vec{r}}} < \infty$;

2. Let $D \subset C^N$ be a symmetric Siegel domain of type II, $p \in \left(\frac{2N}{2N+1}, 1\right)$, $r \in R^l$, $r^i_j > \left(\frac{n_i+2}{2(2d-q)_i}\right)$; $i = 1,...,l$.

If (A), (B) holds then $f_i \in A^{p,\vec{r}}(D)$, $\vec{r}_i = \left(r^i_1,...,r^i_l\right)$, $i = 1,...,m$ and there is $\left\{\lambda^j_i\right\}$, $j = 1,...,m$, $i = 1,...,m$.

So that

$$(f_j)(z) = \sum_{i=1}^{\infty} \lambda^j_i \left(b^{\frac{\alpha}{p}}(z,z_i)\right) b^{\frac{1+r_j-\alpha}{p}}(z_i,z_i), z \in D, j = 1,...,m,$$

where $\{z_i\}$ is a lattice in D and the following estimates are valid

$$c_2\left(\|f_j\|_{A^{p,r_j}}\right) \le \sum_{i=1}^{\infty} \left|\lambda^j_i\right|^p \le c_1\left(\|f_j\|_{A^{p,r_j}}\right),$$

where $\vec{\alpha} > \alpha_0$ for some fixed large enough α_0.

Remark 2 Note putting $m = 1$ in theorem 4 and using known Bergman representation formula with large index for Functions from Bergman class (see [10],[12])which remove additional integral condition we obtain known results for atomic decomposition of one functional Bergman spaces (see, for example, [10,12] and theorem C)

Same results with same proofs are valid in spaces of n harmonic functions in the unit polydisk. First we give some basic definitions (see for example [17,18] and references there).

Let $U^n = \{z \in C^n : |z_k| < 1, 1 \le k \le n\}$ be the unit polydisk and by m_{2n} we denote the volume measure on U^n, by $h(U^n)$ we denote the space of all harmonic functions in U^n.

Let also $f \in h(U^n)$, and let $\left(M_p^p\right)(f,r) = \int_{T^n} |f(r\zeta)|^p dm_n(\zeta)$, $r \in I^n$, $0 < p \le \infty$, where $I^n = (0,1)^n$, $m_n(\zeta)$ is Lebeques measure on T^n,

$$T^n = \{z \in C^n : |z_k| = 1,\ k = 1,...,n\}$$

The quasinormed space $L(p,q,\alpha)$, $0 < p,q < \infty$, $\alpha = (\alpha_1,\ldots,\alpha_n)$, $\alpha_j \ge 0$, $j = 1,\ldots,m$ is the space of those functions $f(z)$ measurable in the polydisk U^n for which the quasinorm

$$\|f\|_{p,q,\alpha} = \left\{ \int_{I^n} \prod_{j=1}^n (1-r_j)^{\alpha_j} M_p^q(f,r) \prod_{j=1}^n dr_j \right\}^{\frac{1}{q}},$$

or

$$\left(ess \sup_{r \in I^n} \right) \prod_{j=1}^n (1-r_j)^{\alpha_j} M_p(f,r), q = \infty, 0 < p < \infty$$

is finite.

For the subspace of $L(p,q,\alpha)$ consisting of n-harmonic functions let $h(p,q,\vec{\alpha}) = h(U^n) \bigcap L(p,q,\vec{\alpha})$ (see [17,18]),

$$h^p_{\vec{\alpha}}(U^n) = h(p,p,\vec{\alpha}).$$

We define Poisson kernel (P_α) in the unit disk as usual

$$P_\alpha = (\Gamma(\alpha+1)) \left[Re \left(\frac{2}{\left(1-\bar{\zeta}z\right)^{\alpha+1}} \right) - 1 \right], z \in U, \alpha \ge 0.$$

Let $P_\alpha(z,\zeta) = P_\alpha\left(z,\bar{\zeta}\right)$. For the polydisk we have (see [18,17])

$$P_{\vec{\alpha}}(z,\zeta) = \prod_{j=1}^n P_{\alpha_j}\left(z_j,\zeta_j\right), \alpha_j \ge 0, j = 1,\ldots,n, \zeta \in U^n, z \in U^n.$$

It is well-known that. P_α is n-harmonic by both variables z and ζ and $P_\alpha(z,\zeta) = P_\alpha(\zeta,z) = P_\alpha\left(\bar{z},\bar{\zeta}\right)$. The following assertion is base of our proof. Let

$$\alpha_j > 0, j = 1,\ldots,n, u \in h(p,q,\vec{\alpha}), 0 < p,q, \le \infty, \beta_j > (\beta_0),$$
$$\beta_0 = \beta_0(\vec{\alpha},p,q), j = 1,\ldots,n.$$

Then

$$u(z) = \left(\frac{1}{\prod_{j=1}^n \Gamma(\beta_j)} \right) \int_{U^n} \prod_{j=1}^n (1-|\zeta_j|)^{\beta_j - 1} \left(P_{\vec{\beta}}(z,\zeta)\right)(u(\zeta)) dm_{2n}(\zeta), z \in U^n,$$

(see [17])

We formulate now our atomic theorem in multifunctional Bergman spaces in context of n − harmonic Bergman function spaces.

Remark 3. Note similar theorems with very similar proofs based on same arguments can be probably shown in Bergman spaces of harmonic function in R^{n+1} and R^n.

Theorem 5.

Let α be large enough.Let $\alpha_k > -1$, $k = 1,...,m$. And let also $f_k \in h(U^n)$, $k = 1,...,m$, $m \in \mathbb{N}, 1 < p < \infty$. Let also $\alpha_j > \alpha_0(\alpha_1,...,\alpha_m,p,n)$ for some large enough α_0, $j = 1,...,m$. Let also

$$(f_1,...,f_m)_{h^p_{\vec{\alpha}}} = \int_{U^n} \prod_{k=1}^{m} |f_k(\vec{z})|^p \prod_{j=1}^{n} \left(1-|z_j|\right)^{2(m-1)+\sum_{k=1}^{m}\alpha_k} dm_{2n}(\vec{z}),$$

$$\vec{z}_j \in U^n, j = 1,\ldots,m.$$

If for all $z_j \in U^n$, $j = 1,...,m$.

$$\prod_{j=1}^{m} f_j(z_j) = c_\alpha \int_{U^n} \left(\prod_{j=1}^{m} P_{\frac{\alpha+1-m}{m}}\left(z_j,\vec{\zeta}\right)\right) \times$$

$$\times \left(\prod_{j=1}^{m} f_j\left(\vec{\zeta}\right)\right) \prod_{j=1}^{n} \left(1-|\zeta_j|\right)^{\alpha-1} dm_{2n}(\zeta);$$

$$z_j \in U^n, j = 1,...,m.$$

Then

$$(f_1,...,f_m)_{h^p_{\vec{\alpha}}} \asymp \prod_{k=1}^{m} \|f_j\|_{h^p_{\alpha_j}}.$$

So if each function is from Bergman class then the product of functions is from Bergman space, so the reverse is also true.

Remark 4. The same assertion is valid for multifunctional Bergman spaces of plurisubharmonic functions in $\mathbb{C}^n$ and multifunctional Bergman analytic function spaces in U^n.

2. Proofs of main results

We in this section prove our main results. Note again our proof uses only uniform estimate for A^p_α classes and the Forelly-Rudin estimate.

To prove our first theorem we will show that

$$\int_{B_n} \cdots \int_{B_n} |f_1(z_1)|^p \cdots |f_m(z_m)|^p \left(1-|z_1|^2\right)^{\widetilde{\alpha_1}} \cdots \left(1-|z_m|^2\right)^{\widetilde{\alpha_m}} dv(z_1)\cdots dv(z_m)$$

$$\leq C \int_{B_n} |f_1(z)|^p \cdots |f_m(z)|^p \left(1-|z|^2\right)^r dv(z), \tag{4}$$

for $p > 1$ or $p = 1$ and some $r, \widetilde{\alpha_j}, j = 1,...,m$, and then we will use the well-known one functional result.

Indeed We need to prove that for $p>1$ the following estimate is true.

$$\prod_{k=1}^{m}\int_B |f_k(z_k)|^p\left(1-|z_k|^2\right)^{\alpha_k}dv(z_k)\le C\int_B\prod_{k=1}^{m}|f_k(z)|^p\left(1-|z|^2\right)^{r_1}dv(z)<\infty,$$

where $r_1=(m-1)(n+1)+\sum_{k=1}^{m}\alpha_k>-1$.

Hence according to one functional result (see for example [14, Theorem 2.30]), for every f_k, $k=1,...,m$, there is a sequence

$$\left\{C_j^{(k)}\right\},k=1,...,m,j=1,...,\infty,$$

such that

$$f_k(z)=\sum_{j=1}^{\infty}C_j^{(k)}\frac{\left(1-|a_j|^2\right)^{\frac{(bp-n-1-\alpha_k)}{p}}}{|1-\langle z,a_j\rangle|^b},z\in B, \tag{6}$$

where $p\ge 1$, $\alpha>-1$, $b>\frac{n}{p}+\frac{\alpha_k+1}{p}$, $k=1,...,m$, for some fixed $(a_k)_{k=1}^{\infty}\subset B$ moreover

$$\sum_{j=1}^{\infty}\left|C_j^{(k)}\right|^p<\infty,k=1,...,m.$$

Let

$$\frac{1}{p}+\frac{1}{q}=1,r_1+r_2=\frac{(n+1+\alpha)}{m},r_1,r_2,$$

are positive real numbers, α is big enough. Then from (1) we get

$$\prod_{k=1}^{m}\int_B |f_k(z_k)|^p\left(1-|z_k|^2\right)^{\alpha_k}dv(z_k)\le$$

$$\le C\int_B\cdots\int_B I_p^p\left(1-|z_1|^2\right)^{\alpha_1}\cdots\left(1-|z_m|^2\right)^{\alpha_m}dv(z_1)\cdots dv(z_m)$$

where

$$I_p^p(z,\dots,z_m)=\left(\int_B\frac{|f_1(z)|\cdots|f_m(z)|\,dv_\alpha(z)}{\prod_{k=1}^{m}|1-\langle z,z_k\rangle|^{\frac{(n+1+\alpha)}{m}}}\right)^p.$$

Using Holder"s inequality we get

$$I_p^p\le\left(\int_B\frac{(|f_1(w)|\cdots|f_m(w)|)^p\left(1-|w|^2\right)^{\alpha}dv(w)}{\prod_{k=1}^{m}|1-\langle z_k,w\rangle|^{pr_1}}\right)\times$$

$$\times\left(\int_B\frac{\left(1-|w|^2\right)^{\alpha}dv(w)}{\prod_{k=1}^{m}|1-\langle z_k,w\rangle|^{qr_2}}\right)^{\frac{p}{q}}=L_1\times L_2. \tag{5}$$

Let us estimate L_2 separately now using once more Holder"s inequality for m functions and then the well-known Forelly-Rudin estimate in the unit ball (see ,for example ,[14]). We have

$$L_2 \leq \prod_{k=1}^{m} \left(\int_B \frac{\left(1-|w|^2\right)^{\alpha} dv(w)}{|1-\langle z_k, w\rangle|^{pr_2}} \right)^{\frac{p}{(mq)}} \leq C \prod_{k=1}^{m} \frac{1}{\left(1-|z_k|^2\right)^{r_2 p-(\alpha-n-1)p/(mq)}}$$

$$\leq C \prod_{k=1}^{m} \frac{1}{\left(1-|z_k|^2\right)^{p(r_2-(\alpha-n-1)/(mq))}},$$

$$r_2 > \frac{\alpha+n+1}{mq}, \alpha > -1.$$

After a suitable choice of r_1 and r_2, which will be justified later, by Fubini's theorem and by one more application of Forelly-Rudin estimate m times we have

$$\prod_{k=1}^{m} \int_B |f_k(z_k)|^p \left(1-|z_k|^2\right)^{\alpha_k} dv(z_k) \leq$$

$$\leq C \int_B |f_1(w)|^p \cdots |f_m(w)|^m \left(1-|w|^2\right)^{\alpha} dv(w) \times$$

$$\times \int_B \cdots \int_B \frac{\left(1-|z_1|^2\right)^{r+\alpha_1} \cdots \left(1-|z_m|^2\right)^{r+\alpha_m} dv(z_1) \cdots dv(z_m)}{|1-\langle z_1, w\rangle|^{pr_1} \cdots |1-\langle z_m, w\rangle|^{pr_1}} \leq$$

$$\leq C \int_B \prod_{k=1}^{m} |f_k|^p \left(1-|z|^2\right)^{r_1} dv(z) < \infty, \tag{7}$$

where

$$r_1 = (m-1)(n+1) + \sum_{k=1}^{m} \alpha_k, r = p\left(\frac{\alpha+n+1}{mq} - r_2\right)$$

and $\alpha > \left(n+1+\max_j a_j\right) m-(n+1)$.

If we choose r_1 and r_2 so that

$$0 < \frac{\alpha+n+1}{mq} < r_2 < \min\left\{ \frac{\min_j a_j + 1}{p} + \frac{\alpha+n+1}{mq}, \frac{\alpha+n+1}{m} \right\}$$

and

$$r_1 = \frac{\alpha+n+1}{m} - r_2,$$

then all requirement are satisfied.

Now we will show that the obtained results is sharp in the following sense.

First consider $f_1, \cdots, f_m$ with

$$\int_B \prod_{k=1}^{m} |f_k(z)|^p \left(1-|z|^2\right)^{r_1} dv(z) < \infty,$$

for some finite positive r_1, then from the arguments provided above we see directly that the representation (3) is true for each f_k,if the integral condition (1) holds. Now we show that the reverse is also true. Let us show that if we can represent each f_k function as a sum of functions then the last integral is finite ,so to be more precise (3) imply that

$$\int_B \prod_{k=1}^{m} |f_k(z)|^p \left(1-|z|^2\right)^{r_1} dv(z) < \infty$$

for all $1 \le p < \infty$. Indeed if (3) is valid then each f_k is from Bergman space according to classical onefunctional result we formulated in theorem And hence we only have to show the following inequality.

$$\int_B |f_1(z)|^p \cdots |f_m(z)|^p \left(1-|z|^2\right)^{(m-1)(n+1)+\sum_{k=1}^m \alpha_k} dv(z) \le C \prod_{k=1}^{m} \|f_k\|^p_{A^p_{\alpha_k}}.$$

is also valid.

We use to prove it now simple induction. When $m=1$, this is obvious. Now we assume that the case of $m-1$ is valid. From [14], if $f_k \in A^p_{\alpha_k}$, then we have that (a uniform estimate for Bergman spaces which is valid also in various types of domains in C^n)

$$|f_k(z)| \le \frac{C\|f_k\|_{A^p_{\alpha_k}}}{\left(1-|z|^2\right)^{\frac{\alpha_k+n+1}{p}}}, z \in B, 0<p<\infty, \alpha_k > -1, k=1,\cdots,m, \qquad (8)$$

Therefore we have

$$\int_B |f_1(z)|^p \cdots |f_m(z)|^p \left(1-|z|^2\right)^{(m-1)(n+1)+\sum_{k=1}^m \alpha_k} dv(z) \le$$

$$\le \sup_{z\in B} |f_m|^p \left(1-|z|^2\right)^{\alpha_m+n+1} \int_B |f_1|^p \cdots |f_{m-1}|^p \left(1-|z|^2\right)^{(m-2)(n+1)+\sum_{k=1}^m \alpha_k} dv(z)$$

$$\le C\|f_m\|^p_{A^p_{\alpha_m}} \prod_{k=1}^{m-1} \|f_k\|^p_{A^p_{\alpha_k}} \le C \prod_{k=1}^{m} \|f_k\|^p_{A^p_{\alpha_k}}.$$

Theorem 1 is proved.

Proof of Theorem 2

We easily note our last simple arguments based on induction can be extended easily to various types of domains and Bergman type spaces on them.The only tool we used during the proof is the uniform estimate for Bergman spaces which is well-known and is available in various domains.

Let us turn to situation with T_Λ tubular domains. We repeat arguments we provided in the unit ball. We wish to show first that the following estimate is true

$$\prod_{i=1}^{m} \int_{T_\Lambda} |f_k(z_k)|^p (\Delta(Imz_k))^{\alpha_k} dV(z_k) \le c \int_{T_\Lambda} \prod_{k=1}^{m} f_k(z_k)|^p \Delta(Imz_k))^{\tau_1} dV(z) \qquad (C)$$

where $\tau_1 = (m-1)(\frac{2n}{r}) + \sum\limits_{k=1}^{m} \alpha_k > -1$.

We discuss how this estimate and Forelly-Rudin estimate solves similarly the problem of atomic decomposition of multifunctional Bergman spaces in tubular domains.

The general problem of multifunctional Bergman spaces in the tubular domain is the following.

Let

$$\int\limits_{T_\Lambda} (|f_1|^{q_1})\dots(|f_m|^{q_m})\Delta^{\sum\limits_{k=1}^{m}(\alpha_k)}(Imz)dv(z) < \infty,$$

where $\sum\limits_{k=1}^{m} \alpha_k > -1, q_i \in (1,\infty), j = 1,\dots,m$.

Then can we say that there is a atomic decomposition for each $\{f_j\}, j = 1,\dots,m$? The answer is true when $m = 1$ (see theorem B). Our goal is to show that when $q_j = p, j = 1,\dots,m, p \in (0,\infty)$ the answer is also true that is each function $f_j, j = 1,\dots,m$ can be decomposed into atoms under the following simple integral condition. which vanishes for onefunctional case according to known result .
(additional integral condition)

$$\prod_{i=1}^{m} f_i(\omega_i) = c_\alpha \int\limits_{T_\Lambda} \frac{f_1(z)\dots f_m(z)dv_\alpha(z)}{\prod\limits_{j=1}^{m} \Delta(\frac{\omega_j - z}{i})^{\frac{\frac{2n}{r}+\alpha}{m}}} \qquad (D)$$

where α parameter is large enough. (we put all parameters in our integral condition equal to each other ,the proof of general case is very similar). To prove this we show that

$$\int\limits_{T_\Lambda}\cdots\int\limits_{T_\Lambda} \prod_{j=1}^{m} |f_j(z_j)|^p (\Delta(Imz_j)^{\widetilde{\alpha}_j} dv(z_j) \le c \int\limits_{T_\Lambda} \prod_{j=1}^{m} |f_j(z)|^p (\Delta^{\tau}(Imz))dv(z);$$

$$dV_\alpha(z) = (\Delta^{\alpha}(Imz))dv(z);$$

for $1 \le p < \infty$, and some $\tau, \widetilde{\alpha}_j, j = 1,\dots,m$, and then we will use the known one functional result (see [6], [7]).

We return now to estimate (C), and we will show that estimate using rather elementary calculations and arguments repeating arguments we provided in the unit ball.

This solves the mentioned problem as it is easy to see. Let further

$$\frac{1}{p} + \frac{1}{q} = 1, and \tau_1 + \tau_2 = \frac{\frac{2n}{r}+\alpha}{m}; \tau_1, \tau_2 > 0,$$

We also assume that α is big enough. Then using (D) we have the following chain of estimates (which similarly can be extended even to more general Siegel domains of second type)

$$\prod_{k=1}^{m} \int\limits_{T_\Lambda} |f_k(z_k)|^p \Delta^{\alpha_k}(Imz_k)dV(z_k) \le \widetilde{c} \int\limits_{T_\Lambda}\cdots\int\limits_{T_\Lambda} (I_p^p) \prod_{k=1}^{m} (\Delta(Imz_k))^{\alpha_k} dV(z_k);$$

where

$$(I_p^p)=\Big(\int\limits_{T_\Lambda}\frac{\prod\limits_{j=1}^{m}|f_j(z)|dV_\alpha(z)}{\prod\limits_{k=1}^{m}|\Delta(\frac{\vec{z}-z_k}{i})^{\frac{(\frac{2n}{r}+\alpha)}{m}}|}\Big)^p$$

Using Holder's inequality we get

$$I_p^p\le\Big(\int\limits_{T_\Lambda}\frac{\big(\prod\limits_{i=1}^{m}|f_i(\omega)|\big)^p\Delta^\alpha(Im\omega)dV(\omega)}{\prod\limits_{k=1}^{m}\big|\big(\Delta(\frac{z_k-\overline{\omega}}{i})\big)^{p\tau_1}\big|}\Big)$$

$$\int\limits_{T_\Lambda}\frac{\Delta^\alpha(Im\omega)dV(\omega)}{\prod\limits_{k=1}^{m}\big|\big(\Delta(\frac{z_k-\omega}{i})\big)^{q\tau_2}\big|}\Big)^{\frac{p}{q}}=L_1L_2;$$

Using again Holder"s inequality for m functions we have

$$L_2\le\prod_{k=1}^{m}\Big(\int\limits_{T_\Lambda}\frac{(\Delta(Im\omega))^\alpha}{\big|\Delta(\frac{z_k-\overline{\omega}}{i})\big|^{mq\tau_2}}\Big)^{\frac{p}{mq}}\le$$

$$\le c\prod_{k=1}^{m}\frac{1}{\big|(\Delta(Imz_k))^{p(\tau_2-\frac{(\alpha+\frac{2n}{r})}{mq})}\big|};$$

$$\tau_2>\frac{\alpha+\frac{2n}{r}}{mq};\alpha>-1;$$

We have using appropriate choices of τ_1 and τ_2 by Fubini's theorem

$$\prod_{k=1}^{m}\int\limits_{T_\Lambda}|f_k(z_k)|^p\Delta(Imz_k)^{\alpha_k}dV(z_k)\le$$

$$\le c\int\limits_{T_\Lambda}\prod_{k=1}^{m}\Delta^\alpha(Im\omega)V(\omega)\int\limits_{T_\Lambda}\cdots\int\limits_{T_\Lambda}\frac{\prod\limits_{j=1}^{m}\Delta(Imz_j)^{\tau+\alpha_j}dV(z_j)}{\prod\limits_{j=1}^{m}\big|\big(\Delta(\frac{z_j-\omega}{i})\big)^{p\tau_1}\big|}\le c\int\limits_{T_\Lambda}\prod_{k=1}^{m}|f_k|^p(\Delta^{\tau_1}(Imz)dV(z),$$

where

$$\tau_1=(m-1)(\frac{2n}{r})+\Big(\sum_{j=1}^{m}\alpha_k\Big);\tau=p\Big(\frac{\alpha+\frac{2n}{r}}{mq}-\tau_2\Big);r_3>\Big(\frac{\alpha+\frac{2n}{r}}{mq}\Big)$$

$$\alpha>(\frac{2n}{r}+\max\alpha_j)m-(\frac{2n}{r});\tau_1+\tau_2=\frac{(\alpha+\frac{2n}{r})}{m};\tau_2\in(r_3;r_4),$$

for some positive parameters r_3,r_4. This estimate is sharp in the following sense. Note first if for each f_k the atomic decomposition is valid then each f_k is from ordinary onefunctional Bergman space according to theorem B. And for

$$\tau_1=(m-1)(\frac{2n}{r})+\sum_{k=1}^{m}\alpha_k;$$

we have

$$\int\limits_{T_\Lambda} \prod_{k=1}^{m} |f_k(z)|^p \Delta^{\tau_1}(Imz) dV(z) < \infty$$

for $p < \infty$. And we have to prove the following inequality

$$\int\limits_{T_\Lambda} \prod_{j=1}^{m} |f_j(w)|^p (\Delta^y(Im(w))dv(w) \le c \prod_{k=1}^{m} ||f_k||^p_{A^p_{\alpha_k}};$$

for some y positive parameter which was provided above.

This follows as in the unit ball case directly from ordinary induction and the following known uniform estimate. (see for example[6,7])

$$|f_k(z)| \le \frac{c||f_k||_{A^p_{\alpha_k}}}{\Delta^v(Imz)};$$

where $v = \frac{\alpha_k + \frac{n}{r}}{p} - \frac{2n}{r};$, $\alpha_k > \frac{n}{r} - 1$, $z \in T_\Lambda; 1 < p < \infty, k = 1, \dots, m$.

So we have proved similar to the unit ball atomic decomposition theorem for multifunctional Bergman spaces in tubular domains over symmetric cones.Theorem is proved.

Similarly this theorem can be shown for bounded strongly pseudoconvex domains with smooth boundary and in Siegel domains of second type by repetition of arguments and by simple substitution of uniform estimates and Forelly-Rudin estimates for these domains.

For pseudoconvex domains we refer to [5].The case of analytic Bergman spaces in the unit polydisk can be covered easily using same approaches. We refer to [18],[17] for all mentioned tools in polydisk which are needed for such proofs.

Since these tools and proofs in various domains are very similar we leave some of them to interested readers.

Similar results are valid for Bergman spaces in the minimal ball, where all mentioned tools used in our proofs are also available (see for example [19] and various references there.)

Список литературы/References

[1] Coifman R., Rochderg R., "Representation theorems for holomorphic and harmonic functions L^p", *Asterisque*, 1980, №77, 11-66.

[2] Rochderg R., Semmes S., "A decomposition theorem for BMO and applications", *Jour. of Func. Analysis.*, 1986 67, 228-263.

[3] Luecking D., "Representations and duality in weighted spaces of analytic functions", *Indiana Univ. Math. Journal*, **34**:2 (1985), 319-336.

[4] Li S., Shamoyan R., "O nekotorykh rasshireniyakh teorema ob atomnykh razlozheniyakh prostranstv Bergmana i Blokha v yedinichnom share i svyazannykh s nimi zadachakh [On some extensions of theorems on atomic decompositions of Bergman and Bloch spaces in the unit ball and related problems]", *Zhurnal ellipticheskikh uravneniy i kompleksnykh peremennykh [Journal of Elliptic equations and Complex Variables]*, 2010 (In Russ.).

[5] Shamoyan R., Arsenovic M., "On distance estimates and atomic decompositions in spaces of analytic functions on stricty pseudoconvex domains", *Bulletin Korean Math. Society*, 2015.

[6] Bekolle D., Kagou A. T., “Reproducing properties and L^p estimates for Bergman projections in Siegel domains of second type”, *Studia Math.*, **115**:3 (1995).

[7] Bekolle D., Bonami A., Garrigos G., Ricci F., Sehba B. Analytic Besov spaces and symmetric cones, *Jour. Fur seine and ang.*, 2010, №647, 25-56.

[8] Yamaji S., *Some properties of Bergman kernel in minimal bounded homogeneous domain*, Arxiv, 2013.

[9] Yamaji S., *Essential norm estimates for positive Toeplitz operators on the weighted Bergman space*, Arxiv, 2013.

[10] Bekolle D., Kagou A., “Molecular decomposition and interpolation”, *Int. Equat. Oper. Theory*, **31**:2 (1998), 150-177.

[11] Bonami A., Bekolle D., Garrigos G., “Lecture notes on Bergman projections in tube domains over symmetric cones”, *Yaonde Proc. Int. Workshop*, 2001, 75 pp.

[12] Kagou A., *Temgoua Domaines de Siegel de type II noyau de Bergman*, These de 3 cycle, Yaounde, 1995.

[13] Gheorghi L. G., “Interpolation of Besov spaces and applications”, *Le Mathematiche*, **LV**:1 (2000), 29-42.

[14] Zhu K., *Spaces of holomorphic functions in the ball*, N-Y, Springer, 2005.

[15] Rochberg R., “Decomposition theorems for Bergman spaces and applications”, *Operator theory and function theory*, 1985, 225-277.

[16] Krantz S., Li S.-Y., “On decomposition theorems for Hardy spaces in domains in $\mathbb{C}^n$ and applications”, *Journal of Fourier analysis and applications*, 1995.

[17] Shamoyan F., “O teoremakh vlozheniya i sledakh H^p prostranstv Khardi na diagonali [On embedding theorems and traces of H^p Hardy spaces on diagonal]”, *Matematika Sbornik [Math. Sbornik]*, 1978 (In Russ.).

[18] Shamoyan F., Djrbashian A., “Temy teorii prostranstv A^p_α [Topics in the theory of A^p_α spaces]”, *Teubner Texte zur Math.*, 1988..

[19] Mengotti G., “The Bloch space for minimall ball”, *Studia Math.*, **148**:2 (2001), 131-142.

Список литературы (ГОСТ)

[1] Coifman R., Rochderg R. Representation theorems for holomorphic and harmonic functions L^p // Asterisque. 1980. no. 77. pp. 11-66.

[2] Rochderg R., Semmes A decomposition theorem for BMO and applications // Jour. of Func. Analysis. 1986. no. 67. pp. 228-263.

[3] Luecking D. Representations and duality in weighted spaces of analytic functions // Indiana Univ. Math. Journal. 1985. vol. 34. no. 2. pp. 319-336.

[4] Li S., Shamoyan R. O nekotorykh rasshireniyakh teorema ob atomnykh razlozheniyakh prostranstv Bergmana i Blokha v yedinichnom share i svyazannykh s nimi zadachakh [On some extensions of theorems on atomic decompositions of Bergman and Bloch spaces in the unit ball and related problems] // Zhurnal ellipticheskikh uravneniy i kompleksnykh peremennykh [Journal of Elliptic equations and Complex Variables], 2010. (In Russ.)

[5] Shamoyan R., Arsenovic M. On distance estimates and atomic decompositions in spaces of analytic functions on stricty pseudoconvex domains // Bulletin Korean Math. Society, 2015.

[6] Bekolle D., Kagou A. T. Reproducing properties and L^p estimates for Bergman projections in Siegel domains of second type // Studia Math. 1995. vol. 115. no 3.

[7] Bekolle D., Bonami A., Garrigos G., Ricci F., Sehba B. Analytic Besov spaces and symmetric cones // Jour. Fur seine and ang. 2010. no. 647. pp. 25-56.

[8] Yamaji S. Some properties of Bergman kernel in minimal bounded homogeneous domain // Arxiv, 2013.

[9] Yamaji S. Essential norm estimates for positive Toeplitz operators on the weighted Bergman space // Arxiv, 2013.
[10] Bekolle D., Kagou A. Molecular decomposition and interpolation // Int. Equat. Oper. Theory. 1998. vol. 31. no. 2. pp. 150-177.
[11] Bonami A., Bekolle D., Garrigos G. Lecture notes on Bergman projections in tube domains over symmetric cones // Yaonde Proc. Int. Workshop. 2001. 75 p.
[12] Kagou A. Temgoua Domaines de Siegel de type II noyau de Bergman // These de 3 cycle, Yaonde, 1995.
[13] Gheorghi L. G. Interpolation of Besov spaces and applications // Le Mathematiche. 2000. vol. LV. no. 1. pp. 29-42.
[14] Zhu K. Spaces of holomorphic functions in the ball. N-Y: Springer, 2005.
[15] Rochberg R. Decomposition theorems for Bergman spaces and applications // Operator theory and function theory. 1985. pp. 225-277.
[16] Krantz S., Li S.-Y. On decomposition theorems for Hardy spaces in domains in $\mathbb{C}^n$ and applications // Journal of Fourier analysis and applications, 1995.
[17] Shamoyan F. O teoremakh vlozheniya i sledakh H^p prostranstv Khardi na diagonali [On embedding theorems and traces of H^p Hardy spaces on diagonal] // Matematika Sbornik [Math. Sbornik]. 1978. (In Russ.)
[18] Shamoyan F., Djrbashian A. Temy teorii prostranstv A^p_α [Topics in the theory of A^p_α spaces] // Teubner Texte zur Math., Leipzig, 1988.
[19] Mengotti G. The Bloch space for minimall ball // Studia Math. 2001. vol. 148. no. 2. pp. 131-142.

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