RESEARCHES ON SYSTEMIC STRUCTURING AND STATISTIC MODELLING OF BIOMASS PELLETING PROCESSES
/
CERCETĂRI PRIVIND STRUCTURAREA SISTEMICA SI MODELAREA STATISTICA A PROCESELOR DE PELETIZARE A BIOMASEI

Math. Cârdei P.[1)], Ph.D. Stud. Eng. Găgeanu I.[1)]*, Prof. Ph.D. Eng. Voicu Gh.[2)], Ph.D. Eng. Matache M.[1)1]

[1)]National Institute of Research - Development for Machines and Installations Designed to Agriculture and Food Industry - INMA Bucharest / Romania; [2)] University Politehnica of Bucharest / Romania

E-mail: iulia.gageanu@gmail.com

DOI: 10.35633/INMATEH-58-27

***Keywords:** biomass sawdust, pelleting, statistical modelling, polynomial regression*

ABSTRACT

The paper presents the results on the systemic and statistic modelling of the process of pelleting fir tree sawdust powder. A systemic model is presented, specifying the inputs, commands and outputs. Based on experimental result, it is then attempted to perform a statistical modelling only using the technique of linear and nonlinear regressions. Results for the performances of qualitative modelling and estimations of system (pelleting process) behaviour are displayed, as well as some limitations on using statistic modelling based on regression technique. Although some results can appear as interesting, it is shown that any interesting point in the space of input and command parameters (point that is convenient or even optimal in exploitation) should be experimentally checked in the sense of verifying the properties highlighted by the functions found. This means that any theoretically convenient parametric combination should be experimentally validated through high resolution experiments.

REZUMAT

Articolul prezintă rezultate privind modelarea sistemică şi statistică a procesului de peletizare a pulberii de rumeguş de brad. Se prezintă un model sistemic precizându-se intrările, comenzile şi ieşirile. Pe baza rezultatelor experimentale se încearcă apoi modelarea statistică folosind însă numai tehnică regresiilor liniare şi neliniare. Se expun rezultate privind performanţele de modelare şi estimare calitativă a comportamentului sistemului (procesului de peletizare), precum şi unele limite ale utilizării modelării statistice bazată pe tehnica regresiilor. Deşi unele rezultate pot să apară ca fiind interesante, se arată ca orice punct interesant din spaţiul parametrilor de intrare şi comandă (punct convenabil în exploatare sau chiar optimal) trebuie verificat experimental în sensul ca verifica proprietăţile pe care funcţiile găsite le pun în evidenţă. Aceasta înseamnă ca orice combinaţie parametrică teoretic convenabilă trebuie validate experimental prin experienţe de înaltă rezoluţie.

INTRODUCTION

Biomass densification or compression thus obtaining pellets represents an essential process for producing biofuels. Grinded biomass particles behave differently under the action of different forces applied (*Adapa et al., 2009*). Therefore, it is important to investigate the changes in density and volume of the compressed material when applying pressures. One of the main reasons of introducing the experimental data in an equation is the development of diagrams in order to make comparisons more easily between different sets of data (*Comoglu, 2007*).

The development of a model is a systemic analysis through which a conceptual (abstract) representation of a system is developed or an expected representation of the system is described. In another formulation, building the model represents the process (usually iterative) through which researchers attempt to develop an appropriate model of a physical or biological system. An essential part of this process is to verify a candidate model in comparison to experimental in order to determine if there is a serious inadequacy.

According to *Voicu M. (2008*), a **system** is defined as a *complex of interacting elements*. The system is differentiated from the environment through its structure and internal structure. The differentiation is vaguely enough, but it still allows fulfilling some addressing conditions in the narrow limits of science. Also in *Voicu M., (2008*), is stated that the behaviour of a system depends not only on the properties of its components but especially on the *interactions between them*. *Real systems* process *substance, energy and information*.

[1] *Cârdei P., Math.; Găgeanu I., Ph.D. Stud. Eng.; Voicu Gh., Prof. Ph.D. Eng., Matache M., Ph.D. Eng.*

They are connected to the environment through *cause quantities* or *inputs* and *effect quantities* or *outputs*, according to *Voicu M., (2008*) also states that between these quantities, there is a causal relation, representable in an abstract form through a *transfer operator*. The experimental study of real systems implies the interaction with the analysed object and has, in some situations, limited applicability. Usually, *modelling methods* are used together with experimental procedures, allowing to develop the transfer operator through a *mathematical model*. The pelleting process is regarded as a system in all modern papers, such as (*Shaw M., 2008*).

Compression models help to reveal the biomass particle behaviour during the compaction process and can help optimize the values of parameters needed to obtain good quality pellets. Pellets are formed through the use of pressure which forces the biomass particles to join and stick together.

MATERIALS AND METHODS

In order to formulate the problematic of biomass pelleting in a strict frame of system theory, it needs to be reminded that the mathematical modelling of such processes is already a several decades-old activity. In *(Cardei and Gageanu, 2017)* we noted and modified such models.

The formulation of the problematic in the terms of system theory, imposes a sorting of parameters that mathematically describe the system, of the same type as those presented in Table 1.

Table 1

Parameters of the pelleting process and of the pellets obtained

Parameter type	Parameter name	Notation	Unit	Physical dimension
Input	Raw material granulation	g	m	L
	Raw material density	ρ_0	kg/m^3	ML^{-3}
	Sawdust moisture	U_i	%	-
	Raw material initial volume	V_0	m^3	L^3
Command	Die diameter	$\O_m$	m	L
	Maximum applied force	F_{max}	kN	MLT^{-2}
	Piston movement speed	v	m/s	LT^{-1}
	Die temperature	θ	^{o}C	
Output	Consumed energy	E_c	Wh	ML^{2}T^{-2}
	Pellet length	L	M	L
	Pellet density	ρ_p	kg/m^3	ML^{-3}
	Pellet moisture	U_p	%	-
	Pellet volume	V_p	m^3	L^3

Table 1 lists all the parameters that describe, in the view of the authors, in a first approximation, a system for the pelletisation of biomass in powder state, by compressing it in a cylindrical die, through the means of a piston. The experiments and main results are synthesised in *Gageanu et al. (2019*). These parameters were also highlighted in *Cardei and Gageanu (2017*), but in Table 1 of this paper, they are structured on categories specific to system theory: *input, command and output parameters*. These are the three groups of parameters highlighted in table 1. Not all input or command parameters were varied within the experimental procedures.

Therefore, **powder** granulation was constant and **die diameter** only had two values, insufficient to make a satisfactory data interpolation in relation to these parameters. The parameters that were varied are described in the shaded categories in Table 1. Also, the influence of **binders** and **die walls shape** were not considered in the experimental plan. In the field of biomass compression, there are resources of binders, even from the structure of the raw material, binders that, in certain conditions, can contribute to the durability, cohesiveness and lack of cracks in the final product. (*Mani et al. (2003*) highlighted a series of papers that specially tackled these aspects. Therefore, Table 1 containing the parameters of the system for processing biomass through compression can be completed with **binder concentration and type** and **die shape parameters**, if needed.

All output parameters are parameters characterizing the quality of the pelleting process. Still, there are a series of other quality parameters that characterize the quality of the process that cannot be taken into account within the experimental frame developed in this study, such as working capacity or wears and frictions.

System structure also imposes the general form of output functions having as arguments the input and command parameters. The general form of an output parameter function is, in the case of this study, a certain function with maximum six arguments (input and command parameters). A selection of input and command parameters depending on their influence on output parameters can reduce the number of arguments of each output parameter function. The measure of influence can be taken, as first approximation as the value of correlation between the numerical series given by them during the experiments. Estimations of correlations between output parameters and input and command parameters considered in this study are given in Table 2.

Table 2

Input/output correlation matrix

	ρ_0	U_i	V_0	F_{max}	v	θ
E_c	-0.236000	-0.242000	0.232000	0.644000	-0.112000	-0.027000
L	0.763000	0.78000	-0.753000	-0.305000	0.134000	0.023000
ρ_p	-0.779655	-0.796201	0.770537	0.33332	-0.111758	-0.007409
U_p	0.937586	0.948841	-0.93105	-0.098442	0.085247	-0.08906
V_p	0.768073	0.785466	-0.75853	-0.301463	0.134581	0.027897

Normally, is accepted that the absolute value 1 for correlation represents a perfect linear relation, an absolute value of the correlation higher than 0.7 but lower than 1 signifies a strong dependency between the two parameters or variables, an absolute value situated between 0.5 and 0.7 translates in a moderate dependency between the variables, an absolute correlation value between 0.3 and 0.5 signifies a low correlation, and absolute values of the correlation situated between 0 and 0.3 are interpreted as nonsignificant dependencies between the two variables (*https://www.dummies.com/education/math/statistics/how-to-interpret-a-correlation-coefficient-r/*).

Taking into account the values of correlations in Table 2 and the observation according to (*https://www.dummies.com/education/math/statistics/how-to-interpret-a-correlation-coefficient-r/*), it is reasonable to consider the following dependencies for the output parameters:

$$E_c = E_c(F_{max}) \quad (1)$$

$$L = L(U_i, \rho_0, V_0, F_{max}) \quad (2)$$

$$\rho_p = \rho_p(U_i, \rho_0, V_0, F_{max}) \quad (3)$$

$$U_p = U_p(U_i, \rho_0, V_0) \quad (4)$$

$$V_p = V_p(U_i, \rho_0, V_0, F_{max}) \quad (5)$$

Both Table 2 and the general form predicted for the output functions describing the quality parameters of the experimentally studied pelleting process, show that a major influence on the quality parameters of the process is that of input parameters describing the state of the material (biomass powder) introduced in the process. Among the command parameters, only the maximum compression force has a low influence, the other two parameters having, according to the correlation selection criteria, a negligible influence on the qualitative parameters. These observations refer to the 6-dimension interval of input and command parameters considered in the experiments in (*Gageanu et al., 2019*) and, also, to the set of parameters describing the process. If the 6-dimension interval of the input and command parameters is extended, or if the set of considered parameters increases, some of the conclusions can be updated but, in general, they cannot suffer major changes.

As a confirmation, a multitude of empiric nature compression laws, reviewed in *Cardei and Gageanu, (2017*), refer only to the output parameter called pellet density and only consider a dependent in the form:

$$\rho_p = \rho_p(\rho_0, P_{max}) \quad (6)$$

or relative to pellet volume:

$$V_p = V_p(V_0, P_{max}) \tag{7}$$

where the relation between force and pressure is relatively simple:

$$P = \frac{4F_{max}}{\pi \emptyset^2} \tag{8}$$

The experimental results in (*Gageanu et al., 2019*) highlight the influence of biomass moisture (that was not taken into account for metallic, ceramic and pharmaceutical powders). The influence of pellet volumes, densities and lengths are connected by mass conservation laws:

$$M_{0i} = \rho_{0i} \cdot V_{0i} = \rho_{pi} \cdot V_{pi} = M_{pi} \tag{9}$$

where by M_{0i} and M_{pi} we expressed the masses of raw material introduced in the process and the mass of pellets, for each experiment using the index i.

RESULTS

Applying the method described in the previous chapter has allowed explaining the output parameters of the pelleting process as functions of input and command parameters. In order to study the consequences of choosing the number of arguments of the output parameters functions (quality parameters of the system) we studied both the version with a reduced number of arguments (allowed in the virtue of ranking the correlation between outputs and inputs and commands), as well as the version with a complete number of arguments.

R1) *Polynomial regression for pellet density*

Considering the structure of the pellet density function in the form (3) and using the features of Mathcad Program (*https://www.ptc.com/en/products/mathcad*), the expression of linear regression is obtained:

$$\rho_p(U_i, \rho_0, V_0, P) = -18784.909561 - 357.765614U_i + 171.589847\rho_0 - 9.504088V_0 + 0.000000341P \tag{10}$$

By estimating the intensity of the members of linear sum (10), it is found that the most important member is the one containing raw material density, contributing to the increase of pellet density, the second in intensity being the one containing moisture, but in a negative sense, namely in the sense of decreasing pellet density. Formula (10) is remarkable as precision compared to the classical formulas of the powder compaction, the calculated errors (RMS of the difference between the theoretical and empirical values, and de maximum difference of the same difference) having lower values. Also, to the support of prediction achieved with formula (10) pleads the determination coefficient $r^2 = 0.862$. Linear regression is calculated in relation to all six input and command variables varied within the experimentation program. The following expression is obtained:

$$\rho_p(U_i, \rho_0, V_0, P, v, \theta) = -18737.057502 - 357.765614U_i + 171.589847\rho_0 - 9.504088V_0 + 0.000000341P - 19414.676017v - 0.096605\theta \tag{11}$$

The global error of regression (11) decreases to the value $\varepsilon_g = 0.00242$, and the maximum error increases to the value $\varepsilon_{max} = 0.29087924$, in relation to the regression on the reduced variable system (10). The determination coefficient is better for regression (11) than the one for regression (10), $\varepsilon_{max} = 0.29087924$.

Also, for the density of pellets exiting the compression process, it can be attempted to calculate the second-degree polynomial for the version with reduced variable set. Its expression is:

$$\rho_p(U_i, \rho_0, V_0, P) = -17391.251071 + 245.906496U_i + 210.16553\rho_0 - 3.876632V_0 + 0.000018P - 1.667714U_i\rho_0 + 0.041295U_iV_0 + 0.0000002025078U_iP + 0.01008\rho_0V_0 - 0.0000001335\rho_0P + 0.000000068V_0P - 3.573361U_i^2 - 0.550131\rho_0^2 - 0.000107V_0^2 \tag{12}$$

The quadrate member of pressure is negative, but multiplied with a negligible factor, 10^{-15}. Square regression (12) achieves the following performances: $\varepsilon_g = 0.002244, \varepsilon_{max} = 0.26145721, R^2 = 0.892$. Evidently, the performances achieved by regression (12) are the best for the function giving pellet density at the exit of the compression process. Extending the regression from the second degree to the entire experimental domain of the process parameters, namely to all the six variables, therefore considering a second degree regression polynomial in all input and output process parameters varied, increases again with the precision: $\varepsilon_g = 0.001703, \varepsilon_{max} = 0.26425, R^2 = 0.938$. The second-degree regression polynomial with six variables has 28 members.

R2) *Polynomial regression for the consumed energy*

Statistical models similar to those for pellet density are made for the other output (quality) parameters of the process of pelleting fine biomass powders.

For the consumed energy, the reduced first-degree polynomial regression has a single argument, according to the correlation selection (table 2), being given by formula:

$$E_c(P) = 2.65 + 0.00000001251P \quad (13)$$

with connection between the maximum force and the maximum pressure given by (8). The precision of this formula is given by values $\varepsilon_g = 0.017, \varepsilon_{max} = 0.6439, R^2 = 0.414$. The formula for the second-degree regression polynomial is basically also of first-degree, because the coefficient of the second-degree member for the compression pression is below the negligible limit of 10^{-15}.

$$E_c(P) = 8.37037037 - 0.000000041403088P \quad (14)$$

The performances of this formula are measured by values: $\varepsilon_g = 0.014, \varepsilon_{max} = 0.6854725, R^2 = 0.575$. Considering the linear regression dependent of all six arguments varied experimentally does not improve precision: $\varepsilon_g = 0.016, \varepsilon_{max} = 0.6101372, R^2 = 0.503$. Better performances are obtained when considering the second-degree polynomial regression, for the entire set of variables: $\varepsilon_g = 0.011, \varepsilon_{max} = 0.56400485, R^2 = 0.742$.

R3) *Polynomial regression for pellet length*

One of the quality parameters for the products of the pelleting process is represented by pellet length. Results in table 2 show that pellet length, when exiting the process, depends directly on the moisture and density of the raw material and inversely on its volume and on the maximum pressing force.

Linear regression for the parameter representing pellet length when exiting the compression process, for the reduced set of variables, is given by expression:

$$L(U_i, \rho_0, V_0, P) = -3.664 - 0.000002978U_i + 0.013\rho_0 + 10226.285V_0 - 0.00000000001P \quad (15)$$

with the performances estimated by values: $\varepsilon_g = 0.003028, \varepsilon_{max} = 0.30457, R^2 = 0.831$.

The first order linear regression for pellet length when exiting the compression process, for the entire set of variables considered, is given by equation:

$$L(U_i, \rho_0, V_0, P, v, \theta) = 0.726 + 0.0134U_i - 0.006042\rho_0 + 0.000334V_0 - 0.00000000001P + 0.778v + 0.000010007\theta \quad (16)$$

The performances of this model are described by the following characteristics: $\varepsilon_g = 0.00283, \varepsilon_{max} = 0.28575897, R^2 = 0.852$.

Second degree linear regression for the reduced set of variables is characterized by the following values: $\varepsilon_g = 0.00283, \varepsilon_{max} = 0.28575897, R^2 = 0.852$. For the second-degree linear regression for the complete set of variables, the following precision estimations are obtained: $\varepsilon_g = 0.002851, \varepsilon_{max} = 0.32654358, R^2 = 0.85$.

R4) *Polynomial regression for pellet moisture*

According to the results in Table 2, pellet moisture when existing the compression process, depends essentially on three input variables: raw material moisture, raw material density and volume.

Linear regression for pellet moisture has the expression:

$$U_p(U_i, \rho_0, V_0) = 231.414 + 4.416U_i - 1.961\rho_0 + 1.11V_0 \quad (17)$$

and precision evaluation: $\varepsilon_g = 0.002549, \varepsilon_{max} = 0.16885892, R^2 = 0.95$. The precision of quadratic regression for pellet moisture, in the version of the reduced set of parameters, has a low precision evaluation and cannot be taken into consideration.

Linear regression on the complete set of variables has the form:

$$U_p(U_i, \rho_0, V_0, P, v, \theta) = 232.954 + 4.416U_i - 1.962\rho_0 + 0.111V_0 - 0.0000000017P + 259.31v - 0.02\theta \quad (18)$$

Precision evaluation for formula (18) is: $\varepsilon_g = 0.001804, \varepsilon_{max} = 0.12492327, R^2 = 0.975$.

Second degree quadratic regression is characterised by an even better precision: $\varepsilon_g = 0.001588, \varepsilon_{max} = 0.11079441, R^2 = 0.981$.

R5) *Polynomial regression for pellet volume*

According to Table 2, pellet volume depends directly on the moisture of the raw material introduced in the process, on its density and inversely on raw material volume, as well as on the maximum force applied in the process. Linear regression of pellet volume at the exit of the compression process, for the reduced set of variables, has the form:

$$V(U_i, \rho_0, V_0, P) = -0.0002986 - 0.00000000023U_i + 0.000001\rho_0 + 8.333V_0 \quad (19)$$

Precision performances of formula (19) are given by: $\varepsilon_g = 0.002957, \varepsilon_{max} = 0.3042955, R^2 = 0.838$. For the second-degree regression, having the reduced set of variable (that has 15 members), the precision evaluation is given by the following values: $\varepsilon_g = 0.00277, \varepsilon_{max} = 0.28586421, R^2 = 0.857$.

First-degree linear regression with the complete set of variables, for pellet volume at process exit has the expression:

$$V(U_i, \rho_0, V_0, P, v, \theta) = 0.000059 + 0.000001U_i - 0.0000004917\rho_0 + 0.00000002719V_0 + 0.00006352v + 0.000000000988\theta \quad (20)$$

Performances of regression (20) are given by: $\varepsilon_g = 0.002774, \varepsilon_{max} = 0.32612782, R^2 = 0.857$. Second degree linear regression with the complete set of variables has better performances: $\varepsilon_g = 0.001957, \varepsilon_{max} = 0.28689592, R^2 = 0.929$.

Table 3 concentrates the precision characteristics for the interpolation formulas investigated, for each output parameter as function of input and command parameters.

Table 3

Precision estimators for the dependency laws of output (qualitative), input and command parameters

Parameter	Regression degree	Version (formula)	ε_g	ε_{max}	R^2
ρ_p	1	Reduced (10)	0.002538	0.27598956	0.862
		Complete (11)	0.002420	0.29087924	0.874
	2	Reduced (12)	0.002244	0.26145721	0.892
		Complete	0.001703	0.26425000	0.938
E_c	1	Reduced (13)	0.017000	0.64390000	0.831
		Complete	0.016000	0.61013720	0.503
	2	Reduced (14)	0.014000	0.68547250	0.575
		Complete	0.011000	0.56400485	0.742
L	1	Reduced (15)	0.003028	0.30457000	0.831
		Complete (16)	0.002830	0.28575897	0.852
	2	Reduced	0.002830	0.28575897	0.852
		Complete	0.002851	0.32654358	0.850

Table 3
(continuation)

U_p	1	Reduced (17)	0.002549	0.16885892	0.950
		Complete (18)	0.001804	0.12492327	0.975
	2	Reduced	-	-	-
		Complete	0.001588	0.11079441	0.981
V	1	Reduced (19)	0.002957	0.3042955	0.838
		Complete (20)	0.002774	0.32612782	0.857
	2	Reduced	0.002770	0.28586421	0.857
		Complete	0.001957	0.28689592	0.929

Graphical representations of relations characterizing the pelleting process

The results presented in this subchapter are obtained through a few of the many possible graphical representations for the relations characterizing the studied pelleting process. Graphical representations allow to highlight some aspects of the physical process and, simultaneously, to choose the functions with which to go further in any improvement or optimization studies.

Figure 1 shows the graphical representation of pellet density dependency on raw material moisture, in the version of linear and quadratic regression, with the reduced and the complete set of arguments. In can be noticed that all curves show a monotonous decrease in pellet density when increasing raw material density. Linear regression is not an adequate calculation representation for further researches because towards the ends of the moisture variation interval, the values of pellet densities are far from reality. Therefore, for researching the pelleting phenomenon, one of the second-degree regressions will be taken, fitting well within the limits of the experimental values. The fact that quadratic regressions highlight a maximum point in the moisture variation interval considered experimentally, is a fact that should be verified experimentally by refining the experimentation network around the maximum point.

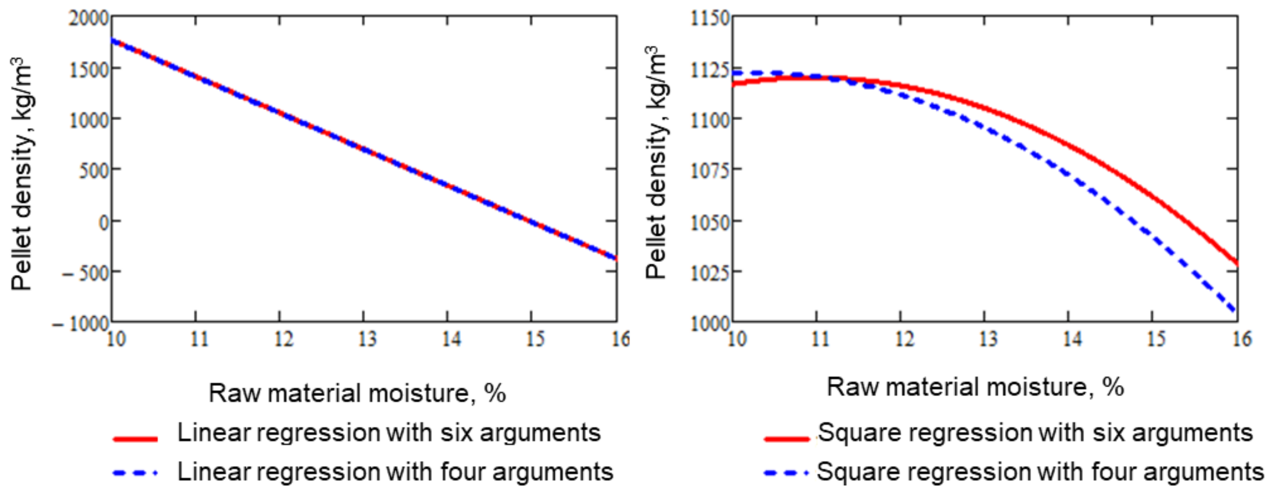

Fig. 1 – The dependency of pellet density on raw material moisture, in various regression versions, for constant values: $\rho_0 = 140\frac{kg}{mc}, V_0 = 0.00001837\ mc, P = 300MPa, v = 0.002\frac{m}{s}, \theta = 80\ ^oC$.

Figure 2 represents graphically the dependency of pellet density on raw material density, in the version of linear and quadratic regression, with the reduced and the complete set of arguments. In this case, it is interesting that the linear regression indicates an increase tendency in pellet density, while the quadratic regression gives a reverse result.

In order to choose, we have two arguments: linear regression is inadequate because it moves further away from the real end values in a non-permitted manner, it also receives negative values and they do not have a physical meaning, and a third reason is that the quadratic regression with the complete set of arguments has a better precision.

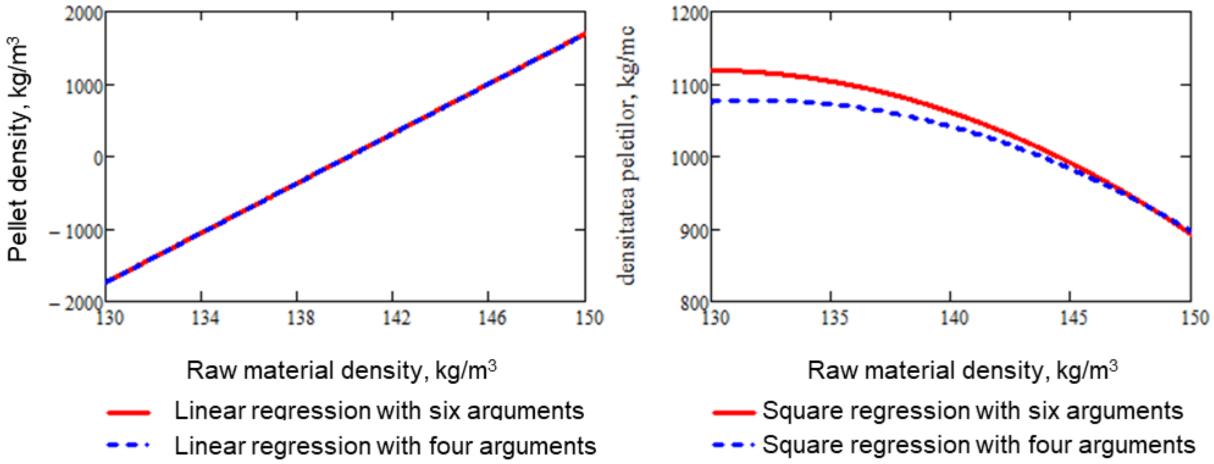

Fig. 2 – The dependency of pellet density on raw material density, in various regression versions, for constant values: $h_0 = 15\%, V_0 = 0.00001837\ mc, P = 300MPa, v = 0.002\frac{m}{s}, \theta = 80\ ^oC$.

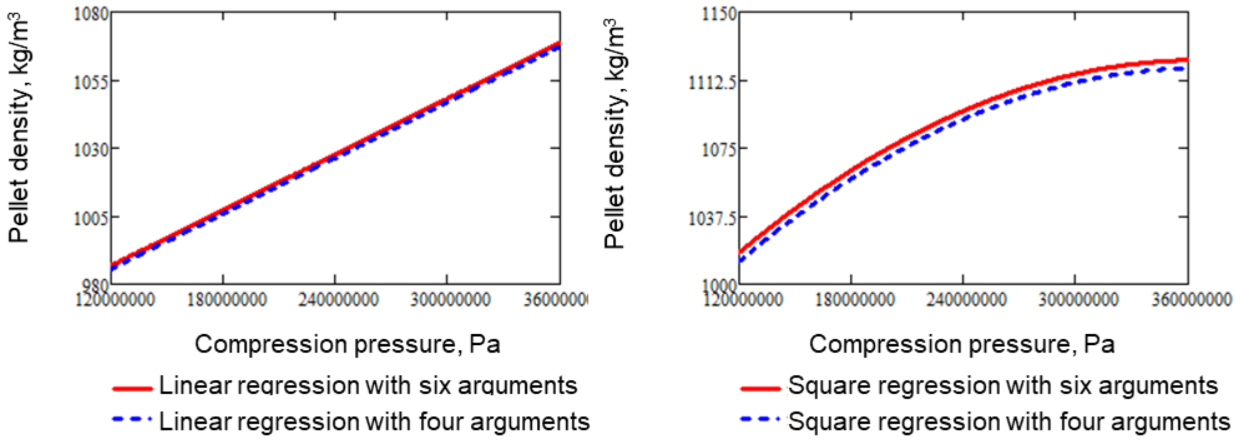

Fig. 3 – The dependency of pellet density on raw material compression pressure, in various regression versions, for constant values : $h_0 = 12\%, V_0 = 0.00001837\ mc, P = 300MPa, v = 0.002\frac{m}{s}, \theta = 80\ ^oC$.

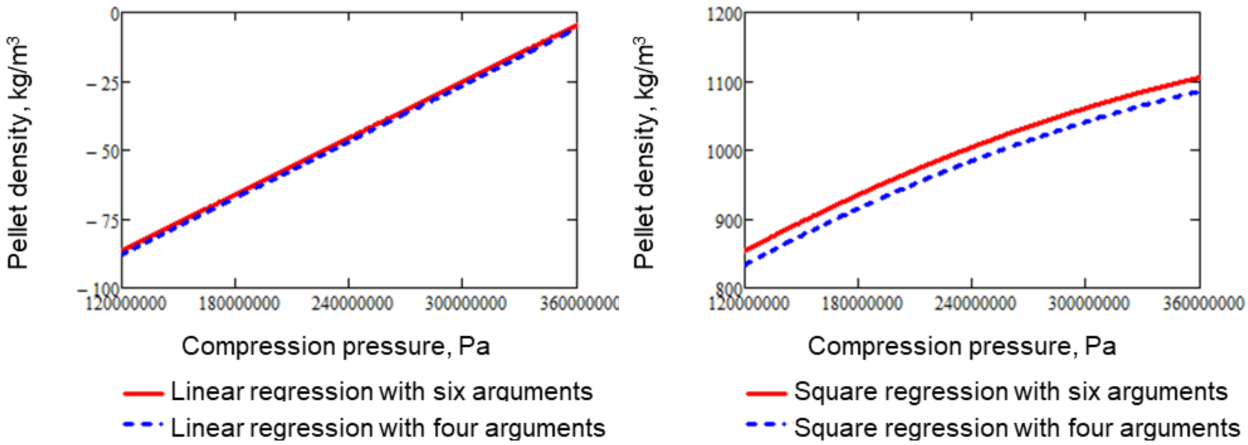

Fig. 4 - The dependency of pellet density on raw material compression pressure, in various regression versions, for constant values: $h_0 = 15\%, V_0 = 0.00001837\ mc, P = 300MPa, v = 0.002\frac{m}{s}, \theta = 80\ ^oC$.

An interesting variation example is given in Figures 3 and 4, where for two sets of fixed variables, different only in the values of raw material moisture, it is noticed that linear regression should be avoided because, in one of the cases (15% moisture) it yields negative values, without physical sense for density. Here is noticed, not only a reason to reject the linear model, but also that there is a necessity to control the entire model, even being nonlinear, on the whole definition domain. Otherwise, there is a possibility for erroneous prediction, not only numerically, but also physically. This control needs to be done for all relations established through interpolation.

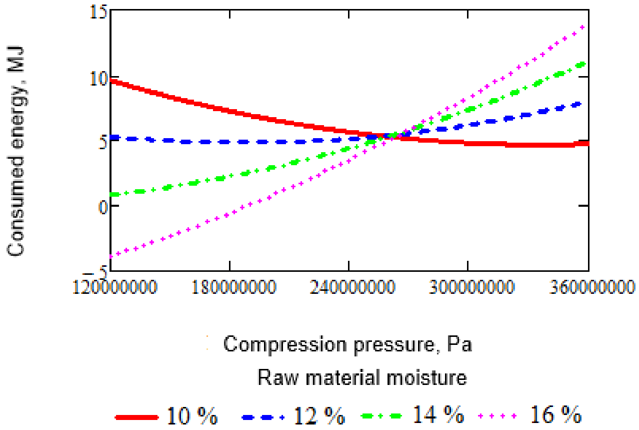

Fig. 5 – Dependency of consumed energy on the maximum pressure applied, for various values of moisture, the other variable remaining constant: $V_0 = 0.00001837\ mc, P = 300MPa, v = 0.002\frac{m}{s}, \theta = 80\ ^oC$

The energy consumed during the pelleting process depends most strongly on the compression pressure (Table 2), but, according to the correlation coefficient, it only has a moderate dependency. Still, moisture also influences the consumed energy in a significant way. Figure 5 presents the variation of consumed energy with the working pressure (implicitly the compression force), for four values of raw material moisture. It is noticed that interpolation does not work very well for high moistures, where there is an interval of reduced pressure where the energy would be negative. The curves represented in Figure 5 were drawn from the second-degree interpolation polynomial with a complete set of arguments (the six input and command parameters given in Table 1).

Evidently, for practical applications, technologies and installations of concrete dimensions, it is possible to attempt to make interpolations of quality parameters with higher degree polynomials. Thus, precision will increase and will be sought to eliminate work intervals where there are values without physical meaning. The disadvantage of these high-degree interpolation formulas is that they become very particular. On the other hand, the physical significance of coefficients is difficult to explain. Still, in the context of increasing production efficiency, the elaboration of an operation algorithm based on high-degree polynomial functions represents a seldom encountered alternative in industry. Installations are therefore computer assisted in order to obtain products with a desired quality (ρ_p, L, U_p, V) and a lower energy cost (E_c).

A possible result of statistic modelling through interpolation is to improve or, if possible, even to optimise the pelleting process. A possibly usable function in such a study is the energy consumption, E_c, in one of the versions of nonlinear regression. We showed above that there are possibilities to minimize the energy consumed, in certain conditions ever to optimize it (minimums exist). Because this case was displayed above, we will further study a more complex example. The function consumption energy per pellet mass unit is formed, by combining three qualitative functions from the set of five that we have at our disposal:

$$E_{cmp}\,(U_i, \rho_0, V_0, P, v, \theta) = \frac{E_c(U_i,\rho_0,V_0,P,v,\theta)}{\rho_p\,(U_i,\rho_0,V_0,P,v,\theta)\cdot V(U_i,\rho_0,V_0,P,v,\theta)} \quad (21)$$

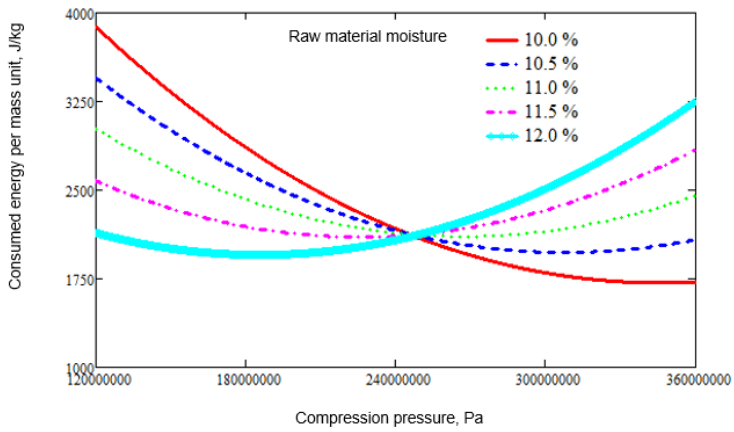

Fig. 6 – Variation of energy consumption specific to pellet mass unit, E_{cmp} with the compression pressure for five values of raw material moisture. Functions in definition (21) are each a second-degree regression

Same as for the consumed energy function, the function for specific energy consumption per pellet mass unit can vary monotonously with the applied pressure, but can also have minimum points. This behaviour is visible in the graphical representation in Figure 6. A bidimensional representation of the same dependency is given in Figure 7.

If the first-degree interpolation functions are used in definition (21), then the representations in Figure 6 take the form shown in Figure 8, and the bidimensional representation in Figure 7 takes the form shown in Figure 9. As it can be easily observed, aspects noticed regarding the function *specific energy consumption per pellet mass* formed by second-degree regressions are only partially found in the version of the same function formed by linear regressions (compare Figures 6 and 8). The same conclusion is visible by comparing the bidimensional representations of the same two versions of the function *specific energy consumption per pellet mass* (Figures 7 and 9). In reality, the situation is even more difficult because the interpolated denominator can have zeros that can induce surprizing asymptotical varieties to the surface of consumed energy specific to mass unit. Function (21) is, through its formulation, nonlinear even in the case when it is formed by linear regressions. Therefore, it is recommended to avoid optimization in such conditions, when the objective function has singularities. Moreover, any determined optimum, even on the qualitative system parameters, in various interpolation versions (linear, nonlinear or nonpolynomial) can be validated only after conducting a new high-resolution experimental study around the suspected optimal point.

Polynomial interpolation formulas are also used in the foundation of compressed powders dynamics, for example, ceramic powders (*Vogler et al., 2007*). Authors *Vogler et al. (2007*) use second-degree polynomial curves for the interpolation of results.

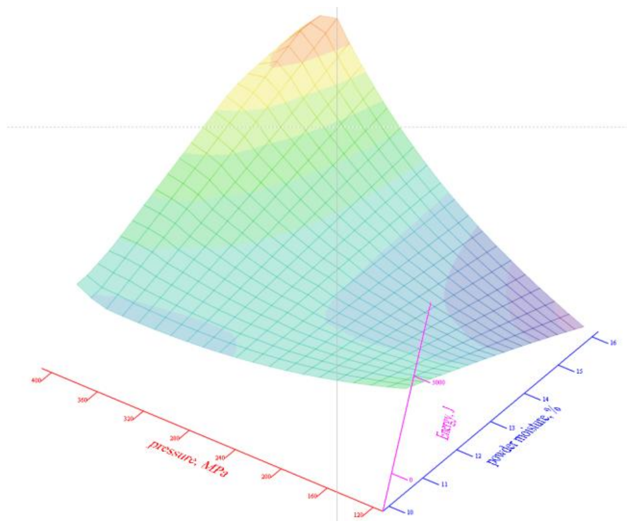

Fig. 7 - Variation of energy consumption specific to pellet mass unit, E_{cmp} with compression pressure and on raw material moisture. Functions in definition (21) are each a second-degree regression

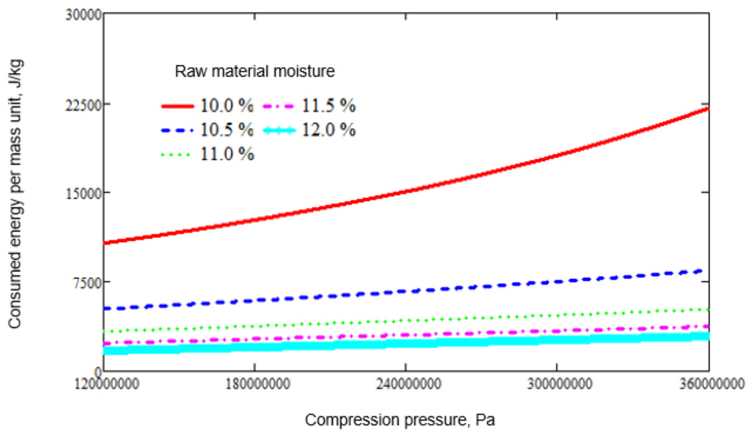

Fig. 8 - Variation of energy consumption specific to pellet mass unit, E_{cmp} with compression pressure for five values of raw material moisture. Functions in definition (21) are each a first-degree regression

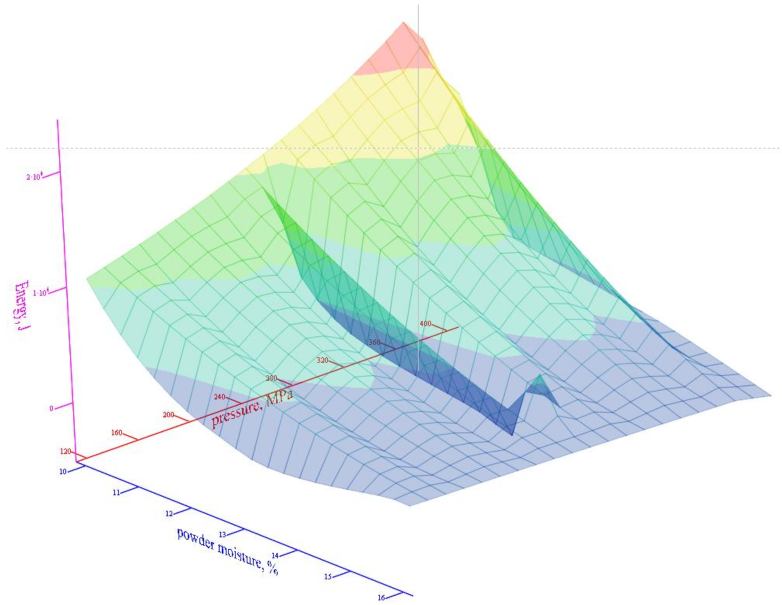

Fig. 9 – Variation of energy consumption specific to pellet mass unit, E_{cmp} on compression pressure and on raw material moisture. Functions in definition (21) are each a first-degree regression

In figures 6 and 8 was made the representation for moisture in the narrow interval between 10-12%, in order to not highlight the fact that there are high moisture values that in certain conditions lead to negative values for volume and/or density, leading to the emergence of before mentioned asymptotic varieties for the *specific energy consumption per pellet mass* function.

CONCLUSIONS

The systemic structure with the help of which we have attempted to describe the process of pelleting fir tree sawdust in a cylindrical closed die is a modern and appropriate description for the technological processes because it provides prediction and optimization tools useful in the rational exploitation of technologies.

The construction of the system presented in this paper took into account the experimental availability. For these reasons, there are essential parameters that have been neglected or for which a low number of values was reached and it was not possible to sense their influence on the process (on other parameters).

The interaction between various parameters is studied using the statistic modelling method, more precisely of polynomial regressions. Correlations between output parameters (the ones indicating the quality of the pelleting process) and the input and command parameters, show that the five output parameters are in strong relation to only a part of the input and command parameters. Parameters L, ρ_p, U_p, V strongly depend on parameters ρ_0, U_i, V_0 and low on F_{max} , with an exception for U_p which does not show any relation to F_{max} , according to the correlation calculation. E_c depends moderately on F_{max} and insignificantly on all other input and command variables (parameters). The dependencies of most of the quality parameters are in relation to the input parameters and less to the command ones (as far as raw material moisture is regarded as an input parameter and does not become a command one). The dependency on compression speed and die temperature are low for all parameters.

The statistic interpolation study shows that good precisions are possible using reduced, but also complete sets of variables. Going through the polynomial regression study highlights the fact that quality parameters vary monotonously in relation to input and command parameters, in the limits of the work domain covered experimentally. Precision performances can direct researchers and result users toward choosing a convenient interpolation version in the purpose of continuing the investigations for improving working regimes, possibly by introducing the results of researching the dependency of pellet durability on the input and command parameters used.

By using the statistic mathematical models obtained through interpolation, it is noticed that nonlinear regressions are better modelling the phenomenon (from the point of view of precision), despite the reduction of variables set given by the correlation criteria. Still, the best version is the one where the dependencies are expressed on the whole set of variables, in general. Nevertheless, high caution is recommended in using mathematical models based on regressions, because there are possible traps leading to false optimal values or to combinations of critical values without physical meaning. It is recommended to perform a high-resolution examination of interpolated functions behaviour and only after eliminating subdomains of definition on which they are not behaving satisfactory, to move to qualitative investigations of the studied process, using these functions.

ACKNOWLEDGEMENT

This work was supported by a grant of the Romanian Research and Innovation Ministry, through Programme 1 – Development of the national research-development system, subprogramme 1.2 – Institutional performance – Projects for financing excellence in RDI, contract no. 16PFE.

REFERENCES

[1] Adapa P. K., Tabil L. G., Schoenau G., (2009), Compression Characteristics of Selected Ground, Agricultural Biomass, *Agricultural Engineering International*: CIGR Ejournal, Manuscript 1347, 11;
[2] Cardei P., Gageanu I., (2017), A Critical Analysis of Empirical Formulas Describing the Phenomenon of Compaction of The Powders, J. *Modern Technology & Engineering*, vol. 2, No. 1, pp. 1-20;
[3] Celik M., (2016), *Pharmaceutical Powder Compaction Technology*, Drugs and Pharmaceutical Sciences, vol. 197, CRC Press, ISBN:9781420089172, 344 p.;
[4] Comoglu, T., (2007), An Overview of Compaction Equations, *Journal of Faculty of Pharmacy*, Ankara, 36(2), pp. 123–133;
[5] Gageanu I., Cardei P., Matache M., Voicu Gh., (2019), Description of the experimental data of the pelleting process using elementary statistics, *Proceedings of the Sixth International Conference "Research People and Actual Tasks on Multidisciplinary Sciences",* 12-15 June 2019, Lozenec, Bulgaria, pp. 437-445;
[6] German R.M., (2013), History of sintering: empirical phase, *Powder Metallurgy*, vol. 56, issue 2, pp. 117-123, https://www.tandfonline.com/doi/abs/10.1179/1743290112Y.0000000025;
[7] Mani S., Tabil L.G., Sokhansanj S., (2003), An Overview of Compaction of Biomass Grinds, *Powder handling & processing*, vol. 15, no.3, pp. 160-168;
[8] Nadon G.C., (1998), Magnitude and timing of peat-to-coal compaction, *Geology*, 26(8), pp. 727-730;
[9] Shaw M., (2008) Feedstock and process variables influencing biomass densification, Thesis for Degree of Master in Science, March, Department of Agricultural and Bioresource Engineering University of Saskatchewan Saskatoon, Saskatchewan;
[10] Vogler T.J., Lee M.Y., Grady D.E., (2007), Static and dynamic compaction of ceramic powders, *International Journal of Solids and Structures*, vol. 44, issue 2, pp. 636-658;
[11] Voicea I., Voicu Gh., Cardei P., Vladut V., Gageanu I., Daraban A., (2016), Theoretical and experimental research on the process of biomass briquetting, *U.P.B. Scientific Bulletin,* Series D, Vol. 78, Iss. 3, pp. 203-214;
[12] Voicu M., (2008), *System theory (Teoria sistemelor)*, Romanian Academy Publishing House, Bucharest;
[13] *https://www.dummies.com/education/math/statistics/how-to-interpret-a-correlation-coefficient-r/*;
[14] *https://www.ptc.com/en/products/mathcad*;
[15] *https://en.wikipedia.org/wiki/Biomass_briquettes*.