A NOVEL APPROACH FOR SOLVING GEAR TRAIN OPTIMIZATION PROBLEM

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1. INTRODUCTION

From a research and practical aspect, there is a need for constant improvement in theoretical and practical segments in order to justify the use of gear trains. As a basic problem is achieving better working characteristics, performances, in terms of mass, volume costs, etc. Chong et al. [1] presented a general methodology for optimizing gear ratios, sizes and housing volume for multi-stage gear trains in preliminary design phases. Marjanovic et al. [2] developed a practical approach to optimizing gear trains with spur gears based on a selection matrix of optimal materials, gear ratios and shaft axes positions. Golabi et al [3] presented gear train volume/weight minimization optimizing single and multistage gear trains' gear ratios. Mendi et al. [4] aimed to optimize gear train component dimensions to achieve minimal volume comparing GA results to analytic method parameter volume. Savsani et al. [5] described gear train weight optimization comparing various optimization methods to genetic algorithm (GA) result values. Gologlu and Zeyveli [6] performed preliminary design automation through optimization of gear parameters and properties using a GA based approach. Pomrehn and Papalambros, [7] optimized gear train volume varying gear thicknesses, distances between centers, pitch diameters, number of teeth, ratios, etc. for a specific four stage setup. Deb and Jain [8] used similar principles for optimizing multi-speed gearboxes using multi-objective evolutionary algorithm optimizing volume in relation to power.

In order to facilitate improvement in this field, a great deal of experience and use of novel and alternative methods is required. Optimization presents a way of effectively achieving desired characteristics of gear trains. This paper is concentrated only on gear trains with parallel shaft axes. Aside from knowing reducer construction, the processes and method of optimization, it is necessary to develop a mathematical model which can be representative of the problem, and improve performances. Modern approaches of solving this problem represents the use of a heuristic approach, as an alternative to optimization. A heuristic approach allows for achieving adequately optimal solutions with having very little input data.

Motivation for this research is to find a means, and volume optimization method for gear trains using a heuristic approach, where a minimal amount of input data is necessary in order to form a universal method and approach for solving this specific problem for any gear train of this type. Furthermore this method should lead to achieving minimal volume, and with that a lower mass of the construction, smaller dimensions,

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savings on materials, expenses and other benefits. The realization of this problem is oriented on mimicking natural occurrences. This heuristic approach considers using a golden spiral, which represents the correlation of growth and development in nature, with the assumption that it would have the same effect in the process of gear train design.

2. PROBLEM DEFINITION

Gear trains with parallel shaft axes are commonly designed to have all exes in the same plane. This is most frequently the horizontal plane. Such a setup of gears takes up a large space which could be decreased by optimizing the gear train. An optimal solution defines a new position of axes which in turn causes the decrease in overall volume. Gear train volume is defined by the product of length width and height based on which a mathematical model is created. As the width can be considered constant due to calculated values of gear dimensions and other construction dimensions, the optimization problem is reduced to the optimization of area as a product of length (L) and height (H) of the gear train. Optimal volume can be achieved by forming a mathematical model, as was done in some research papers [1-3, 7]. There is a tendency to avoid complex mathematics which represents the goal function and constraints. For this problem it is possible to achieve an optimal solution by combining the trajectory of the golden spiral and positions of the gear train axes. By placing the axes on the contour of the golden spiral it is possible to achieve an alternative solution for minimizing gear train volume, or more precisely minimizing its area in the plane normal to the shafts.

2.1 Golden spiral definition

Growth and development of natural structures and processes always has the same, unique dependency. This dependency can be mathematically formulated and presented by the golden section (1.6180339887), golden angle (137.508°), and golden spiral. The golden spiral, due to its complexity, can be approximated by a line which connects the corners of tiles which are the size of the Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21...). Natural dependencies formed in a spiral as the Fibonacci tiles are presented in picture 1, where a correlation can be easily seen. The Fibonacci spiral has a very small divergence from the golden spiral, and it is easy to use. This is the reason for the golden spiral being commonly considered as the Fibonacci spiral, which is also the case in this paper.

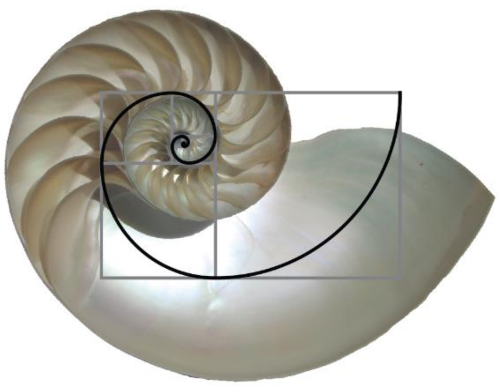

Figure 1 Fibonacci tiling and spiral found in nature

The golden spiral has a large influence as it represents a natural solution and depicts natural occurrences. The use of these occurrences from nature presents an analogy to natural solutions. When applied, these solutions seem completely natural and logical, while their possible implementation is not always easily noticed.

2.2 A novel approach (novel method)

This research is oriented towards analyzing the problem of minimizing gear train volume from a shaft axes position angle aspect. A heuristic approach to positioning axes along the contour of a golden spiral was developed, and the methodology of such an alternative solution for optimization is defined. This approach represents mimicking natural occurrences and processes of development in nature. Nature in itself achieves optimal processes, therefore this research was started under the hypothesis of it converging towards an optimum.

Forming a golden spiral is done according to the distance between two axes. In order to avoid overlap among gear train elements it is necessary for the distance between the first gear pairs axes be the same as the diagonal of the third tile of the Fibonacci spiral (1).

$$a_1 = f_3\sqrt{2} \tag{1}$$

where:

a_1 - distance between axes of the first gear pair,

f_3 - size of the side of the third Fibonacci tile.

The presented equation (1) is the first condition of this approach. The rest of the shaft axes are placed along the golden spiral in order. This approach can be applied to all multi stage gear trains, regardless of the number of stages. A planar representation of how a multistage gear train design should look is shown in picture 2.

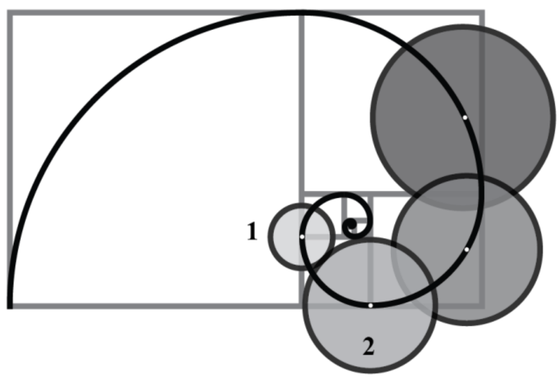

Figure 2 Planar view of gear placement along Golden spiral

The number of stages of the gear train does not influence the methodology of approach, as in can be seen in picture 2. In case there is overlap of gear train elements in this setup, the starting point of the first gear axes should be moved to point 2 (Picture 2).

As the width of the reducer is constant die to calculated values, the volume depends exclusively on the length and height of the gear train. This reduces the volume optimization problem to a planar area minimization problem. In the case of a contemporary concept the housing is a lot larger than when the axes are placed at different angles from the horizontal plane. The complete methodology of gear train volume optimization is developed based on this fact. A simplified diagram of which angles are optimized is shown in picture 3.

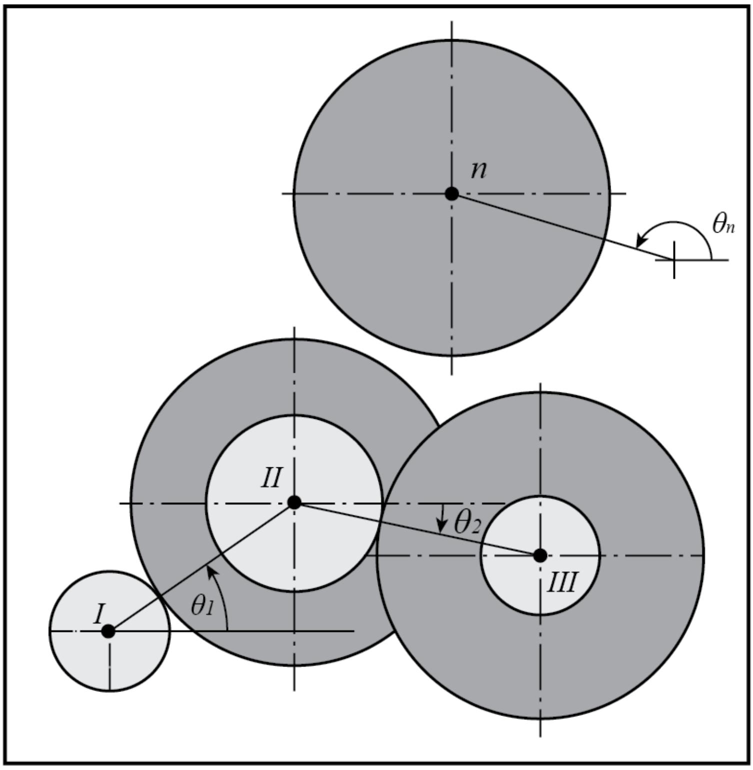

Figure 3 Gear train optimization of axes angles for gear train with n gear pairs

The approach is completely original and therefore is presented as a novel method for solving gear train volume optimization problems. The benefits of this methodology are that there are no complex equations which define the mathematical model, there is no need for using optimization software, and the solution is achieved in a quick and simple manner. The methodology is generalized, meaning that it can be used on all gear trains with parallel shaft axes.

3. TEST EXAMPLES

In order to verify this research, and its practical application verification was conducted on examples from literature. Two examples were used, where example 1 [2] is a two stage, and example 2 [1] is a four stage gear train. For input data gear diameters are necessary. All gear train data is shown in table 1.

Table 1 Input values for testing for two examples

	Example 1(2 stage)			Example 2 (4 stage)			
Stage	1	2		1	2	3	4
Module (mm)	2	3		1.5	2	3	4
Number of teeth in pinion	20	23		14	18	20	25
Number of teeth in gear	83	67		77	79	74	82
Pitch diameter of pinion (mm)	40	69		21	36	60	100
Pitch diameter of gear (mm)	166	201		115.5	158	222	328
Face width (mm)	103	135		68.25	97	141	214
Net length, height, width	388.5	231	110.6	724.75	358	153	

Presented values in table 1 are dimensions of the gear train and using these values initial volume of the conventional setup can be calculated. Gear train volume is calculated with clearances of 15mm on all sides from the gear train housing and between gear train elements, and the calculations are done according to [2]. Initial volume of the two stage gear train (Example1) is 9 925 631.1mm3, and the four stage (Example 2) is 39 697 456.5mm3. Picture 4 shows concepts of example 1 and example 2, for the conventional and optimal shaft axes positions.

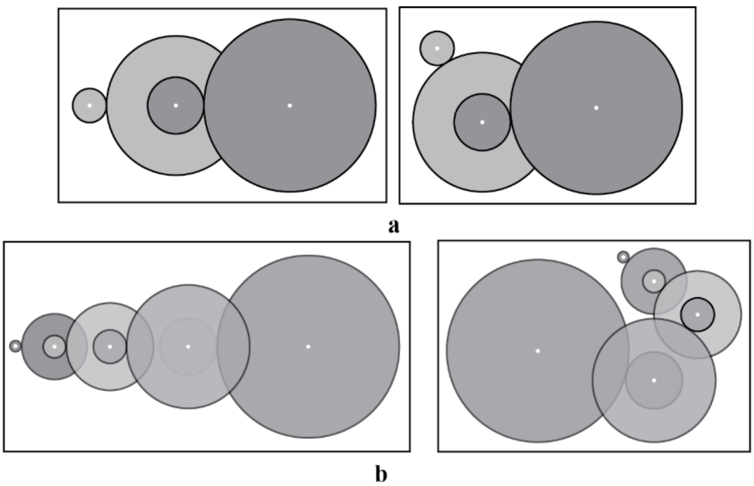

Figure 4 Conventional and optimal concepts a) Example 1; b) Example 2

Optimal gear train volume for Example 1 is 8 874 585.6 mm^3, and for Example 2 is 31 767 763.4 mm^3. These values are achieved by optimization and need to be compared to volumes achieved using the golden spiral approach, using the same width of each gear train respectively.

4. RESULTS

In order to achieve an optimal solution a large number of complex equations needs to be developed, which create the mathematical model. The optimization method needs to be utilized and adapted to the problem or adapt the problem to the optimization method. All this resents a long and arduous process in which there is always room for error and wrong conclusions. The idea of the golden spiral approach is that that process is avoided, and still achieve acceptable, improved solutions. These solutions can be treated as optimal as there is a minimal effort put into them, with a minimal amount of input information, achieving notable benefits and minimizing volume.

For achieving results, table one is used for optimal concepts. The first step is to have the first gear pair placed freely in the vertical plate, after that a golden (Fibonacci) spiral is constructed, so that the diagonal od the third Fibonacci tile is the distance between the shaft axes of the first gear pair. The other axes are placed along the spiral contour in order. The improvements which can be achieved through optimization in this case depend mainly on the diameters of the gear pairs.

With conventional optimization approaches shaft axes (the planes in which they lay) in relation to the horizontal plane are at a specific angle, and the optimal area and volume can be achieved using more than one angular setup. For the golden spiral approach it is not possible to change angles.

The golden spiral approach comparison to optimal solutions is in calculation the areas and volumes of both and comparing their deviations. Values for the conventional, optimal [2], and golden spiral dimensions and volumes are given in Table 2 for both examples.

Table 2 Values for conventional, optimal, and golden spiral designs

		L [mm]	H [mm]	B [mm]	V [mm³]
Conventional	Two stage gear train	388.5	231	110.6	9925631.1
	Four stage gear train	724.8	358	153	39700195.2
Optimal	Two stage gear train	347.4	231	110.6	8875583.6
	Four stage gear train	555.6	373.7	153	31767041.2
Golden spiral	Two stage gear train	347.4	231.1	110.6	8879425.9
	Four stage gear train	559.7	384.2	153	32900621.2

From table 2 it is obvious that values achieved using the new approach are very close to optimal values, which directly indicates the validity of the new developed approach. Optimal solutions have values in acceptable ranges for the same value of volume, as shown in picture 5.

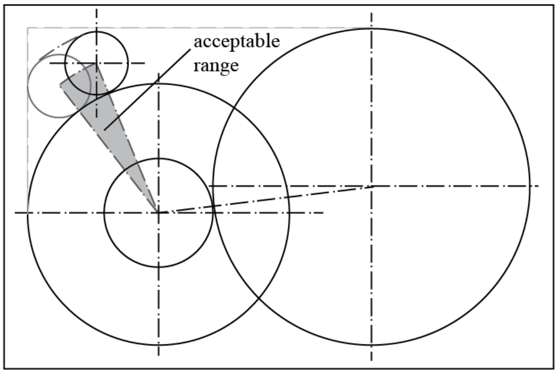

Figure 5 Acceptable angle ranges for the optimal solution of Example 1

For solutions achieved using the new approach the use of the golden spiral has a singular solution. A schematic visualization of the new solutions is shown in picture 6.

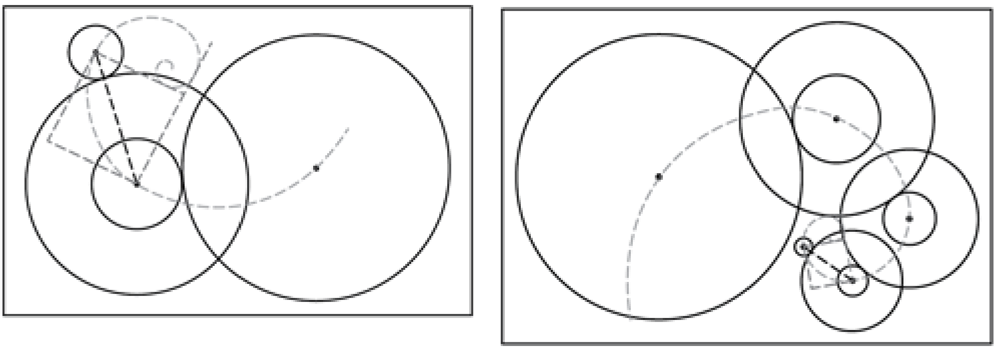

Figure 6 Optimal gear train setups using new method

Based on achieved results and table 2 it can be concluded that the reduced gear train volume by optimization gives a 10.58% decrease from the conventional design, while the new approach yields a 10.54% decrease for Example 1. The decrease on volume for Example 2 using optimization is 19.98%, while the new approach gives a 17.13% decrease.

It is obvious that optimization yields somewhat better results; however the effort put into achieving those results is much greater than for the novel approach. As an optimal solution is one which is achieved with the least amount of effort while achieving maximal gain, the novel approach can be treated as optimal. This methodology presents an alternative for practical use in gear train design due to its simplicity and benefits in terms of volume minimization.

5. CONCLUSIONS

The novel approach presented by this paper has been verified on examples, as functional and the results have been compared to optimal solutions from literature.

The optimization process requires the development of numerous equations and constraints for the mathematical model, whose number increases even further with the number of stages of the gear train which is optimized. Using the golden spiral novel approach the complex mathematic model is avoided all together, thereby shortening the time needed to develop the design of such a setup and minimizing the risk of error in the process. In addition to these benefits, the implementation of the novel method into software is very simple as opposed to optimization methods which require the creation of, or modification of existing software in order to use.

Results achieved using the novel method is very close in values to optimal solutions, and their deviation in terms of percentage is acceptable for engineering practice. Given the simplification of achieving an improved design concept by using the novel approach, the difference in volume minimization compared to optimization is negligible.

Achieved results of minimized volume are directly linked to savings, not only in space used, but also in material for the gear train housing, costs, speed of forming design documentation, etc.

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