OPTIMAL CONTROL OF THE MACROECONOMY WITH THE APPLICATION TO 2001 CRISIS OF TURKEY

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Abstract

This paper concentrates on the application of optimal control theory to highlight some aspects of Turkish economy. First the setup is given for Turkey to grow over the balanced path. Then the optimal control problem is identified. The control and state variables are mentioned. The objective is the maximization of life-time discounted utility of the society through optimal choice of consumption which automatically determines investment. We make use of Bellman's principle to guarantee optimality. We make necessary assumptions (technical assumptions) to make use of calculus techniques for a solution. Some functions to represent utility and production are specified. I used the econometric techniques to estimate some parameters of the functions to decide upon the optimal level of investment for steady-state in Turkey over the period including 2001 crisis. The corresponding differential equations are obtained as a result of the Hamiltonian defined. The phase diagram is prepared to analyse different trajectories.

Keywords: *Optimal Control, Growth, Turkey, CRRA, Cobb-Douglas.*

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Introduction

Literature on growth holds a vast volume in macroeconomic theory. This subject is at the center of many macroeconomic research discussions. Contemporary theory on growth can be traced back to Solow (1956) for his contributions to the so-called exogeneous growth theory. He has made use of the basics developed by even Adam Smith. Later contributions to theory formalized by Solow were towards repairing some shortcomings of this theory on several fronts and these contributions have led to the endogeneous growth where the saving rates of the consumers can somehow be internalized within the economy. This innovation in theory made the framework much closer to real consumption behaviors of individuals and households.

Much earlier studies on growth belong to the classical school of thought. Adam Smith (1904) proposed that growth is supply-sided and total output is a function of labor, capital and land. Therefore, growth was stated to be driven by population, investment and land progress as well as technological advances. Smith thought that all these factors were endogenous and he had given emphasis to specilaization as a particular engine to fuel growth. He suggested that there is a limit to growth as a result of an ultimate halt of capital accumulation and population increase.

The other major profile in the classical school of thought, David Ricardo, made a remarkable contribution to the discourse of Smith by saying that land is limited and cannot be cultivated as Smith suggested. And Malthus had stated that population growth depends on the food supply and would ultimately limit economic growth thereby leading to the poorness of individuals.

The neoclassical school of thought could not add much to the pile of the classical school. It was early 1900s at which Schumpeter completed his Theory of Economic Development and had mentioned the steady state as his reference point. Schumpeter assumed population growth was exogenous as opposed to Smith and has taken the enterpreneurs as the main driving force of development (Schumpeter's arguments were around development instead of growth).

It was the middle of the 20th century when Robert M. Solow had presented his growth model which became famous by his name, Solow or neoclassical growth model. Solow appeared as the most prominent economist in the growth literature. He proposed a model that handled the capital output ratio fixed and this ratio guranteed the system to move back to steady state. He assumed that there are CRS, perfect competition, perfect information flow and no externalities. These were technical assumptions instead of being realistic. He had to make these assumptions to guarantee the steady state mathematically. And this is the reason of why Solow's Model lacked empirical evidence in many studies. Furthermore, Solow had assumed that the individual savings and technology were exogeneous. In the same year Swan had made a similar study so that the model is sometimes called the Solow-Swan Model. Solow had introduced the concept of growth accounting through which growth is decomposed into its capital accumulation, labor force increase and the residual that handles all others including technology.

There are two main shortcomings of the neoclassical growth models. First of all, it lacks empirical evidence. Many of the empirical studies have disproved the model. Secondly, technology, which is the key determinant of growth was taken exogeneous. The model could no longer be used. Therefore, the model was modified by two economists, Kass (1965) and Koopmans (1967). They had made use of the earlier studies by Ramsey (1928) who had taken the saving rate endogenous. That is why the model is known as the Ramsey-Kass-Koopmans Model.

The progress of the growth theory in last decades are mainly due to the studies of Romer (1986) and the Nobel Laurate Robert E. Lucas (1988) that popularized the earlier study of Romer. This streamline of growth theory is called “New Growth Theory” and questions the results and assumptions of the previous one. Basically, the returns to scale are taken increasing and the competition is assumed imperfect which reflect the facts of real economies in a better way.

This paper superficially summarizes the very well-known literature of the growth theory in the first section, Introduction. The second section introduces the model and the basics of the optimal control theory to resolve this model. Third section includes the phase diagram based on the optimal control analysis. Section 4 estimates the parameters of the econometric model which handles the macroeconomy of Turkey. Section 5 lists down the concluding remarks.

Optimal Control Problem of Growth

We assume a production function with the arguments of capital (K), labor (L), and others, (A).

$$Y(t) = F(K(t), L(t)A(t)) \quad [1]$$

We have the population growth rate of n and technology growth rate of g. Both growths are continuous: $L(t) = e^{nt}$ and $A(t) = e^{gt}$.

What is not depreciated and not consumed out of production is the increment of the capital stock:

$$\dot{K}(t) = F(K(t), L(t)A(t)) - \delta K(t) - C(t) \quad [2]$$

Letting $c(t) = \frac{C(t)}{L(t)}$, $Z(t) = \frac{C(t)}{A(t)L(t)}$, and $\kappa(t) = \frac{K(t)}{A(t)L(t)}$ we can write the equation in its intensive form:

$$\dot{\kappa}(t) = f(\kappa(t)) - Z(t) - (n + \delta + g)\kappa(t) \quad [3]$$

The society wants to maximize the lifetime utility function where the life of the society is known to be infinity:

$$\max_{c(t)} \int_0^\infty e^{-\rho t} U(c(t))dt \quad [4]$$

where ρ is the discount factor.

We assume that the utility function is in the form of Constant Relative Risk Aversion (CRRA):

$$U(c(t)) = \frac{c(t)^{1-\theta}}{1-\theta} \quad [5]$$

when $\theta \neq 1$ and when $\theta = 1$ we assume that $U(c(t)) = \ln c(t)$. There is no empirical research to figure out the numerical value of θ to the best of my knowledge. The numerical value is assumed to be 0.355. This number follows from the empirical research of Holt and Laury (2002).

Substituting this utility function into the maximization problem leads to:

$$\max_{c(t)} \int_0^\infty e^{-\rho t} \frac{c(t)^{1-\theta}}{1-\theta} dt$$

Substituting $Z(t)e^{gt}$ for $c(t)$ and rearranging give:

$$\max \int_0^\infty e^{-(\rho - g(1-\theta))t} \frac{Z(t)^{1-\theta}}{1-\theta} dt \quad [6]$$

Defining the utility function U in terms of Z, and using a to handle the power of e the problem turns out to be:

$$\max \int_0^\infty e^{-at} U(Z(t))dt \quad [7]$$

Both output and consumption levels can be figured out when κ and Z are worked and figured out. We have the typical social planner in charge one more time. The social planner is directly responsible for the life-time discounted utility maximization of the whole society.

So that the social planner has the problem of:

$$\max \int_0^\infty e^{-at} U(Z(t))dt$$

st

$$\dot{\kappa}(t) = f(\kappa(t)) - Z(t) - (n + \delta + g)\kappa(t)$$

and

$$\kappa(0) = \kappa_0$$

This problem is much different than an ordinary maximization problem because the function to be maximized is the integral and the constraint is a differential equation. This is a typical optimal control problem. Pontryagin is the prominent Russian mathematician to study these kinds of problems. Dorfman (1969) applied the theory to economic problems. One can read Kamien and

Schwarts (1981), or Stokey et. al. (1989) for very complete exposure of the material on optimal control theory.

This problem can be solved with the help of Pontryagin's Maximum Principle. The allocation that is found as a result of this problem is at the same time Pareto Optimum. The state variable is $\kappa(t)$ and the control variable is $Z(t)$. We define the co-state variable λ to form the present value Hamiltonian, H.

$$H(\kappa, Z, \lambda, t) = e^{-at}U(Z(t)) + \lambda f(\kappa(t) - Z(t) - (n+\delta+g)\kappa(t) \text{ [8]}$$

The sufficient conditions for an optimal solution are:

$$\lambda(t) = e^{-at}U'Z(t) \text{ [9a]}$$

$$\dot{\lambda}(t) = -f'(\kappa(t)) - (n+\delta+g)\lambda(t) \text{ [9b]}$$

$$\lim_{t\to\infty} \lambda(t)\kappa(t) = 0 \text{ [9c]}$$

In order to solve this system of differential equations one would rather get rid of the co-state variable, λ, first. Differentiating Eq. (9a) with respect to time gives:

$$\dot{\lambda}(t) = U''(Z(T))\dot{Z}(t) - ae^{-at}U'Z(t) \text{ [10]}$$

Dividing both sides of this equation by both sides of Eq. (9a):

$$\frac{\dot{\lambda}(t)}{\lambda(t)} = \frac{\dot{Z}(t)U''Z(t)}{U'Z(t)} - a$$

Incorporating Eq. (9b), and multiplying both sides of the equation by Z(t):

$$\dot{Z}(t) = \frac{1}{\theta}(f'(\kappa(t)) - (n+\delta+g+a))Z(t) \text{ [11]}$$

Here θ is substituted for $-\frac{Z(t)U''(Z(t))}{U'(Z(t))}$.

Now the two differential equations are obtained, Eq. (11) above, and Eq. (12) below.

$$\dot{\kappa}(t) = f(\kappa(t)) - Z(t) - (n+\delta+g)\kappa(t) \text{ [12]}$$

Phase Diagram

Phase diagrams are very beneficial tools that are employed in analyzing the behaviors of functions that appear in systems of differential equations especailly in cases where the set of differential equations cannot be solved directly. In our situation the set of equations include some functions that are not identified, so we have no chance to solve these differential equations. But we know some characteristics of them to draw some conclusions about them over the phase diagrams.

Let's start by writing down the two differential equations in hand one more time:

$$\dot{Z}(t) = \frac{1}{\theta}(f'(\kappa(t)) - (n + \delta + g + a))Z(t) \text{ [11]}$$

$$\dot{\kappa}(t) = f(\kappa(t)) - Z(t) - (n + \delta + g)\kappa(t) \text{ [12]}$$

We assign Z to vertical axis and κ to horizontal axis. Let's indicate the steady-state values of Z and κ by Z^* and κ^*, respectively.

Let's go through Eq. (11) first. $\dot{Z} = 0$ when the right-hand-side (RHS) of this equation is equal to 0. This is only when $f'(\kappa(t)) - (n + \delta + g + a) = 0$ or $f'(\kappa(t)) = (n + \delta + g + a)Z(t)$. This is when $\kappa(t)$ is at its steady-state condition, $\kappa(t) = \kappa^*$. We draw a strict line at $\kappa(t) = \kappa^*$ to indicate the points at which $\dot{Z}(t) = 0$.

What happens to $Z(t)$ off this line? Now let's look at Eq. (11) one more time. Let's pick a point $\kappa(t) > \kappa^*$ over the phase-diagram plane. For this point $f'(\kappa(t))$ is smaller since f'' is a decreasing function of $\kappa(t)$. That is why $f'(\kappa(t)) - (n + \delta + g + a)$ is negative and $\dot{Z}(t)$ (which is equal to $f'(\kappa(t)) - (n + \delta + g + a)$) is also negative since we are in the 1st quadrant and $Z(t)$ is positive for sure. This means $Z(t)$ is declining (since the derivative of $Z(t)$ is negative). That is why we put downward directed arrows to indicate this decline in $Z(t)$. Similarly when $\kappa(t) < \kappa^*$, $\dot{Z}(t) > 0$. And this time the directions of the arrows are reversed, ie. they are directed upward.

Coming to Eq. (12), $\dot{\kappa}(t) = 0$ when $f(\kappa(t)) - Z(t) - (n + \delta + g)\kappa(t) = 0$, or $Z(t) = f(\kappa(t)) - (n + \delta + g)\kappa(t)$. $Z(t) = 0$ when $\kappa(t) = 0$. So that the shape of the curve for which $\dot{\kappa}(t) = 0$, passes through the origin.

Let's think about the derivative of $Z(t) = f(\kappa(t)) - (n + \delta + g)\kappa(t)$ to see how the function behaves. Since f is concave in $\kappa(t)$ the slope will be positive. But the slope will decline for higher values of $\kappa(t)$. So that the shape of the curve is like an unusual bell. The maximum arises at κ^g which is larger than the steady state value of $\kappa(t)$, κ^*. This gives us the shape of Figure 1.

Coming to the dynamics on some sides of this bell-shaped curve, let's consider a point which is above the borderline, for this point $Z(t) > f(\kappa(t)) - (n + \delta + g)\kappa(t)$ so that $\dot{\kappa}(t) = f(\kappa(t)) - Z(t) - (n + \delta + g)\kappa(t) < 0$, and $\kappa(t)$ declines. We have the arrows directed to left. For the points surrounded by the bell-shape we have $Z(t) < f(\kappa(t)) - (n + \delta + g)\kappa(t)$ and $\dot{\kappa}(t) > 0$. The directions of the arrows at this region will be directed to right.

These arrows indicate where the economy will let consumption and capital stock to move. As the figure indicates there is just one saddle-path that leads the economy to the steady-state. This saddle-path is shown by the trajectory of the additional arrows, ie from top-right to bottom-left. If the economy starts at any other location it will go astray.

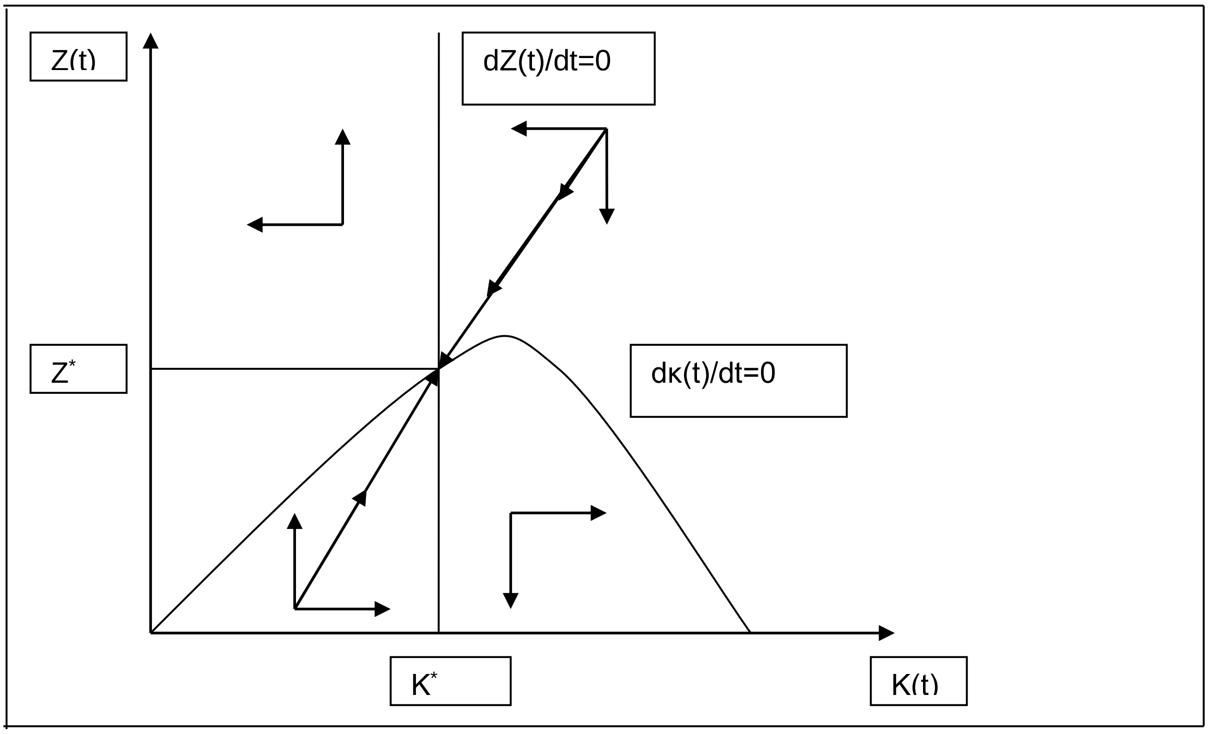

Figure 1. Phase diagram for Z(t) and κ(t).

If Turkey is positioned over the trajectory then she will be enjoying the travel over the saddle-path towards the golden route of growth. The main characteristic of the intersection of $\dot{Z}(t) = 0$ vertical line and $\dot{\kappa}(t) = 0$ bell-shape is that at this intersection both $Z(t)$ and $\kappa(t)$ will have no tendency to change over time.

In this last section of the paper we estimate parameters to intend to apply the theory to Turkish economy around her 2001 crisis. We start by stating the additional condition of the steady state: $\dot{Z}(t) = 0$ and $\dot{\kappa}(t) = 0$.

Now let’s rewrite the essential equations under this condition:

$$\dot{Z}(t) = \frac{1}{\sigma}(f'(\kappa(t)) - (n + d + g + a))Z(t)$$

so;

$$0 = f'(\kappa(t)) - (n + \delta + g + a)$$

and;

$$f'(\kappa(t) = n + \delta + g + a \ [11a]$$

If one assumes the Cobb-Douglas type production function in its intensive from then:

$$f(\kappa) = \kappa^{\alpha} \text{ and } f'(\kappa) = \alpha\kappa^{\alpha-1}.$$

Substituting this in Eq. (11a):

$$\alpha\kappa^{\alpha-1} = n + \delta + g + a$$

So

$$\kappa^{*} = (\frac{n + \delta + g + a}{\alpha})^{\frac{1}{\alpha-1}} \ [13]$$

This is the steady-state level of capital stock.

Indeed, the consumption maximizing level of capital stock and the steady-state capital stock are different than each other. The consumption maximizing capital stock is the projection of the peak of the bell shape which is higher than the level of capital stock at the intersection of $\dot{Z}(t) = 0$ and $\dot{\kappa}(t) = 0$ trajectories apparently.

Estimation of Parameters

In this part of the study, I implement the finding to the Turkish economy over one of the severest crisis of its last century, namely the 2001 crisis. One has to numerically evaluate all parameters appearing on the RHS of Eq. (13) in order to assess the balanced growth path of Turkey over the mentioned crisis. The purpose of this last section is this assessment.

Since $L(t) = e^{nt}$, taking logarithms of both sides: $\ln L(t) = nt$. This kind of a specification of the function automatically normalizes the population at time 0 to 1, ie. $L(0) = e^{0t} = 1$. This equation resembles the simple regression function of type: $Y = \beta X + \varepsilon$ where Y corresponds to $\ln L(t)$, β corresponds to n and the error term is assumed to satisfy the classical assumptions of ordinary regression. But since what we are handling is time-series data we have to take care of stationarity.

We gather annual data from the official website of the Central Bank, www.tcmb.gov.tr, from 1968 to 2004 for this estimation. We form a new series by setting 1968 to 0 and that year's population to 1. The series proves to be stationary as consequence of the Augmented Dickey-Fuller Unit Root Test executed by EViews. The estimated value of n appears to be 0.02222. The Eviews outputs are postponed to Appendix 1.
The depreciation coefficient is assumed to be 0.05 since we assume that the capital used in manufacturing depreciates completely in 20 year's time.

We estimate g just like how we estimate n. Recall, $A(t) = e^{gt}$. After some similar arrangements and manipulations, we estimate $Y = \beta X + \varepsilon$, one more time for this case. We use the National Technology Index, as a proxy for $A(t)$. The original data set is obtained from the same web site and raw data set is offered on a daily basis over 12.11.2001-13.9.2005. We make the necessary arrangements to have the consistency of data sets about their frequency. Again the series proves to be stationary as a result of the Augmented Dickey-Fuller Test. The regression output reveals that g is estimated to be 0.000232 and is highly significant, ie. p-value is 0.000. The EViews outputs are in Appendix 2.

Coming to the estimation of a that stands for $\rho - g(1-\theta)$. Since we assumed that $\rho = 0.90$ and $\theta = 0.355$ and estimated $\hat{g} = 0.000232$, the estimate of a turns out to be 0.90 due to the very small numerical value of $\hat{g}$.

The last parameter to be estimated is α of the Cobb-Douglas Production Function in its intensive form.

Remembering:
$Y = F(K, L) = K^{\alpha}(AL)^{1-\alpha}$ is written in its intensive form as: $y = \kappa^{\alpha}$ where $\kappa = \dfrac{K}{AL}$. The rest is arranging data and running a similar regression of $\ln y$ over $\ln \kappa$. The coefficient found is the estimate of α. Both series proved to be stationary at their levels. OLS estimate of α turned out to be approximately 0.98 where t-statistic showed that the coefficient is highly significant. The regression results are given in Appendix 3. Table 1 below lists down numerical values of parameters to estimate κ^* in Eq. (13).

Table 1. Numerical values of parameters, by assumption or estimation.

n	δ	g	a	α
0.0222	0.0500	0.000232	0.90	0.98

Substituting all these numerical values give the optimal level of capital per ideal labor force, κ^*.

Concluding Remarks

- This paper attempted to apply the optimal control theory to figure out the level of consumption, and thereby investment, to the economy of Turkey around her 2001 crisis. Real data sets of Turkey are used to find the optimizing policies.
- Unfortunately, there are not many theoretical applications of macroeconomics to Turkish economy. One has to recall significant contributions of Prof. Merih Celasun (2002) as well as Metin-Özcan, et. al (2001).
- One can make more explicit analysis after completely specifying the utility function, production function and all parameters belonging to both of these functions. The growth accounting can then be handled to observe the contributions of factors of production to growth individually. This issue stays as an open area for further research.
- In the last section of the paper some parameters are estimated and some are assumed to have certain numeric values in order to conclude κ^* which leads to the conclusion of Z^*, consumption. These estimates can be used by policy-makers in various developments of policies.
- Last but not the least, theoretical findings of the paper can be applied to all developing countries.

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Appendix 1. Unit Root Test results and the regression output:

ADF Test Statistic	-2.146026	1% Critical Value*	-3.6289
		5% Critical Value	-2.9472
		10% Critical Value	-2.6118

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LABOR)
Method: Least Squares
Sample(adjusted): 3 37
Included observations: 35 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LABOR(-1)	-0.012265	0.005715	-2.146026	0.0396
D(LABOR(-1))	0.032330	0.174769	0.184985	0.8544
C	0.025305	0.005154	4.909369	0.0000

R-squared	0.148377	Mean dependent var	0.020984
Adjusted R-squared	0.095151	S.D. dependent var	0.007146
S.E. of regression	0.006797	Akaike info criterion	-7.062774
Sum squared resid	0.001478	Schwarz criterion	-6.929459
Log likelihood	126.5985	F-statistic	2.787656
Durbin-Watson stat	1.999600	Prob(F-statistic)	0.076552

Dependent Variable: LABOR
Method: Least Squares
Sample: 1 37
Included observations: 37
LABOR=C(2)*TIME

	Coefficient	Std. Error	t-Statistic	Prob.
C(2)	0.022221	0.000168	132.2684	0.0000

R-squared	0.991312	Mean dependent var	0.406809
Adjusted R-squared	0.991312	S.D. dependent var	0.229443
S.E. of regression	0.021387	Akaike info criterion	-4.825444
Sum squared resid	0.016466	Schwarz criterion	-4.781906
Log likelihood	90.27071	Durbin-Watson stat	0.109222

Appendix 2. Unit Root Test results and the regression output:

ADF Test Statistic	-1.845245	1% Critical Value*	-3.4399
		5% Critical Value	-2.8650
		10% Critical Value	-2.5686

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(TEK)

Method: Least Squares

Sample(adjusted): 6 999

Included observations: 963

Excluded observations: 31 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
TEK(-1)	-0.009073	0.004917	-1.845245	0.0653
D(TEK(-1))	-0.037789	0.032409	-1.165991	0.2439
D(TEK(-2))	0.022148	0.032326	0.685159	0.4934
D(TEK(-3))	0.045252	0.032131	1.408356	0.1594
D(TEK(-4))	-0.026804	0.032124	-0.834412	0.4043
C	-0.005615	0.003241	-1.732292	0.0835

R-squared	0.008484	Mean dependent var	0.000183
Adjusted R-squared	0.003304	S.D. dependent var	0.024495
S.E. of regression	0.024455	Akaike info criterion	-4.577769
Sum squared resid	0.572322	Schwarz criterion	-4.547426
Log likelihood	2210.196	F-statistic	1.637783
Durbin-Watson stat	1.997377	Prob(F-statistic)	0.147277

Dependent Variable: TEK

Method: Least Squares

Sample: 1 999

Included observations: 973

Excluded observations: 26

TEK=C(1)+C(2)*TIME

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.757620	0.009768	-77.55940	0.0000
C(2)	0.000232	1.67E-05	13.83073	0.0000

R-squared	0.164580	Mean dependent var	-0.639958
Adjusted R-squared	0.163719	S.D. dependent var	0.163747
S.E. of regression	0.149744	Akaike info criterion	-0.957721
Sum squared resid	21.77310	Schwarz criterion	-0.947689
Log likelihood	467.9312	Durbin-Watson stat	0.027124

Appendix 3. Unit Root Test results and the regression output.

ADF Test Statistic	-2.922172	1% Critical Value*	-4.0113
		5% Critical Value	-3.1003
		10% Critical Value	-2.6927

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LNY)

Method: Least Squares

Sample(adjusted): 3 16

Included observations: 14 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LNY(-1)	-1.340020	0.458570	-2.922172	0.0139
D(LNY(-1))	0.174648	0.302724	0.576922	0.5756
C	28.01735	9.582192	2.923898	0.0138

R-squared	0.577508	Mean dependent var	0.017310
Adjusted R-squared	0.500691	S.D. dependent var	1.283887
S.E. of regression	0.907218	Akaike info criterion	2.830542
Sum squared resid	9.053495	Schwarz criterion	2.967483
Log likelihood	-16.81379	F-statistic	7.517984
Durbin-Watson stat	2.065442	Prob(F-statistic)	0.008750

ADF Test Statistic	-2.579003	1% Critical Value*	-4.0113
		5% Critical Value	-3.1003
		10% Critical Value	-2.6927

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LNK)

Method: Least Squares

Sample(adjusted): 3 16

Included observations: 14 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LNK(-1)	-0.997358	0.386722	-2.579003	0.0256
D(LNK(-1))	0.163368	0.289323	0.564658	0.5836
C	21.27366	8.252885	2.577724	0.0257

R-squared	0.445479	Mean dependent var	0.033733
Adjusted R-squared	0.344657	S.D. dependent var	1.671197
S.E. of regression	1.352888	Akaike info criterion	3.629770

Sum squared resid	20.13337	Schwarz criterion	3.766711
Log likelihood	-22.40839	F-statistic	4.418471
Durbin-Watson stat	1.943716	Prob(F-statistic)	0.039043

Dependent Variable: LNY

Method: Least Squares

Sample: 1 16

Included observations: 16

LNY=C(2)*LNK

	Coefficient	Std. Error	t-Statistic	Prob.
C(2)	0.977570	0.017414	56.13713	0.0000

R-squared	-2.477597	Mean dependent var	20.93775
Adjusted R-squared	-2.477597	S.D. dependent var	0.798666
S.E. of regression	1.489377	Akaike info criterion	3.695054
Sum squared resid	33.27365	Schwarz criterion	3.743341
Log likelihood	-28.56043	Durbin-Watson stat	1.774899