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Research Article

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Hydrodynamics of Single Shear between Porous Plates in an Inclined Magnetic Field

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Abstract In this paper, the motion of a two dimensional steady flow of a viscous is investigated, electrically conducting, incompressible fluid flowing between porous plates under the influence of an inclined magnetic field. The resulting coupled equation of motion was solved analytically using undetermined coefficient method subject to Beavers-Joseph boundary conditions. The analytic expression for the fluid velocity obtained is expressed in terms of Hartmann Number. The effects of different magnetic inclinations, porous parameters and Hartmann number on the velocity are discussed graphically.

Keywords magnetohydrodymamics, angle of inclination, porous plate and Beavers-Joseph conditions

1. Introduction

In some years back, several simple flow problems associated with classical hydrodynamics have gained momentum within the general context of magneto hydrodynamics (MHD). Magnetohydrodynamics is the mechanics of electrically conducting fluids. Some of these fluids include liquid metals such as mercury, molten irons and ionized gases known as plasma by physicists, an example being the solar atmosphere. The fundamental concept behind magnetohydrodynamics (MHD) is that magnetic field can induce currents in moving conductive fluid, which in turn polarizes the fluid and create forces on the fluid that changes the magnetic field. The production of these current has led to design of among other devices the MHD generators for electricity production.

The study of the flow of an electrically conducting viscous fluid between porous parallel plates under the influence of an inclined magnetic field has instant applications in numerous devices such as accelerators, magnetohydrodynamics pumps, magnetohydrodynamics power generators for electricity production. Al-Hadhrami *et al*., [1] examined control volume method (CVM) with the staggered grid system which was utilized to solve the two-dimension Brinkman equation for different shapes of porous media in a parallel channel. Ingham *et al*., [2] studied the fluid flow through few composite channels with the physical parameter that are suitable for flows in geological applications. Specifically, the fluid flow through a composite channel that has undergone a vertical fracture was considered. Ganesh *et al*., [3] examined the unsteady stokes flow of a viscous, electrically conducting and incompressible fluid between porous plates under the influence of a transverse magnetic field when the fluid was extracted through both walls of the channel at same rate. Kuiry & Bahadhr [4] investigated the magnetohydrodynamics behaviour of two dimensional poiseuille flows under the influence of an inclined magnetic field and fixed pressure gradient of an electrically conducting, viscous and incompressible fluid between two horizontal plates of which the lower plate was considered to be porous. Siddiqui *et al*., [5] investigated the magnetohydrodynamics flow of an incompressible, electrically conducting, Burger's fluid in an orthogonal rheometer. Kuiry & Bahadhr [6] examined unsteady magnetohydrodynamics Couette flow of a viscous, electrically conducting and incompressible fluid bounded by two parallel insulating porous plates under the influence of transverse uniform magnetic field and constant pressure gradient with heat transfer. Paresh &

Archana [7] studied entropy generation in radiative dissipative couette flow of an electrically conducting, incompressible, Newtonian fluid in a horizontal plate channel whose upper impermeable wall moves with constant velocity while the lower permeable wall was stationary. Seth *et al*., [8] examined unsteady hydromagnetic convective flow of an incompressible, viscous and electrically conducting heat generating or absorbing fluid with a horizontal plate rotating channel in a uniform porous medium under slip boundary conditions. Fasano *et al*., [9] reviewed a series of problems arising in the field of flows through porous media which are highly non trivial either due to presence of mass exchange between the fluid and the porous matrix. Uddin [10] examined chemically reactive solute transfer over a plate in porous medium in the presence of suction. Punnamchandar & Iyengar [11] examined the pulsating flow of slightly conducting, and incompressible micropolar fluid between two homogeneous permeable beds in an inclined uniform magnetic field.

2. Formulation of the Problem

An electrically conducting fluid moving with velocity (V) perpendicular to this flow is considered. The strength of an applied magnetic field is represented by the vector B. We shall assume that the fluid flow variables are independent of the time t. This condition is majorly for analytic reasons so that no macroscopic charge density is being built up at any place in the system and all currents are fixed in time. An electric field vector denoted E is induced perpendicularly to both V and B, because of the interaction of velocity and magnetic field. This electric field is given by

$$E = V \times B \tag{1}$$

Where $\times$ stands for cross product of the two vectors V and B. If we assume that the conducting fluid is isotopic in spite of the magnetic field, we can denote the electrical conductivity of the fluid by a scalar σ. According to Ohm's law, the density of the current induced in the conducting fluid denoted by J is given by

$$J = \sigma (V \times B) \tag{2}$$

Simultaneously occurring with the induced current is the Lorentz force F given by

$$F = J \times B \tag{3}$$

Consider an incompressible, viscous, steady and electrically conducting fluid flowing between two infinite parallel plates both kept at a fixed distance $2h$ between them. Both plates of the channel are fixed with no motion. This is plane hydromagnetic fluid flow driven by a constant pressure gradient. The equations of motion are the continuity equation.

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \tag{4}$$

and the Navier – stokes equations

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = \frac{1}{\rho} f_{Bx} - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + \nu\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) \tag{5}$$

$$\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = \frac{1}{\rho} f_{By} - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{y}} + \nu\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) \tag{6}$$

Where ρ is the fluid density, $f_{Bx}, f_{By}, \bar{u}, \bar{v}$, are the components of the body force per unit mass of the fluid and the velocity in $\bar{x}$ and $\bar{y}$ directions respectively, μ is the fluid viscosity and p is the pressure acting on the fluid. The flow is practically horizontal, in which the axis of the channel formed by the two plates is considered as the $\bar{x}$-axis and assumed that flow is in this direction.

Since $\bar{u}$ is a function of $\bar{y}$ alone and $\bar{v} = 0$, it implies that continuity equation collapses to

$$\frac{\partial \bar{u}}{\partial \bar{x}} = 0$$

Hence equations (5) and (6) become;

$$0 = \frac{1}{\rho} f_{Bx} - \frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{x}} + \frac{\mu}{\rho}\frac{\partial^2 \overline{u}}{\partial \overline{y}^2} \tag{7}$$

$$0 = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{y}} \tag{8}$$

Combining (7) and (8) gives

$$0 = \frac{1}{\rho} f_{Bx} - \frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{x}} + \frac{\mu}{\rho}\frac{\partial^2 \overline{u}}{\partial \overline{y}^2} \tag{9}$$

The $\overline{x}$ -component of the Lorentz force in equation (9) can be expressed as follow;

$$\frac{1}{\rho} f_{Bx} = \frac{\sigma}{\rho}\left[\left(\overline{u}i \times jB_0\right) \times jB_0\right] = -\frac{\sigma}{\rho} B_0^2 \overline{u} \tag{10}$$

Where B_0 is the magnetic field strength component assumed to be applied to a direction perpendicular to fluid motion. Equation (10) is achieved by using the expressions of force and induced current in equations (2) and (3) together with the fact that for any three vectors A, B and C it can be shown that ($A \times B$) $\times C$ = ($A \cdot B$) C – ($B \cdot C$) A. In equation (10) i and j refer to unit rectangular vector i and j hence equation (9) becomes

$$\frac{d^2\overline{u}}{d\overline{y}^2} - \frac{\sigma B_0^2 \overline{u}}{\mu} = -\frac{1}{\mu}\frac{d\overline{p}}{d\overline{x}} \tag{11}$$

Now, we want to examine the effect of magnetic inclination of the magnetic field on the velocity of the fluid flow. Angle of inclination is introduced to the second term in equation (11) as follows;

$$\frac{d^2\overline{u}}{d\overline{y}^2} - \frac{\sigma B_0^2 \sin\theta \overline{u}}{\mu} = \frac{1}{\mu}\frac{d\overline{p}}{d\overline{x}} \tag{12}$$

Where θ is the angle between V and B. in equation (12), it is assume that the two fields are inclined to each other at an angle θ lying in the range $0 < \theta < \frac{\pi}{2}$ and the equation is solved subject to Beavers- Joseph boundary conditions when $y = \pm h$. If the characteristics length of the flow is small the magnetic Reynolds number cannot exceed unity unless the flow is turbulent.

3. Solution to the Problem

Introduce the following non-dimensional quantities

$$x = \frac{\overline{x}}{h},\ y = \frac{\overline{y}}{h},\ p = \frac{\overline{p}h^2}{\rho v^2},\ u = \frac{\overline{u}h}{v} \tag{13}$$

Using these quantities in equation (12) gives

$$\frac{d^2u}{dy^2} - \frac{\sigma}{\mu} B_0^2 h^2 \sin^2\theta u = \frac{dp}{dx} \tag{14}$$

Where $M = M^* \sin\theta,\ M^* = hB_0\sqrt{\frac{\sigma}{\mu}} = Ha$. Ha is the Hartman number defined by $(Ha)^2 = \frac{B_0^2 h^2 \sigma}{\mu}$, in order to eliminate the pressure term, differentiating equation (14) gives;

$$\frac{d^3u}{dy^3} - M^2\frac{du}{dy} = 0 \tag{15}$$

Using these conditions (16) to (18) in solving (15) gives (19)

$$\frac{du}{dy} = -\alpha\sigma(u_{B1} - Q_1), \quad at \quad y = 1, \quad \frac{du}{dy} = \alpha\sigma(u_{B2} - Q_1) \quad at \quad y = -1 \tag{16}$$

Where $\sigma = \frac{h}{\sqrt{k}}$ is a non-dimensional number

$$u_{B1} = u_B \quad at \quad y = 1, \quad u_{B2} = u_B \quad at \quad y = -1 \tag{17}$$

$$\int_{-1}^{1} u(y)dy = \frac{n_f}{h} \tag{18}$$

$$u(y) = \frac{n_f}{2h} + \frac{\alpha\sigma}{2h}\left(\frac{(n_f - 2hQ_1)\chi}{(M^2 - \alpha\sigma)\sinh M + \alpha\sigma M\cosh M}\right) \tag{19}$$

Where $n_f = \frac{2h^3}{\sigma M^2}\left(\frac{\psi_1 + \psi_2}{M^2 \sinh M + \alpha\sigma M \cosh My}\right)\left(-\frac{1}{\mu}\frac{\partial p}{\partial x}\right)$

$\chi = \sinh M - M\cosh My,\ \psi_1 = (\alpha M^2 + \sigma M^2 + \alpha\sigma^2)\sinh M,\ \psi_2 = \alpha\sigma^2 M\cosh M$

4. Results and Discussion

The present discussion analyzed the effects of magnetic inclinations (θ), porous parameter ($\alpha\sigma$) and Hartmann number (M) on the flow velocity. Fig.1 shows the velocity profile for some selected magnetic angles of inclination $\theta = 15^\circ, 45^\circ\ and\ 90^\circ$ .It was observed that when $\theta = 15^\circ$ the velocity profile is minimum at the center of the channel, it decreases when $\theta = 45^\circ\ and\ 90^\circ$ respectively. Fig. 2 and fig. 3 show the velocity profile for different porous parameter and Hartmann number for some selected $\theta = 15^\circ, 45^\circ\ and\ 90^\circ$.The velocity profile is minimum at the center of the channel, it decreases as both porous parameter and Hartmann number were increased.Fig.4 shows the velocity profile for different magnetic angles of inclination $\theta = 30^\circ, 60^\circ\ and\ 75^\circ$.It was observed that when $\theta = 30^\circ$ the velocity profile is maximum at the center of the channel, it decreases when $\theta = 60^\circ\ and\ 75^\circ$. Fig.5 and fig.6 shows the velocity profile for different porous parameter and Hartmann number for some selected $\theta = 30^\circ, 60^\circ\ and\ 75^\circ$ .The velocity profile at the center of the channel is maximum and increases as both porous parameter and Hartmann number were increased.

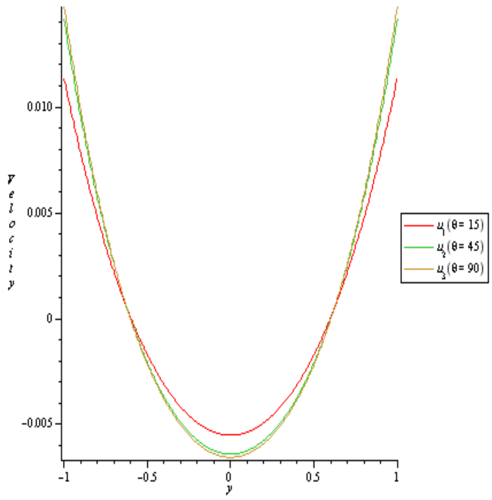

Figure 1: Velocity profile for some selected maganetic angles of inclination when M=2*

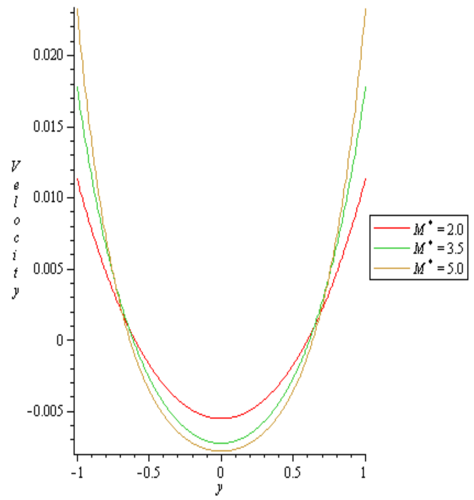

Figure 2: Velocity profile for different values of M when M=M* sin15*

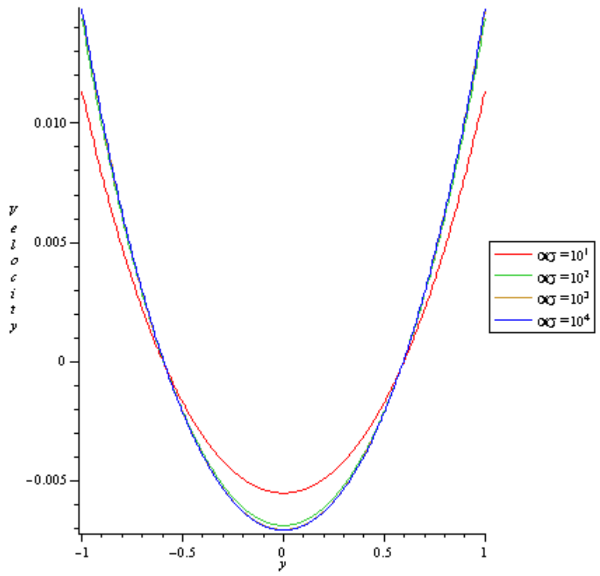

Figure 3: Velocity Profile for different porous parameters when M=2sin15

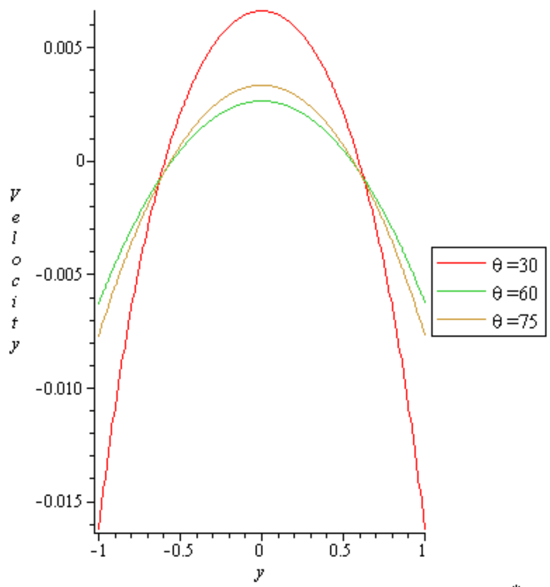

Figure 4: Velocity Profile for some selected maganetic angles of when M=2*

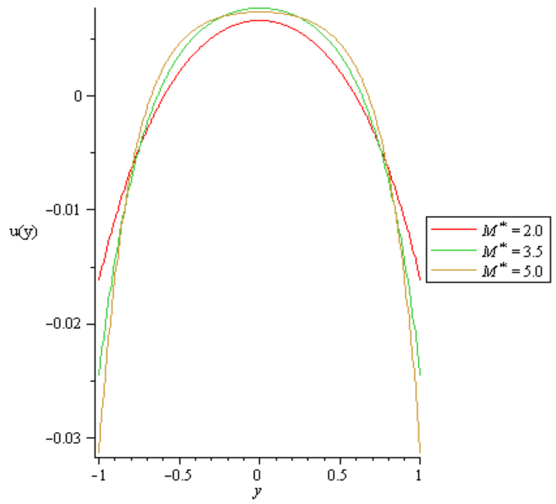

*Figure 5: Plot of u(y) against y when M=M*sing30 for different values of M**

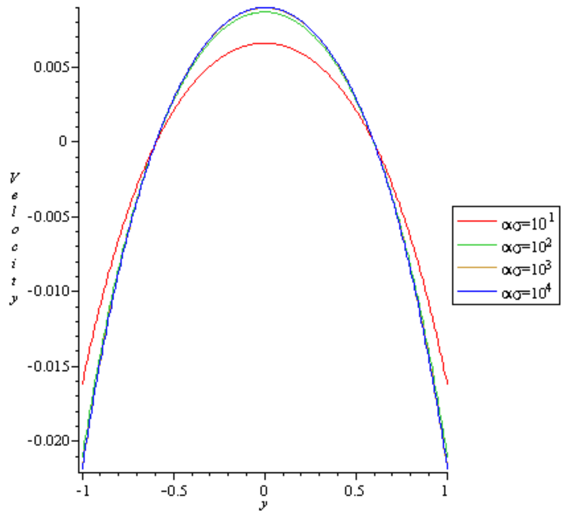

Figure 6: Velocity Profile for different porous parameters when M=2sin30

Conclusion

This work investigates the effect of porous parameter, Hartmann number and magnetic angles of inclination on the steady hydrodynamics of single shear between porous plates in an inclined magnetic field. Field Engineers should be careful with their choice of magnetic angle of inclination because it plays an important role in production of MHD pumps or generators for minimum or maximum performance.

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