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Analysis of Covariance of Sudoku Square Design Models

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Abstract Sudoku square designs have been studied under the analysis of variance, but no literature was traced to the Analysis of Covariance of Sudoku square design. This paper proposes inclusion of concomitant variables into existing Sudoku square design models. However, the least square method was used for the derivation of sum of squares and cross products for the various effects of the proposed models as well as analysis of covariance of Sudoku square design.

Keywords Sudoku Square, Least Squares, Concomitant variables, Covariance

1. Introduction

Sudoku puzzle is a very popular game, with the objective of filling the 9×9 grid with digits from 1 to 9. Each digit must appear once only in each row, each column and each of the 3×3 boxes [1].

Sudoku puzzle was considered as a designed experiment having k treatments and k replications [2]. Detail of description on how to design and randomizing a Sudoku Square are presented in Hui-Dong and Ru-Gen [2]. However, the Sudoku design presented by Hui-Dong and Ru-Gen [2] does not contain row-blocks or column-blocks effects. Subramani and Ponnuswamy [3] extended the design to include row-blocks and column-blocks effects in which they called Sudoku designs-Type I. Methods of constructing the Sudoku design and analysis are discussed [2-4].

Donovan, Haaland and Nott [6] and Kumar, Varghese and Jaggi [7] studied the Sudoku based space filling designs. Recently, Danbaba [5] proposed combined analysis of multi-environment experiments conducted via Sudoku square designs of odd order where the treatments are the same to the whole set of experiments in the analysis, ANOVA was used. Danbaba [4] proposed a simple method of constructing Samurai Sudoku designs and orthogonal (Graeco) Samurai Sudoku design. He also discussed linear models and methods of data analysis for these designs. The linear models and methods of data analysis for all the Sudoku designs considered so far are on univariate analysis of variance. Subramani [8] discussed the orthogonal Sudoku square. Danbaba [5] used ANOVA to carry out the combined analysis of Sudoku square designs with same treatments.

The analysis of covariance is a typically used to adjust or control for the difference between the groups based on another, typically interval level variable called the covariate. Fisher [9] stated that ANCOVA is an extension of ANOVA that majorly provides a way of statistically controlling for the effects of continuous or scale variables that one concerned about but that not independent variables in the study. The major role ANCOVA played in this research is that, that would reduce the probability of a type II error when tests are made of main effects. Since the probability of a Type II error is inversely related to statistical power, the ANCOVA is considered more powerful than its ANOVA method presuming that other things are held constant and that a good covariate has been used within the ANCOVA Cochran [10]. Though some authors like [11-12] have used ANCOVA method to control excessive error variance incurred by the used analysis of variance which actually improve the precision of the results.

All these authors mentioned used analysis of variance (ANOVA) to carry out their analyses on Sudoku square design and the use of ANCOVA is still missing, in view of this, this paper proposes inclusion of concomitant variable into each model of the four univariate Sudoku square design models given by Subramani and Ponnuswamy [3]. The derivation of sum of squares and cross products would be carried out as well as analysis of covariance for each proposed models would be given.

2. Proposed Models

ANCOVA Sudoku Square Model I

We will assume that row, column, and treatments effect as in the latin square, also in addition to the assumption, Row block, column block and square effects.

$$Y_{ij\,(k,l,p,q)} = \mu + \alpha_i + \beta_j + \tau_k + C_p + \gamma_l + s_q + \beta^*\left(x_{ij\,(k,l,p,q)} - \bar{x}_{\ldots}\right) + e_{i,j\,(k,l,p,q)} \quad (1)$$

where $i, j = 1 \cdots m \quad k, l, p, q = 1 \cdots m^2$

ANCOVA Sudoku Square Model II

The model assumed that row effects are nested in the row block effect and the column effects are nested in the column block effects.

$$Y_{ij\,(k,l,p,q)} = \mu + \alpha_i + \beta_j + \tau_k + \gamma(\alpha)_{l(i)} + C(\beta)_{p(j)} + s_q + \beta^*\left(x_{ij\,(\;)} - \bar{x}_{\ldots}\right) + e_{i,j\,(k,l,p,q)} \quad (2)$$

where $i, j\; l, p = 1 \cdots m \quad k, q = 1 \cdots m^2$

ANCOVA Sudoku Square Model III

The model assumes that the horizontal square effects are nested in the row block and vertical Square effects are nested in the column block effects.

$$Y_{ij\,(k,l,p,q,r)} = \mu + \alpha_i + \beta_j + \tau_k + \gamma_l + c_p + s(\alpha)_{q(i)} + \pi(\beta)_{r(j)} + \beta^*\left(x_{ij\,(\;)} - \bar{x}_{\ldots}\right) + e_{i,j\,(k,l,p,q,r)} \quad (3)$$

where $i, j, q, r = 1 \cdots m \quad k, l, p = 1 \cdots m^2$

ANCOVA Sudoku Square Model IV

In the model below, it is assumed that the row effects and horizontal square effects are nested in the row block and the column effects and the vertical square effects are nested in the column block effects

$$Y_{ij\,(k,l,p,q,r)} = \mu + \alpha_i + \beta_j + \tau_k + \gamma(\alpha)_{l(i)} + c(\beta)_{p(j)} + s(\alpha)_{q(i)} + \pi(\beta)_{r(j)} + \beta^*\left(x_{ij\,(\;)} - \bar{x}_{\ldots}\right) + e_{i,j\,(k,l,p,q,r)} \quad (4)$$

where $i, j, l, p, q, r = 1 \cdots m \;\; and \;\; k = 1 \cdots m^2$

$\mu =$ general mean

$\alpha_i = ith$ Row block effect

$\beta_j = jth$ Column block effect

$\tau_k = kth$ treatment effect

$s_q =$qth square effect

$\gamma(\alpha)_{l(i)} = l$th Row effect nested in ith row block effect

$c(\beta)_{p(j)} = p$th Column effect nested in jth column block effect

$s(\alpha)_{q(i)} = qth$ Horizontal square effect nested in ith Row block effect

$\pi(\beta)_{r(j)} = rth$ vertical square effect nested in the jth column block effect

$\beta^* =$ is the slope of the regression

$e_{i,j\,(k,l,p,q,r)}$ = is the error component assumed to have mean zero and constant variance σ^2.

$x_{ij\,(k,l,p,q,r)}$ is the concomitant variable

$\bar{x}_{\ldots}$ is the overall mean of the concomitant variable

In the study, we assume that e_{ijk} is normally identically, distributed with zero mean and constant variance σ^2, we also assume that there is linear relationship between y_{ijk} and x_{ijk} and regression coefficient for each treatment are identical, the treatment effect sum to zero and the concomitant variable is not affected by the treatment.

3. Method of Estimating Parameters of ANCOVA models

The parameters in the model would be estimated with the method of Principle of least square estimate, the method make use of second order polynomial of residual effect.i.e,

$$S({\varepsilon_{ijk}}^2) = \sum(\mu + \alpha_i + \beta_j + \tau_k + C_p + \gamma_l + s_q + \beta^*(x_{ij\,(k,l,p,q)} - \bar{x}_{...}) + e_{i,j\,(k,l,p,q)})^2 \quad (5)$$

This equation may be refered to score function, to estimate the parameter of the model, we differentiate the score function with respect to each of the parameters and setting the differentials to zero, the estimated values would be called least square estimators. From the estimators, we obtain sum of squares error, total sum of square, treatment square and sum of square regression and will be denoted by E, T and S respectively Error sum of square of x, and sum of square of y and error sum of product of x and y will be estimated and denoted by $E_{xx} E_{yy}$ and E_{xy} respectively.

Total sum of square of x, and sum of square of y and error sum of product of x and y will be estimated and denoted by $S_{xx} S_{yy}$ and S_{xy} respectively. Sum of square treatment of x, and sum of square of y and error sum of product of x and y will be estimated and denoted by $T_{xx} T_{yy}$ and T_{xy} respectively.

The three quantities could be connected by this relation as $E = S - T$, the same is translated to the components of the quantities E, T and S i.e xx, yy, xy.

The assumption on model equation (1) be made that no treatment effect, then least square estimators are obtained for the reduced model, the error sum of square are equally obtained. The difference between the error sum of square in a full model (SS_f) and error sum of square in reduced SS_r) model, therefore this differences provides a sum of squares with a degree of freedom corresponding to treatment effect for testing the hypothesis of no treatment effect. In the end we test the null hypothesis $H_0: \tau_0 = 0$ and compute

$$F_0 = \frac{SS_r - SS_f}{degree\ \ of\ freedom\ \ of\ difference} / (\frac{SS_f}{degree\ \ of\ freedom\ \ of\ full\ model}) \quad (6)$$

If $F_0 > F_{\alpha, degree\ of\ freedom\ of\ difference\ , degree\ of\ freedom\ of\ full\ model}$ we reject the null Hypothesis.

Finally, we test the hypothesis of no regression coefficient and compute the statistics

$$F_0 = ((E_{xy})^2 / E_{xy}) / MS \quad (7)$$

The same procedure is repeated for the estimation of the parameters of the proposed models Equations (2 -4).

4. Result

4.1 Derivation

The section shows the derivation of sum of squares and products of the proposed ANCOVA Sudoku square design models and respective tables of analysis of covariance are given.

Ancova Sudoku Square Design Model I

The score function of the equation (1)is given in (5)

$$S(e^2) = \sum_{i=1}^{m^2} \sum_{j=1}^{m^2} \left[Y_{ij\,(k,l,p,q)} - \mu - \alpha_i - \beta_j - \tau_k - C_p - \gamma_l - s_q - \beta^*(x_{ij\,(k,l,p,q)} - \bar{x}_{...})\right]^2 \quad (8)$$

We obtain the derivatives of (8) under the constraints

$$\left.\begin{array}{r} \sum_i \alpha_i = 0, \\ \sum_j \beta_j = 0, \\ \sum_k \tau_k = 0, \\ \sum_p C_p = 0, \\ \sum_q s_q = 0, \\ \sum_l \gamma_l = 0 \\ \sum_{i=1}^{m^2} \sum_{j=1}^{m^2} (x_{ij\,(k,l,p,q)} - \bar{x}_{...}) = 0 \end{array}\right\} \quad (9)$$

$$\frac{\partial(S(e^2))}{\partial\mu} = 0$$

$$\sum_{i=1}^{m^2} \sum_{j=1}^{m^2} \left[Y_{ij\,(k,l,p,q)} - \mu - \alpha_i - \beta_j - \tau_k - C_p - \gamma_l - s_q - \beta^*(x_{ij\,(k,l,p,q)} - \bar{x}_{...})\right]$$

$$\sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{ij(k,l,p,q)} - \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}\mu - \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}\alpha_i - \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}\beta_j - \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}\tau_k - \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}\gamma_l - \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}C_p - \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}s_q - \beta^*\sum_{i=1}^{m^2}\sum_{j=1}^{m^2}(x_{ij(k,l,p,q)} - \bar{x}_{...}) = 0$$

Applying the contraint (9) to the above derivative we have,

$$Y_{...} - m^4\hat{\mu} = 0$$

$$\hat{\mu} = \frac{Y_{...}}{m^4} = \bar{y}_{...} \quad (10)$$

$$\hat{\alpha}_i = \frac{y_{i..}}{m^3} - \hat{\mu} - \hat{\beta}^*(\bar{x}_{i..} - \bar{x}_{...})$$

$$\hat{\alpha}_i = \bar{y}_{i...} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{i..} - \bar{x}_{...}) \quad (11)$$

$$\hat{\beta}_j = \bar{y}_{.j(.)} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{.j.} - \bar{x}_{...}) \quad (12)$$

$$\hat{\tau}_k = \bar{y}_{..(k)} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{k..} - \bar{x}_{...}) \quad (13)$$

$$\hat{\gamma}_l = \bar{y}_{l..} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{l..} - \bar{x}_{...}) \quad (14)$$

$$\hat{C}_p = \bar{y}_{p..} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{p..} - \bar{x}_{...}) . \quad (15)$$

$$\hat{s}_q = \bar{y}_{q...} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{q..} - \bar{x}_{...}) \quad (16)$$

$\frac{\partial(S(e^2))}{\partial\hat{\beta}^*} = 0$ yeilds,

$$\sum_{i=1}^{m^2}\sum_{j=1}^{m^2}\left[Y_{ij(k,l,p,q)} - \hat{\mu} - \hat{\alpha}_i - \hat{\beta}_j - \hat{\tau}_k - \hat{C}_p - \hat{\gamma}_l - \hat{s}_q - \hat{\beta}^*(x_{ij()} - \bar{x}_{...})\right]\left[x_{ij(\ )} - \bar{x}_{...}\right] = 0 \quad (17)$$

Subtituting equtations (10– 16) into the above derivative of $\hat{\beta}^*$(17)

$$\sum_{i=1}^{m^2}\sum_{j=1}^{m^2}(x_{ij(\ )} - \bar{x}_{...})Y_{ij(k,l,p,q)} - \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}(x_{ij(\ )} - \bar{x}_{...})\bar{y}_{...}$$
$$- \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}(x_{ij(\ )} - \bar{x}_{...})(\bar{y}_{i...} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{i..} - \bar{x}_{...}))$$
$$- \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}(x_{ij(\ )} - \bar{x}_{...})(\bar{y}_{.j(.)} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{.j.} - \bar{x}_{...}))$$
$$- \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}(x_{ij(\ )} - \bar{x}_{...})(\bar{y}_{..(k)} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{k..} - \bar{x}_{...}))$$
$$- \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}(x_{ij(\ )} - \bar{x}_{...})(\bar{y}_{l..} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{l..} - \bar{x}_{...}))$$
$$- \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}(x_{ij(\ )} - \bar{x}_{...})(\bar{y}_{p..} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{p..} - \bar{x}_{...}))$$
$$- \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}(x_{ij(\ )} - \bar{x}_{...})(\bar{y}_{q...} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{q..} - \bar{x}_{...}))$$
$$-\hat{\beta}^*\sum_{i=1}^{m^2}\sum_{j=1}^{m^2}(x_{ij(\ )} - \bar{x}_{...})^2 = 0$$

$$S_{xy} - (T_{xy}^i + T_{xy}^j + T_{xy}^k + T_{xy}^l + T_{xy}^p + T_{xy}^q) = \hat{\beta}^*(S_{xx} - (T_{xx}^i + T_{xx}^j + T_{xx}^k + T_{xx}^l + T_{xx}^p + T_{xx}^q)$$

If we let $T_{xy} = (T_{xy}^i + T_{xy}^j + T_{xy}^k + T_{xy}^l + T_{xy}^p + T_{xy}^q)$

$T_{xx} = \left(T_{xx}^{i} + T_{xx}^{j} + T_{xx}^{k} + T_{xx}^{l} + T_{xx}^{p} + T_{xx}^{q}\right)$

$S_{xy} - T_{xy} = \hat{\beta}^* (S_{xx} - T_{xx})$

$$\hat{\beta}^* = \frac{S_{xy} - T_{xy}}{S_{xx} - T_{xx}} = \frac{E_{xy}}{E_{xx}}. \quad (18)$$

Where the notations represent the following

$$S_{xy} = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}(x_{ij\,(\,)} - \bar{x}_{...})(Y_{ij\,(k,l,p,q)} - \bar{y}_{...}) \quad (19)$$

$$S_{xx} = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}\left(x_{ij\,(\,)} - \bar{x}_{...}\right)^2 \quad (20)$$

$$T_{xy}^{i} = m^3 \sum_{i=1}^{m}(\bar{x}_{i.(.)} - \bar{x}_{...})(\bar{y}_{i...} - \bar{y}_{...}) \quad (21)$$

$$T_{xx}^{i} = m^3 \sum_{i=1}^{m}(\bar{x}_{i.(.)} - \bar{x}_{...})^2 \quad (22)$$

$$T_{xy}^{j} = m^3 \sum_{j=1}^{m}(\bar{x}_{.j\,(.)} - \bar{x}_{...})(\bar{y}_{.j\,(.)} - \bar{y}_{...}) \quad (23)$$

$$T_{xx}^{j} = m^3 \sum_{j=1}^{m}(\bar{x}_{.j\,(.)} - \bar{x}_{...})^2 \quad (24)$$

$$T_{xy}^{k} = m^2 \sum_{k=1}^{m^2}(\bar{x}_{..(k)} - \bar{x}_{...})(\bar{y}_{..(k)} - \bar{y}_{...}) \quad (25)$$

$$T_{xx}^{k} = m^2 \sum_{k=1}^{m^2}(\bar{x}_{..(k)} - \bar{x}_{...})^2 \quad (26)$$

$$T_{xy}^{l} = m^2 \sum_{l=1}^{m^2}(\bar{x}_{..(l)} - \bar{x}_{...})(\bar{y}_{..(l)} - \bar{y}_{...}) \quad (27)$$

$$T_{xx}^{l} = m^2 \sum_{l=1}^{m^2}(\bar{x}_{..(l)} - \bar{x}_{...})^2 \quad (28)$$

$$T_{xy}^{p} = m^2 \sum_{p=1}^{m^2}(\bar{x}_{..(p)} - \bar{x}_{...})(\bar{y}_{..(p)} - \bar{y}_{...}) \quad (29)$$

$$T_{xx}^{p} = m^2 \sum_{p=1}^{m^2}(\bar{x}_{..(p)} - \bar{x}_{...})^2 \quad (30)$$

$$T_{xy}^{q} = m^2 \sum_{q=1}^{m^2}(\bar{x}_{..(q)} - \bar{x}_{...})(\bar{y}_{..(q)} - \bar{y}_{...}) \quad (31)$$

$$T_{xx}^{q} = m^2 \sum_{q=1}^{m^2}(\bar{x}_{..(q)} - \bar{x}_{...})^2 \quad (32)$$

We obtain as follows

Note that $S = T + E$ where the symbols S,T and E are used to denote sums of squares and cross-product for total, treatments and error, respectively. The sums of squares for x and y must be non negative; however, the sums of cross-products (xy) may be negative.

Using the solution to the normal equation, we may find the estimated or fittted values of $y_{ij\,(k,l,p,q)}$ as

$$\hat{y}_{i,j\,(k,l,p,q)} = \hat{\mu} + \hat{\alpha}_i + \hat{\beta}_j + \hat{\tau}_k + \hat{\gamma}_l + \hat{c}_p + \hat{s}_q + \hat{\beta}^*\left(x_{ij\,(k,l,p,q)} - \bar{x}_{...}\right) \quad (33)$$

Subtituting equations (8– 16)into (7) we have

$$\begin{aligned}\hat{y}_{i,j\,(k,l,p,q)} = {} & \bar{y}_{...} + \left[\bar{y}_{i...} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{i..} - \bar{x}_{...})\right] + \left[\bar{y}_{.j\,(.)} - \bar{y}_{...} - \hat{\beta}^*\left(\bar{x}_{.j.} - \bar{x}_{...}\right)\right] + \\ & \left[\bar{y}_{..(k)} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{k..} - \bar{x}_{...})\right] + \left[\bar{y}_{l..} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{l..} - \bar{x}_{...})\right] + \\ & \left[\bar{y}_{p..} - \bar{y}_{...} - \hat{\beta}^*\left(\bar{x}_{p..} - \bar{x}_{...}\right)\right] + \left[\bar{y}_{q...} - \bar{y}_{...} - \hat{\beta}^*\left(\bar{x}_{q..} - \bar{x}_{...}\right)\right] \\ & + \left[\hat{\beta}^*\left(x_{ij\,(k,l,p,q)} - \bar{x}_{...}\right)\right]\end{aligned}$$

$$\begin{aligned}\hat{y}_{i,j\,(k,l,p,q)} = {} & \bar{y}_{i...} + \bar{y}_{.j\,(.)} + \bar{y}_{..(k)} + \bar{y}_{l..} + \bar{y}_{p..} + \bar{y}_{q...} - 5\bar{y}_{...} \\ & - \frac{E_{xy}}{E_{xx}}\left[x_{ij\,(k,l,p,q)} + \bar{x}_{i..} + \bar{x}_{.j.} + \bar{x}_{k..} + \bar{x}_{l..} + \bar{x}_{p..} + \bar{x}_{q..} - 7\bar{x}_{...}\right]\end{aligned} \quad (34)$$

The estimated residual $\hat{e}_{i,j\,(k,l,p,q)} = y_{ij\,(k,l,p,q)} - \hat{y}_{i,j\,(k,l,p,q)}$. (35)

We may express the reduction in the total sum of squares due to fitting the model equation (1) as

$$\begin{aligned}R(\mu,\tau,\beta,\gamma,s,\hat{\beta}^*) \\ = {} & \hat{\mu}y_{...} + \sum_{i=1}^{m}\hat{\alpha}_i y_{i..} + \sum_{j=1}^{m}\hat{\beta}_j y_{.j.} + \sum_{k=1}^{m^2}\hat{\tau}_k y_{..k} + \sum_{l=1}^{m^2}\hat{\gamma}_l y_{...i} + \sum_{p=1}^{m^2}\hat{c}_p y_{...p} + \sum_{q=1}^{m^2}\hat{s}_q y_{..q} \\ & + \hat{\beta}^*\left(x_{ij\,(k,l,p,q)} - \bar{x}_{...}\right)\left(y_{ij\,(k,l,p,q)} - \bar{y}_{...}\right)\end{aligned}$$

Subtitute the estimators of $\hat{\mu}, \hat{\alpha}, \hat{\beta}_j, \hat{\tau}_k, \hat{\gamma}_l, \hat{c}_p, \hat{s}_q, \hat{\beta}^*$

$$= \bar{y}_{...}y_{...} + \sum_{i=1}^{m}(\bar{y}_{i...} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{i..} - \bar{x}_{...}))y_{i..} + \sum_{j=1}^{m}\left(\bar{y}_{.j\,()} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{.j.} - \bar{x}_{...})\right)y_{.j.}$$

$$+ \sum_{k=1}^{m^2}\left(\bar{y}_{..(k)} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{k..} - \bar{x}_{...})\right)y_{..k} + \sum_{l=1}^{m^2}(\bar{y}_{l..} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{l..} - \bar{x}_{...}))y_{l...}$$

$$+ \sum_{p=1}^{m^2}\left(\bar{y}_{p..} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{p..} - \bar{x}_{...})\right)y_{...p} + \sum_{q=1}^{m^2}(\bar{y}_{q...} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{q..} - \bar{x}_{...}))y_{..q} + \hat{\beta}^* S_{xy}$$

$$= \frac{Y_{....}^2}{m^4} + \sum_{i=1}^{m}(\bar{y}_{i..} - \bar{y}_{...})y_{i..} + \sum_{i=1}^{m}(\bar{y}_{.j.} - \bar{y}_{...})y_{.j.} + \sum_{k=1}^{m^2}(\bar{y}_{..k} - \bar{y}_{...})y_{..k} + \sum_{l=1}^{m^2}(\bar{y}_{l..} - \bar{y}_{...})y_{l...}$$

$$+ \sum_{p=1}^{m^2}\left(\bar{y}_{p..} - \bar{y}_{...}\right)y_{...p} + \sum_{q=1}^{m^2}\left(\bar{y}_{..q} - \bar{y}_{...}\right)y_{..q}$$

$$+ \hat{\beta}^*\Bigg[S_{xy} - \sum_{i=1}^{m}(\bar{x}_{i..} - \bar{x}_{...})y_{i..} - \sum_{i=1}^{m}(\bar{x}_{.j.} - \bar{x}_{...})y_{.j.} - \sum_{k=1}^{m^2}(\bar{x}_{k..} - \bar{x}_{...})y_{..k}$$

$$- \sum_{l=1}^{m^2}(\bar{x}_{l..} - \bar{x}_{...})y_{l...} - \sum_{p=1}^{m^2}(\bar{x}_{p..} - \bar{x}_{...})y_{...p} - \sum_{q=1}^{m^2}(\bar{x}_{q..} - \bar{x}_{...})y_{..q}\Bigg]$$

$$= \frac{Y_{....}^2}{m^4} + \left(T_{yy}^i + T_{yy}^j + T_{yy}^k + T_{yy}^l + T_{yy}^p + T_{yy}^q\right) + \hat{\beta}^*\left(S_{xy} - T_{xy}^i - T_{xy}^j - T_{xy}^k - T_{xy}^l - T_{xy}^p - T_{xy}^q\right)$$

We replace $T_{yy} = T_{yy}^i + T_{yy}^j + T_{yy}^k + T_{yy}^l + T_{yy}^p + T_{yy}^q$

$$T_{xy} = T_{xy}^i + T_{xy}^j + T_{xy}^k + T_{xy}^l + T_{xy}^p + T_{xy}^q$$

$$= \frac{Y_{....}^2}{m^4} + T_{yy} + \hat{\beta}^*(S_{xy} - T_{xy})$$

But $\left(S_{xy} - T_{xy}\right) = E_{xy}$ and $\hat{\beta}^* = E_{xy}/E_{xx}$

$$= \frac{Y_{....}^2}{m^4} + T_{yy} + \frac{E_{xy}^2}{E_{xx}} \tag{36}$$

The error sum of square for thereduced model can be given as

$$SSE = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} y_{i,jk,l,p,q)}^2 - R\left(\mu, \tau, \beta, \gamma, c, s, \hat{\beta}^*\right)$$

$$= \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} y_{i,jk,l,p,q)}^2 - \frac{Y_{....}^2}{m^4} - T_{yy} - \frac{E_{xy}^2}{E_{xx}}$$

$$= S_{yy} - T_{yy} - \frac{E_{xy}^2}{E_{xx}}$$

$$S_{yy} = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} y_{i,jk,l,p,q)}^2 - \frac{Y_{....}^2}{m^4}$$

$$= E_{yy} - \frac{E_{xy}^2}{E_{xx}} \quad . \tag{37}$$

With $m^2(m^2 - 1) - 1$ degree of freedom. The experimental error variance is estimated by

$$MSE = \frac{SSE}{m^2(m^2-1)-1} \tag{38}$$

Now consider the model restricted to the null hypothesis that there is no treatment effect $H_0: \tau_1 = \tau_2 = \cdots \tau_k = 0$ The reduced model would then be

$$Y_{ij\,(k,l,p,q)} = \mu + \alpha_i + \beta_j + C_p + \gamma_l + s_q + \beta_1^*\left(x_{ij\,(k,l,p,q)} - \bar{x}_{...}\right) + e_{i,j\,(k,l,p,q)} \quad (39)$$

The score function of the model is

$$S(e^2)^* = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}\left[Y_{ij\,(k,l,p,q)} - \mu - \alpha_i - \beta_j - C_p - \gamma_l - s_q - \hat{\beta}^*\left(x_{ij\,(k,l,p,q)} - \bar{x}_{...}\right)\right]^2 \quad (40)$$

Differentiating equation (39) with respect to each of the parameters in the model and equating to zero.

Applying the contraint (9) to the above derivative we have,

$$\hat{\mu} = \frac{Y_{...}}{m^4} = \bar{y}_{...} \quad (41)$$

$$\hat{\alpha}_i = \bar{y}_{i...} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{i..} - \bar{x}_{...}) \quad (42)$$

$$\hat{\beta}_j = \bar{y}_{j\,(.)} - \bar{y}_{...} - \hat{\beta}^*\left(\bar{x}_{.j.} - \bar{x}_{...}\right) \quad (43)$$

Similarly,

$$\hat{\gamma}_l = \bar{y}_{l..} - \bar{y}_{...} - \hat{\beta}^*(\bar{x}_{l..} - \bar{x}_{...}) \quad (44)$$

$$\hat{C}_p = \bar{y}_{p..} - \bar{y}_{...} - \hat{\beta}^*\left(\bar{x}_{p..} - \bar{x}_{...}\right) \quad (45)$$

$$\hat{s}_q = \bar{y}_{q...} - \bar{y}_{...} - \hat{\beta}^*\left(\bar{x}_{q..} - \bar{x}_{...}\right) \quad . \quad (46)$$

$\frac{\partial(S(e^2)}{\partial\hat{\beta}^*} = 0$ yeilds,

$$\sum_{i=1}^{m^2}\sum_{j=1}^{m^2}\left[Y_{ij\,(k,l,p,q)} - \hat{\mu} - \hat{\alpha}_i - \hat{\beta}_j - \hat{\tau}_k - \hat{C}_p - \hat{\gamma}_l - \hat{s}_q - \hat{\beta}^*\left(x_{ij\,()} - \bar{x}_{...}\right)\right]\left[x_{ij\,(\ )} - \bar{x}_{...}\right] = 0$$

Subtituting equtations 40-45 into the derivative of $\hat{\beta}^*$above

$$S_{xy} - \left(T_{xy}^i + T_{xy}^j + T_{xy}^l + T_{xy}^p + T_{xy}^q\right) = \hat{\beta}^*\left(S_{xx} - \left(T_{xx}^i + T_{xx}^j + T_{xx}^l + T_{xx}^p + T_{xx}^q\right)\right.$$

If we let $T_{xy}^1 = \left(T_{xy}^i + T_{xy}^j + T_{xy}^l + T_{xy}^p + T_{xy}^q\right)$

$$T_{xx}^1 = \left(T_{xx}^i + T_{xx}^j + T_{xx}^l + T_{xx}^p + T_{xx}^q\right)$$

$$S_{xy} - T_{xy}^1 = \hat{\beta}^*\left(S_{xx} - T_{xx}^1\right)$$

$$\hat{\beta}^* = \frac{S_{xy} - T_{xy}^1}{S_{xx} - T_{xx}^1} \quad (47)$$

The solution to the equation (40) reduction in the total sum of squares due to fitting the Reduced model is

$$R(\mu,\beta,\gamma,s,\hat{\beta}^*) = \hat{\mu}y_{...} + \sum_{i=1}^{m}\hat{\alpha}_i y_{i..} + \sum_{j=1}^{m}\hat{\beta}_j y_{.j.} + \sum_{l=1}^{m^2}\hat{\gamma}_l y_{...i} + \sum_{p=1}^{m^2}\hat{c}_p y_{...p} + \sum_{q=1}^{m^2}\hat{s}_q y_{..q} + \hat{\beta}^*\left(x_{ij\,(k,l,p,q)} - \bar{x}_{...}\right)\left(y_{ij\,(k,l,p,q)} - \bar{y}_{...}\right)$$

Subtitute the estimators of $\hat{\mu},\hat{\alpha},\hat{\beta}_j,\hat{\gamma}_l,\hat{c}_p,\hat{s}_q,\hat{\beta}^*$

$$= \frac{Y_{....}^2}{m^4} + \sum_{i=1}^{m}(\bar{y}_{i..} - \bar{y}_{...})y_{i..} + \sum_{i=1}^{m}\left(\bar{y}_{.j.} - \bar{y}_{...}\right)y_{.j.} + \sum_{l=1}^{m^2}(\bar{y}_{l..} - \bar{y}_{...})y_{l...} + \sum_{p=1}^{m^2}\left(\bar{y}_{p..} - \bar{y}_{...}\right)y_{...p} + \sum_{q=1}^{m^2}\left(\bar{y}_{..q} - \bar{y}_{...}\right)y_{..q} + \hat{\beta}^*\left[S_{xy} - \sum_{i=1}^{m}(\bar{x}_{i..} - \bar{x}_{...})y_{i..} - \sum_{i=1}^{m}\left(\bar{x}_{.j.} - \bar{x}_{...}\right)y_{.j.} - \sum_{l=1}^{m^2}(\bar{x}_{l..} - \bar{x}_{...})y_{l...} - \sum_{p=1}^{m^2}\left(\bar{x}_{p..} - \bar{x}_{...}\right)y_{...p} - \sum_{q=1}^{m^2}\left(\bar{x}_{q..} - \bar{x}_{...}\right)y_{..q}\right]$$

$$= \frac{Y_{....}^2}{m^4} + \left(T_{yy}^i + T_{yy}^j + T_{yy}^l + T_{yy}^p + T_{yy}^q\right) + \hat{\beta}^*\left(S_{xy} - T_{xy}^i - T_{xy}^j - T_{xy}^l - T_{xy}^p - T_{xy}^q\right)$$

We replace $T_{yy}^1 = T_{yy}^i + T_{yy}^j + T_{yy}^l + T_{yy}^p + T_{yy}^q$

$$T_{xy}^{1} = T_{xy}^{i} + T_{xy}^{j} + T_{xy}^{l} + T_{xy}^{p} + T_{xy}^{q}$$
$$= \frac{Y_{....}^{2}}{m^{4}} + T_{yy}^{1} + \hat{\beta}^{*}(S_{xy} - T_{xy}^{1})$$
$$= \frac{Y_{....}^{2}}{m^{4}} + T_{yy}^{1} + \hat{\beta}^{*}(S_{xy} - T_{xy}^{1})$$

From equation (46)

$$\hat{\beta}^{*} = \frac{S_{xy} - T_{xy}^{1}}{S_{xx} - T_{xx}^{1}}$$

$$= \frac{Y_{....}^{2}}{m^{4}} + T_{yy}^{1} + \frac{(S_{xy} - T_{xy}^{1})^{2}}{S_{xx} - T_{xx}^{1}} \quad (48)$$

Sum of square error for thereducecd model equation in (40)

$$SSe = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{ij\,(l,p,q)} - \frac{Y_{....}^{2}}{m^{4}} - T_{yy}^{1} - \frac{(S_{xy} - T_{xy}^{1})^{2}}{S_{xx} - T_{xx}^{1}}.$$

$$= S_{yy} - T_{yy}^{1} - \frac{(S_{xy} - T_{xy}^{1})^{2}}{S_{xx} - T_{xx}^{1}} \quad (49)$$

With $(m^4 - 2)$ degree of freedom

$$MSe = SSe/(m^4 - 2) \quad (50)$$

We may now find the appropriate sum of square treatment

$$TSS = SSe - SSE$$

$$= \left(S_{yy} - T_{yy}^{1} - \frac{(S_{xy} - T_{xy}^{1})^{2}}{S_{xx} - T_{xx}^{1}}\right) - E_{yy} + \frac{E_{xy}^{2}}{E_{xx}} \quad (51)$$

With the $m^2 - 1$ degree of freedom. We noted that SSE is somehow smaller than SSe the reason is owning to the fact that the model that generated SSE contains additional parameters τ_k. Therefore, the differences between SSe and SSE, gives a sum of square with square $m^2 - 1$ degrees of freedom for testing the hypothesis of no treatment effects.

To test $H_0: \tau_k = 0$, we consider the statistic below

$$F_0 = \frac{TSS/(m^2-1)}{MSE} \quad (52)$$

Which if the null hypothesis is true , is distributed as $F_{(m^2-1,(m^2(m^2-1)-1}$. Thus we reject $H_0: \tau_k = 0$, if $F_0 > F_{(m^2-1,(m^2(m^2-1)-1}$.

We recall that regression coeffient β^* in the ANCOVA Sudoku model I has been assumed to be nonzero. We may test the hypothesis $H_0: \beta^* = 0$ in this case we assume that $\beta^* = 0$.

The reduced model of equation (1) will be

$$Y_{ij\,(k,l,p,q)} = \mu + \alpha_i + \beta_j + \tau_k + C_p + \gamma_l + s_q + e_{i,j\,(k,l,p,q)} \quad (53)$$

where $i,j = 1 \cdots m \quad k,l,p,q = 1 \cdots m^2$

This is just an ANOVA model of equ.(1)and the score function would be

$$S(e^2)^{!} = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} \left[Y_{ij\,(k,l,p,q)} - \mu - \alpha_i - \beta_j - \tau_k - C_p - \gamma_l - s_q\right]^2 \quad (54)$$

We differentiate eqution (54) with respect to each of the parameters and equating each differential to zero and all estimates of the parameters are evaluated usual and also has the same estimate as those obtained earlier except for β^*.

We may express the reduction in the total sum of squares due to fitting the model equation (53)

$$R(\mu,\tau,\beta,\gamma,s) = \hat{\mu}y_{...} + \sum_{i=1}^{m}\hat{\alpha}_i y_{i..} + \sum_{j=1}^{m}\hat{\beta}_j y_{.j.} + \sum_{k=1}^{m^2}\hat{\tau}_k y_{..k} + \sum_{l=1}^{m^2}\hat{\gamma}_l y_{...l} + \sum_{p=1}^{m^2}\hat{c}_p y_{...p} + \sum_{q=1}^{m^2}\hat{s}_q y_{..q}$$

Subtituting equations(41-46) into equation (53), we have

$$\frac{Y^2_{....}}{m^4} + \sum_{i=1}^{m}(\bar{y}_{i..} - \bar{y}_{...})y_{i..} + \sum_{i=1}^{m}(\bar{y}_{.j.} - \bar{y}_{...})\,y_{.j.} + \sum_{k=1}^{m^2}(\bar{y}_{..k} - \bar{y}_{...})\,y_{..k} + \sum_{l=1}^{m^2}(\bar{y}_{l..} - \bar{y}_{...})y_{l...}$$

$$+\sum_{p=1}^{m^2}\left(\bar{y}_{p..} - \bar{y}_{...}\right)y_{...p} + \sum_{q=1}^{m^2}\left(\bar{y}_{..q} - \bar{y}_{...}\right)y_{..q}$$

$$= \frac{Y^2_{....}}{m^4} + \left(T^i_{yy} + T^j_{yy} + T^k_{yy} + T^l_{yy} + T^p_{yy} + T^q_{yy}\right)$$

$$= \frac{Y^2_{....}}{m^4} + T_{yy}\left(T_{yy} = T^i_{yy} + T^j_{yy} + T^k_{yy} + T^l_{yy} + T^p_{yy} + T^q_{yy}\right)$$

Sum of square of $\hat{\beta}^* = R(full\ model\ equation\ (1) - R(reduced\ model\ equation\ (53)$

$$= \frac{Y^2_{....}}{m^4} + T_{yy} + \hat{\beta}^*\left(S_{xy} - T_{xy}\right) - \left(\frac{Y^2_{....}}{m^4} + T_{yy}\right)$$

$= \hat{\beta}^*\left(S_{xy} - T_{xy}\right)$ but $S_{xy} - T_{xy} = E_{xy}$

$$= \hat{\beta}^* E_{xy}$$

$$= \frac{(E_{xy})^2}{E_{xx}}$$

With 1 degree of freedom. Then the test statistic

$$F_0 = \frac{(E_{xy})^2/E_{xx}}{MSE} \quad (55)$$

Which under the null hypothesis is distributedn as $F_{1,(m^2(m^2-1)-1}$. Thus, we reject

$H_0: \beta^* = 0$ if $F_0 > F_{1,(m^2(m^2-1)-1}$

Table 1: ANCOVA of Sudoku square design model I of order m^2

Source	Sum of square	Degree of freedom	Mean sum of square	F- ratio
Regression	E^2_{xy}/E_{xx}	1	E^2_{xy}/E_{xx}	$\frac{E^2_{xy}}{E_{xx}}/MSE$
Treatment	$TSS = SSe - SSE$	$m^2 - 1$	$\mathrm{TSS}/m^2 - 1$	$\frac{TSS/(m^2-1)}{MSE}$
Error	$SSE = E_{yy} - \frac{E^2_{xy}}{E_{xx}}$	$m^2(m^2-1) - 1$	$\frac{SSE}{m^2(m^2-1)-1}$	
Total	S_{yy}	$m^4 - 1$		

$$T_{yy} = T^i_{yy} + T^j_{yy} + T^k_{yy} + T^l_{yy} + T^p_{yy} + T^q_{yy} \quad (56)$$

$$T_{xy} = T^i_{xy} + T^j_{xy} + T^k_{xy} + T^l_{xy} + T^p_{xy} + T^q_{xy} \quad (57)$$

$$T_{xx} = \left(T^i_{xx} + T^j_{xx} + T^k_{xx} + T^l_{xx} + T^p_{xx} + T^q_{xx}\right) \quad (58)$$

$$T^1_{yy} = T^i_{yy} + T^j_{yy} + T^l_{yy} + T^p_{yy} + T^q_{yy} \quad (59)$$

$$T^1_{xy} = T^i_{xy} + T^j_{xy} + T^l_{xy} + T^p_{xy} + T^q_{xy} \quad (60)$$

$$T^1_{xx} = \left(T^i_{xx} + T^j_{xx} + T^l_{xx} + T^p_{xx} + T^q_{xx}\right) \quad (61)$$

$$TSS = SSe - SSE$$

$$= \left(S_{yy} - T^1_{yy} - \frac{(S_{xy} - T^1_{xy})^2}{S_{xx} - T^1_{xx}}\right) - E_{yy} + \frac{E^2_{xy}}{E_{xx}} \quad (62)$$

The same procedures is repeated for ANCOVA Sudoku square models II –IV However, only the derivation of ANCOVA Sudoku design model I is revealed in the paper.

Ancova Sudoku design model II

Table 2: ANCOVA of Sudoku model II of order m^2

Source	Sum of square	Df	Mean sum of square	F- ratio
regression	E^2_{xy}/E_{xx}	1	E^2_{xy}/E_{xx}	$\frac{E^2_{xy}}{E_{xx}}/MSE$
treatment	$TSS = SSe - SSE$	$m^2 - 1$	$\text{TSS}/m^2 - 1$	$\frac{TSS/(m^2-1)}{MSE}$
Error	$SSE = E_{yy} - \frac{E^2_{xy}}{E_{xx}}$	$m^2(m^2-1)-1$	$\frac{SSE}{m^2(m^2-1)-1}$	
Total	S_{yy}	$m^4 - 1$		

$$T_{yy} = T^i_{yy} + T^j_{yy} + T^k_{yy} + T^{l(i)}_{yy} + T^{p(j)}_{yy} + T^q_{yy} \quad (63)$$

$$T_{xy} = T^i_{xy} + T^j_{xy} + T^k_{xy} + T^{l(i)}_{xy} + T^{p(j)}_{xy} + T^q_{xy} \quad (64)$$

$$T_{xx} = T^i_{xx} + T^j_{xx} + T^k_{xx} + T^{l(i)}_{xx} + T^{p(j)}_{xx} + T^q_{xx} \quad (65)$$

$$T^1_{yy} = T^i_{yy} + T^j_{yy} + T^{l(i)}_{yy} + T^{p(j)}_{yy} + T^q_{yy} \quad (66)$$

$$T^1_{xy} = T^i_{xy} + T^j_{xy} + T^{l(i)}_{xy} + T^{p(j)}_{xy} + T^q_{xy} \quad (67)$$

$$T^1_{xx} = \left(T^i_{xx} + T^j_{xx} + T^{l(i)}_{xx} + T^{p(j)}_{xx} + T^q_{xx}\right) \quad (68)$$

$$TSS = SSe - SSE$$

$$= \left(S_{yy} - T^1_{yy} - \frac{(S_{xy} - T^1_{xy})^2}{S_{xx} - T^1_{xx}}\right) - E_{yy} + \frac{E^2_{xy}}{E_{xx}} \quad (69)$$

Ancova Sudoku design model III

Table 3:ANCOVA of Sudoku square model III of orderm^2

Source	Sum of square	Degree of freedom	Mean sum of square	F- ratio
regression	E^2_{xy}/E_{xx}	1	E^2_{xy}/E_{xx}	$\frac{E^2_{xy}}{E_{xx}}/MSE$
treatment	$TSS = SSe - SSE$	$m^2 - 1$	$\text{TSS}/m^2 - 1$	$\frac{TSS/(m^2-1)}{MSE}$
Error	$SSE = E_{yy} - \frac{E^2_{xy}}{E_{xx}}$	$m^2(m^2-1)-1$	$\frac{SSE}{m^2(m^2-1)-1}$	
Total	S_{yy}	$m^4 - 1$		

$$T_{yy} = T^i_{yy} + T^j_{yy} + T^k_{yy} + T^l_{yy} + T^p_{yy} + T^{q(i)}_{yy} + T^{r(j)}_{yy} \quad (70)$$

$$T_{xy} = T^i_{xy} + T^j_{xy} + T^k_{xy} + T^l_{xy} + T^p_{xy} + T^{q(i)}_{xy} + T^{r(j)}_{xy} \quad (71)$$

$$T_{xx} = T^i_{xx} + T^j_{xx} + T^k_{xx} + T^l_{xx} + T^p_{xx} + T^{q(i)}_{xx} + T^{r(j)}_{xx} \quad (72)$$

$$T^1_{yy} = T^i_{yy} + T^j_{yy} + T^l_{yy} + T^p_{yy} + T^{q(i)}_{yy} + T^{r(j)}_{yy} \quad (73)$$

$$T^1_{xy} = T^i_{xy} + T^j_{xy} + T^l_{xy} + T^p_{xy} + T^{q(i)}_{xy} + T^{r(j)}_{xy} \quad (74)$$

$$T^1_{xx} = T^i_{xx} + T^j_{xx} + T^l_{xx} + T^p_{xx} + T^{q(i)}_{xx} + T^{r(j)}_{xx} \quad (75)$$

$$TSS = SSe - SSE$$

$$= \left(S_{yy} - T^1_{yy} - \frac{(S_{xy} - T^1_{xy})^2}{S_{xx} - T^1_{xx}}\right) - E_{yy} + \frac{E^2_{xy}}{E_{xx}} \quad (76)$$

ANCOVA Sudoku design Model IV

Table 4: ANCOVA of Sudoku square design model IV of order m^2

Source	Sum of square	Degree of freedom	Mean sum of square	F- ratio
regression	E^2_{xy}/E_{xx}	1	E^2_{xy}/E_{xx}	$\frac{E^2_{xy}}{E_{xx}}/MSE$
Treatment	$TSS = SSe - SSE$	$m^2 - 1$	$\text{TSS}/m^2 - 1$	$\frac{TSS/(m^2-1)}{MSE}$
Error	$SSE = E_{yy} - \frac{E^2_{xy}}{E_{xx}}$	$m^2(m^2-1)-1$	$\frac{SSE}{m^2(m^2-1)-1}$	
Total	S_{yy}	$m^4 - 1$		

$$T_{yy} = T_{yy}^{i} + T_{yy}^{j} + T_{yy}^{k} + T_{yy}^{l(i)} + T_{yy}^{p(j)} + T_{yy}^{q(i)} + T_{yy}^{r(j)} \quad (77)$$
$$T_{xy} = T_{xy}^{i} + T_{xy}^{j} + T_{xy}^{k} + T_{xy}^{l(i)} + T_{xy}^{p(j)} + T_{xy}^{q(i)} + T_{xy}^{r(j)} \quad (78)$$
$$T_{xx} = T_{xx}^{i} + T_{xx}^{j} + T_{xx}^{k} + T_{xx}^{l(i)} + T_{xx}^{p(j)} + T_{xx}^{q(i)} + T_{xx}^{r(j)} \quad (79)$$
$$T_{yy}^{1} = T_{yy}^{i} + T_{yy}^{j} + T_{yy}^{l(i)} + T_{yy}^{p(j)} + T_{yy}^{q(i)} + T_{yy}^{r(j)} \quad (80)$$
$$T_{xy}^{1} = T_{xy}^{i} + T_{xy}^{j} + T_{xy}^{l(i)} + T_{xy}^{p(j)} + T_{xy}^{q(i)} + T_{xy}^{r(j)} \quad (81)$$
$$T_{xx}^{1} = T_{xx}^{i} + T_{xx}^{j} + T_{xx}^{l(i)} + T_{xx}^{p(j)} + T_{xx}^{q(i)} + T_{xx}^{r(j)} \quad (82)$$
$$TSS = SSe - SSE$$

$$= S_{yy} - T_{yy}^{1} - \frac{(S_{xy} - T_{xy}^{1})^2}{S_{xx} - T_{xx}^{1}} - E_{yy} + \frac{E_{xy}^2}{E_{xx}} \quad (83)$$

5. Conclusion

This paper proposed four ANCOVA Sudoku Square design models, the procedure for the derivation of all of sum of squares and products were revealed and analysis of covariance for the proposed models were also given as well as test of significant of adjusted treatment and that of regression were given.

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