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Properties of Soft Vector Space Operations

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Abstract In this work, we study the soft vector space introduced by A. Sezgin Sezer et al. in 2014. We define normal soft sub vector space, normalistic soft vector space, trivial normalistic soft vector space and whole normalistic soft vector space. We also present the properties of operations such as union ($\widetilde{\cup}$), intersection ($\widetilde{\cap}$), restricted union ($\cup_R$), extended intersection ($\cap_E$), restricted difference ($\sim_R$), AND ($\widetilde{\wedge}$) and OR ($\widetilde{\vee}$) operations and investigate their inter-relationship between each other.

Keywords Soft set, Vector space, Soft vector space, Operation, Soft Sub vector space

1. Introduction

To solve complicated problems in Engineering, Economics, Environmental Sciences, Medical Sciences, etc. we cannot successfully use classical methods because of different kinds of uncertainties. Theories, such as Probability theory [1], Fuzzy set theory [2], Intuitionistic fuzzy set theory [3], [4], Rough set theory [5], Vague set theory [6] and Interval Mathematics [7], [8] are well known and often useful mathematical approaches for modeling uncertainty. However, what these theories can handle is merely is a proper part of uncertainty. But all these theories have their inherent limitations as pointed out by Molodtsov [9]. The reason for these difficulties is the inadequacies of the parameterization tools of the theories. As a result, Molodtsov [9] initiated the concept of soft set theory as general mathematical tool for dealing with uncertainties about vague concepts which is free from the aforementioned difficulties. This theory has been useful in many different areas such as decision making problems [10], [11], [12], [13], [14], [15], data analysis [16] and simulation [17].

Maji et al. [18] worked extensively on theoretical study of soft set theory and its operations in details. After wards, many researchers have worked on this concept as in [19], [20], [21], [22], [23] and [24].

In this paper, we study the soft vector spaces by investigating and proving some existing results in the background of vector space and further present the properties of operations on soft vector spaces such as union, intersection, restricted union, extended intersection, AND and OR operations on soft vector spaces.

2. Preliminaries and Basic definitions

In this section, we review the notion of vector space and some basic definitions in soft set theory.

Definition 2.1

Let V be a nonempty set with two binary operations:

(a) Vector addition: For any $u, v \in V$, a sum $u + v \in V$.

(b) Scalar multiplication: For any $u \in V,\ k \in K$, the product $ku \in V$.

Then V is called a **vector space** over a field K, if the following axioms hold for any vectors $u, v, w \in V$:

(i) $(u + v) + w = u + (v + w)$.

(ii) $u + v = v + u$.

(iii) There exists a vector in V, denoted by 0 and called the zero vector, such that, for any $u \in V$, $u + 0 = 0 + u = u$.

(iv) For every $u \in V$, there is a vector in V, denoted by $-u$, and is called the negative of u, such that $u + (-u) = (-u) + u = 0$

(v) $k(u + v) = ku + kv$, for any scalar $k \in K$.

(vi) $(a + b)u = au + bu$, for any scalars $a, b \in K$.

(vii) $(ab)u = a(bu)$, for any scalars $a, b \in K$.

(viii) $1u = u$, for the unit scalar$1 \in K$.

Let V be a vector space over a field K and let W be a subset of V. Then W is a subvector space of V if W is itself a vector space over K with respect to the operations of vector addition and scalar multiplication on V.
Mathematically, suppose W is a subset of a vector space V. Then W be a **sub vector space** of V, if the following two conditions hold:

(a) The zero vector 0 belongs to W.

(b) For every $u, v \in W, k \in K$:

 (i) The sum $u + v \in W$.

 (ii) The multiple $ku \in W$.

Definition 2.2. [1]

Soft set is defined in the following way. Let U be an initial Universe set and E be a set of parameters. Let P (U) denotes the power set of U and $A \subset E$.
A pair (F, A) is called a softset over U where F is a mapping given by $F: A \longrightarrow P(U)$.
In other words, a Soft set over U is a parameterized family of subsets of the universe U.
For $e \in A$, $F(e)$ may be considered as the set of $e -$ approximate elements of the Softset(F, A). Obviously, a softset is not a set.

Definition 2.3: Soft subset.

Let (F, A) and (G, B) be two soft set over a common universe U, we say that (F, A) is a softsubset of (G, B) if

i. $A \subset B$

ii. $\forall e \in A, F(e)$ and $G(e)$ are identical approximations. We write$(F, A) \subseteq (G, B)$.

(F, A) is said to be a soft superset of (G, B) if $(G, B)$is a soft subset of (F, A). We denote it by $(F, A) \supseteq (\mathrm{G}, \mathrm{B})$.

Definition 2.4: Equality of two soft sets

Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a softsubset of (G, B) and $(G, B)$is a softsubset of (F, A).

Definition 2.5: Null soft set.

A soft set (F, A) over U is said to be a Null softset denoted by Φ, if $\forall\ \epsilon \in \mathrm{A}$, $F(\epsilon) = \emptyset$, (null-set).

Definition 2.6: Absolute soft set.

A soft set (F, A) over U is said to be absolute softset denoted by
$\tilde{A}$, if $e \in A, F(e) = U$
Clearly, $\tilde{A}^c = \Phi$ and $\Phi^c = \tilde{A}$

Definition 2.7: AND operation on two soft sets

If (F, A) and (G, B) are two softsets then "(F, A) and (G, B)" denoted by $(F, A)\wedge(G, B)$ is defined by $(F, A)\wedge(G, B) = (H, A \times B)$, where
$\mathrm{H}(\alpha, \beta) = F(\alpha) \cap F(\beta), \forall(\alpha, \beta) \in \mathrm{A} \times \mathrm{B}$

Definition 2.8: OR operation on two soft sets

If (F,A) and (G,B) are two soft sets then

"(F,A) OR (G, B)" denoted by (F,A) V (G,B) is defined by (F,A) V (G,B) = (O, A×B), where $O(\alpha,\beta) = F(\alpha) \cup G(\beta), \forall(\alpha,\beta)\epsilon A \times B$.

Definition 2.9.

Let (F, A) and (G, B) be two soft sets over the universes U_1 and U_2 respectively. Then the **Cartesian product** of (F, A) and (G, B), denoted by

$(F,A) \times (G,B)$ is a soft define as :$(F,A) \tilde{\times} (G,B) = (\mathrm{H},\mathrm{C})$, where $\mathrm{C} = \mathrm{A} \times \mathrm{B}$ and $\forall (x,y) \in \mathrm{A} \times \mathrm{B}$ $H(x,y) = F(x) \tilde{\times} \mathrm{G}(y)$.

Definition 2.10. Disjoint Soft set.

Let $(F,A) and (G,B)$ be two softsets over a common universe$U$. Then $(F,A) and (G,B)$ are said to be disjoint if $(F,A) \Cap (G,B) = (H,C)$. Where C =$A \cap B = \emptyset$ and for every$\varepsilon\epsilon C$, H(ε) = F(ε) ∩ G(ε) = ∅

Definition 2.11. Union of two soft sets.

If the union of(F,A) and (G,B) over the common universe U is the soft set (H,C),

Where C= $\mathrm{A} \cup \mathrm{B}$, and $\forall e \in \mathrm{C}$

$$\mathrm{H}(e) = \begin{cases} F(e), & if\ e \in \mathrm{A} - \mathrm{B} \\ G(e), & if\ e \in \mathrm{B} - \mathrm{A} \\ F(e) \cup G(e), & if\ e \in \mathrm{A} \cap \mathrm{B} \end{cases}$$

This relation is denoted by $(F,A) \cup (G,\mathrm{B}) = (\mathrm{H},C)$

Definition 2.12. Intersection of two Soft sets.

Let (F,A) and (G,B) be two softsets over a common universe U. The intersection of (F,A) and (G,B) is denoted by $(F,A) \Cap (G,B)$, and is defined as $(F,A) \Cap (G,B) = (\mathrm{H},C)$, where $C = \mathrm{A} \cap \mathrm{B}$ and for all$c \in C$, $\mathrm{H}(c) = \mathrm{F}(c) \cap \mathrm{G}(c)$.

Definition 2.13. Extended intersection.

Let(F,A) and (G,B) be two softsets over a common universe U. The extended intersection of (F,A) and (G,B) is defined to be the soft set (H,C),

Where C= $\mathrm{A} \cup \mathrm{B}$, and all e $\in \mathrm{C}$,

$$\mathrm{H}(e) = \begin{cases} F(e), & if\ e \in \mathrm{A} - \mathrm{B} \\ G(e), & if\ e \in \mathrm{B} - \mathrm{A} \\ F(e) \cap G(e), & if\ e \in \mathrm{A} \cap \mathrm{B} \end{cases}$$

This relation is written as $(F,A) \cap_{\epsilon} (G,B) = (\mathrm{H},C)$.

Definition 2.14. Restricted difference.

Let (F,A) and (G,B) be two softsets over a common universe U, $such\ that$ $\mathrm{A} \cap \mathrm{B} \neq \emptyset$.

Then the restricted difference of (F,A) $AND\ (G,B)\ is\ denoted\ by$ $(F,\mathrm{A}){\sim_R}(G,\mathrm{B})$, and is defined as $(F,A){\sim_R}(G,B) = (\mathrm{H},C)$, where $C = \mathrm{A} \cap \mathrm{B}$ and for all $c \in C$, $\mathrm{H}(c) = \mathrm{F}(c) - \mathrm{G}(c)$.

Definition 2.15. Restricted Union.

Let (F,A) and (G,B) be two softsets over a common universe U such that $\mathrm{A} \cap \mathrm{B} \neq \emptyset$. The restricted union of (F,A) and $(G,\mathrm{B})$is denoted by $(F,\mathrm{A}) \cup_R (G,\mathrm{B})$and is defined as $(F,A) \cup_R (G,B) = (\mathrm{H},C)$, where $C = \mathrm{A} \cap \mathrm{B}$ and for all $c \in C$, $\mathrm{H}(c) = \mathrm{F}(c) \cup \mathrm{G}(c)$.

Definition 2.16.

Let (F, A) be a soft set over U. Then

The support of (F, A) written **supp (*F*, *A*)** is the set defined as; supp$(F, A) = \{x \in A: F(x) \neq \emptyset\}$.

(i) (F, A) is called a **non-null** soft set if supp $(F, A) \neq \emptyset$.
(ii) (F, A) is called a **relative null** soft set denoted by $\emptyset_A$ $if\ F(x) = \emptyset, \forall x \in A$
(iii) (F, A) is called a **relative whole** soft set, denoted by U_A $if\ F(x) = U\ \forall\ x \in A.$

3. Soft Vector Spaces

Let A be a nonempty set and V be a Vector space. Let R be an arbitrary binary relation between an elements of A and an elements of F, that is , R is a subset of $A \times V$, mathematically written as $R = \{(x, y) \in A \times V: y \in F(x)$. F is a set-valued function $F{:}A \to P(V)$ is defined as $Fx= y \in F{:}x,y \in R$ for all $x \in supp(F, A)$. Then the pair (F, A) is a soft set over V, which is derived from the relation R.

Definition 3.1

Let (F, A) be a soft set over V. Then (F, A) is called a soft vector over V if and only if $F(x)$ is a sub vector space of V denoted by $F(x)\ <_v V$ for all $x \in supp(F,\ A)$.
Let (F, A) and (G, B) be two vector spaces over V. Then (F, A) is a soft sub vector space of (G, B) written as $(F, A)\ \tilde{<}_v\ (G, B)$ if :

(i) $A \subseteq B$,

(ii) $F(x) <_v H(x)$, $\forall x \in supp(F, A)$.

Definition 3.2

Let (F, A) be a soft vector space over V and (G, B) be soft subvector space of (F, A). Then, we say that (G, B) is a normal soft sub vector space of (F, A), written as $(G, B)\ \tilde{\triangleleft}_v\ (F, A)$ if $H(x)$ is a normal sub vector space of $F(x)$ written as $H(x) \triangleleft_v F(x)$, for all $x \in B$.

Definition 3.3

Let V be a vector space and (F, A) be a non-null soft set over V. Then, (F, A) is called a normalistic soft vector space over V if $F(x)$ is a normal sub vector space of V, for all $x \in supp(F, A)$.

Definition 3.5

Let (F, A) and (G, B) be two soft vector spaces over M and N respectively. The product of soft vector spaces (F, A) and (G, B) is define as $(F, A) \times (G, B) = (U, A \times B)$, where $U(x, y) = F(x) \times G(y)$ for all $(x, y) \in A \times B$.

Definition 3.6

Let (F, A) and (G, B) be two soft vector spaces over V. Then (F, A) and (G, B) are called soft equal vector spaces denoted by $(F, A) = (G, B)$, if $(F, A) \subseteq (G, B)$ and $(G, B) \subseteq (F, A)$

Definition 3.7

Let (F, A) and (G, B) be two soft vector spaces over V. Then (F, A) is a soft sub vector space of (G, B) written $(F, A)\ \tilde{<}_v\ (G, B)$ if the following conditions hold:

(i) $A \subseteq B$,

(ii) $F(x) <_v G(x)$, which means $F(x)$ is a sub vector space of $G(x)$ for all $x \in supp(F, A)$.

4. Properties of Operations on soft vector spaces and their relationship with each other

In this section, basic properties of operations on soft vector spaces, such as union, intersection, restricted union, extended intersection, AND and OR operations on soft vector spaces are presented with their relationship with each other.

Proposition. 4.1. If $(F,\ A)$, $(G,\ B)$ and $(H,\ C)$ are three soft vector spaces over V, if it is non-null, then the following hold:

(i) $(F, A)\ \tilde{\cup}\ \big((G, B)\ \tilde{\cup}\ (H, C)\big) = \big((F, A)\ \tilde{\cup}\ (G, B)\big)\ \tilde{\cup}\ (H, C)$

(ii) (F,A) needs not be a soft subset of $(F,\mathrm{A})\ \tilde{\cup}\ (G,\mathrm{B})$. But if
$(F,\mathrm{A}) \tilde{\subseteq} (G,\mathrm{B}), then\ (F,\mathrm{A}) \tilde{\subseteq} (F,\mathrm{A})\ \tilde{\cup}\ (G,\mathrm{B}), moreover$
$(F,\mathrm{A}) =\ (F,\mathrm{A})\ \tilde{\cup}\ (G,\mathrm{B})$.

(iii) $(F,\mathrm{A})\ \tilde{\cup}\ (G,\mathrm{A}) = \Phi \Leftrightarrow (F,\mathrm{A})\ = \Phi\ and\ (G,\mathrm{A}) = \Phi$

(iv) $(F,\mathrm{A})\ \tilde{\cup} \left((G,\mathrm{B}) \Cap (\mathrm{H},C)\right) = \left((F,\mathrm{A})\ \tilde{\cup}\ (G,\mathrm{B})\right) \Cap \left((F,\mathrm{A})\ \tilde{\cup}\ (\mathrm{H},C)\right)$

(v) $(F,\mathrm{A}) \Cap \left((G,\mathrm{B})\ \tilde{\cup}\ (\mathrm{H},C)\right) = \left((F,\mathrm{A})\ \tilde{\cup}\ (\mathrm{H},\mathrm{C})\right) \Cap \left((G,\mathrm{B})\ \tilde{\cup}\ (\mathrm{H},C)\right)$

(vi) $(F,\ A)\ \tilde{\cup}\ (F,A) = (F,\ A)$,

(i) $(F,\ A)\ \tilde{\cup}\ (G,A)_{\emptyset} = (F,\ A)$,

(ii) $(F,\ A)\ \tilde{\cup}\ (G,A)_{V} = (G,A)_{V}$,

(iii) $(F,\ A)\ \tilde{\cup}\ (G,B)_{V} = \begin{cases} (G,B)_{V}, if\ A = B \\ (R,D), otherwise, \end{cases}$ where $D = A \cup B$,

(iv) $(F,\ A)\ \tilde{\cup}\ (G,B)_{\emptyset} = \begin{cases} (F,\ A), if\ A = B \\ (R,D), otherwise, \end{cases}$ where $D = A \cup B$.

proof:

(ii) Let $(F,\mathrm{A})\ \tilde{\cup}\ (G,\mathrm{B}) = (\mathrm{H},C), where\ C = \mathrm{A} \cup \mathrm{B}$ and for all $x \in C$

$$\mathrm{H}(x) = \begin{cases} F(x), & if\ \ x \in \mathrm{A}-\mathrm{B}, \\ G(x), & if\ \ x \in \mathrm{B}-\mathrm{A}, \\ F(x) \cup G(e), if\ x \in \mathrm{A} \cap \mathrm{B}. \end{cases}$$

It is obvious that if $x \in \mathrm{A} \cap \mathrm{B} \neq \emptyset$, then $\mathrm{H}(x) = F(x) \cup G(x)$, thus $F(x)\ and\ \mathrm{H}(x)$ need to be the same approximations. Thus, (F,A) need not be a soft sub vector space of $(F,\mathrm{A})\ \tilde{\cup}\ (G,\mathrm{B})$.

Now let $(F,\mathrm{A}) \tilde{\subset} (G,\mathrm{B})$. Then it is clear that $\mathrm{A} \subset \mathrm{A} \cup \mathrm{B} = \mathrm{A}$.We need to show that $F(x)\ and\ \mathrm{H}(x)$ are the same approximations for all $x \in \mathrm{A}$, then $x \in \mathrm{A} \cap \mathrm{B} = \mathrm{A}$, since $\mathrm{A} \subset \mathrm{B}$ implies $\mathrm{A} - \mathrm{B} = \emptyset$. Thus, $\mathrm{H}(x) = F(x) \cup G(x)\ = F(x) \cup F(x)$
$= F(x),\ \ as\ G(x)\ and\ F(x)$ are the same approximations for all $x \in \mathrm{A}$. This follows that H and F are the same set-valued mapping for all $x \in \mathrm{A}$, as required.

(iii) Suppose that $(F,\mathrm{A})\ \tilde{\cup}\ (G,\mathrm{A}) = (\mathrm{H},\mathrm{A}), where\ \mathrm{H}(x) = \mathrm{F}(x) \cup \mathrm{G}(x)$for all $x \in \mathrm{A}$. since $(\mathrm{H},\mathrm{A}) = \Phi$ from the assumption. $\mathrm{H}(x) = \mathrm{F}(x) \cup \mathrm{G}(x) = \emptyset \Leftrightarrow \mathrm{F}(x) = \emptyset\ and\ \mathrm{G}(x) = \emptyset \Leftrightarrow (F,\mathrm{A})\ = \Phi\ and\ (G,\mathrm{A}) = \Phi$ for all $x \in \mathrm{A}$.

Now assume that $(F,\mathrm{A})\ = \Phi\ and\ (G,\mathrm{A}) = \Phi$ and $(F,\mathrm{A})\ \tilde{\cup}\ (G,\mathrm{A})$
$= (\mathrm{H},\mathrm{A})$. Since $\mathrm{F}(x) = \emptyset\ and\ \mathrm{G}(x) = \emptyset$ for all $x \in \mathrm{A}$. Therefore, $(F,\mathrm{A})\ \tilde{\cup}\ (G,\mathrm{A}) = \Phi$ by definition.

Proposition 4.2. If $(F,\ A)$, $(G,\ B)$ and $(H,\ C)$ are three soft vector spaces over V, if it is non-null, then the following hold:

(i) $(F,\ A)\ \tilde{\cap} \left((G,B)\ \tilde{\cap}\ (H,C)\right) = \left((F,A)\ \tilde{\cap}\ (G,B)\right) \tilde{\cap}\ (H,C)$,

(ii) $(F,\ A)\ \tilde{\cap}\ (F,A) = (F,\ A)$,

(iii) $(F,\ A)\ \tilde{\cap}\ (G,B)_{\emptyset} = \begin{cases} (F,A)_{V}, if\ A = B \\ (R,D), otherwise, \end{cases}$ where $D = A \cup B$,

(iv) $(F,\ A)\ \tilde{\cap}\ (G,B)_{V} = \begin{cases} (F,\ A), if\ A \supseteq B \\ (R,D), otherwise, \end{cases}$ where $D = A \cup B$,

(v) $(F,\ A)\ \tilde{\cap}\ (G,A)_{V} = (F,A)$.

Theorem 4.1: Let $(F,\ A)$, $(G,\ B)$ and $(H,\ C)$ be three soft vector spaces over V, if it is non-null, then the following holds:

Properties of the restricted Union$\cup_R$ operation

Let $(F,\ A)$, $(G,\ B)$ and $(H,\ C)$ be soft vector spaces over V, then

(i) $(F,\mathrm{A}) \cup_R \left((G,\mathrm{B}) \cup_R (\mathrm{H},\mathrm{C})\right) = \left((F,\mathrm{A}) \cup_R (G,\mathrm{B})\right) \cup_R (\mathrm{H},\mathrm{C})$,

(ii) $(F,\mathrm{A}) \not\subseteq\ (F,\mathrm{A}) \cup_R\ (G,\mathrm{B})$ in general. But if,
$(F,\mathrm{A}) \tilde{\subset} (G,\mathrm{B})\ then\ (F,\mathrm{A}) \tilde{\subset} (F,\mathrm{A}) \cup_R\ (G,\mathrm{B}), moreover\ (F,\mathrm{A}) = (F,\mathrm{A}) \cup_R\ (G,\mathrm{B})$.

(iii) $(F,\mathrm{A}) \cup_R (G,\mathrm{A}) = \Phi \Leftrightarrow (F,\mathrm{A}) = \Phi \; and \; (G,\mathrm{A}) = \Phi$.

(iv) $(F,\mathrm{A}) \cup_R \big((G,\mathrm{B}) \Cap (\mathrm{H},C)\big) = \big((F,\mathrm{A}) \cup_R (G,\mathrm{B})\big) \Cap \big((G,\mathrm{B}) \cup_R (\mathrm{H},C)\big)$.

(v) $(F,\mathrm{A}) \Cap \big((G,\mathrm{B}) \cup_R (\mathrm{H},C)\big) = \big((F,\mathrm{A}) \cup_R (\mathrm{H},\mathrm{C})\big) \Cap \big((G,\mathrm{B}) \cup_R (\mathrm{H},C)\big)$.

(vi) $(F,\mathrm{A}) \cup_R \big((G,\mathrm{B}) \cap_\epsilon (\mathrm{H},C)\big) = \big((F,\mathrm{A}) \cup_R (\mathrm{G},\mathrm{B})\big) \cap_\epsilon \big((F,\mathrm{A}) \cup_R (\mathrm{H},C)\big)$.

(vii) $(F,\mathrm{A}) \cap_\epsilon \big((G,\mathrm{B}) \cup_R (\mathrm{H},C)\big) = \big((F,\mathrm{A}) \cup_R (\mathrm{G},\mathrm{B})\big) \cap_\epsilon \big((G,\mathrm{B}) \cup_R (\mathrm{H},C)\big)$.

Proof:

(i) First, we investigate the left hand side of the equality.

Suppose that $(G,\mathrm{B}) \cup_R (\mathrm{H},C) = (T, B \cap C)$, where $\mathrm{T}(x) = G(x) \cup \mathrm{H}(x)$ for all $x \in B \cap \mathrm{C}$. since $(T, B \cap C)$ is a non-null soft set over V. If $x \in supp(T, B \cap C)$, then $\mathrm{T}(x) = G(x) \cup \mathrm{H}(x) \neq \emptyset$. And assume $(F,\mathrm{A}) \cup_R (T, B \cap C) = \big(W, \mathrm{A} \cap (\mathrm{B} \cap C)\big)$, where $W(x) = F(x) \cup T(x) = F(x) \cup G(x) \cup \mathrm{H}(x)$ for all $x \in \mathrm{A} \cap \big((\mathrm{B} \cap C)\big)$. Since, $\big(W, \mathrm{A} \cap (\mathrm{B} \cap C)\big)$ is a non-null soft set. If $x \in supp\big(W, \mathrm{A} \cap (\mathrm{B} \cap C)\big)$, then $W(x) = F(x) \cup T(x) = F(x) \cup G(x) \cup \mathrm{H}(x) \neq \emptyset$.

Therefore, $W(x) = F(x) \cup G(x) \cup \mathrm{H}(x)$

Now, we consider the right hand side of the equality. Suppose that $(F,\mathrm{A}) \cup_R (G,\mathrm{B}) = (M, \mathrm{A} \cap \mathrm{B}), where\ M(x) = F(x) \cup G(x)\ for\ all\ x \in (\mathrm{A} \cap \mathrm{B}), by\ hypothesis,$ let $(M, \mathrm{A} \cap \mathrm{B})$ is a non-null soft set over V. If $x \in supp(M, \mathrm{A} \cap \mathrm{B})$, then $M(x) = F(x) \cup G(x) \neq \emptyset$. Assume that $(F,\mathrm{A}) \cup_R (M, \mathrm{A} \cap \mathrm{B}) = (N, (\mathrm{A} \cap \mathrm{B}) \cap C)$ where $\mathrm{N}(x) = \mathrm{M}(x) \cup \mathrm{H}(x) = \mathrm{F}(x) \cup G(x) \cup \mathrm{H}(x)\ for\ all\ x \in (\mathrm{A} \cap \mathrm{B}) \cap C$. Since, $(N, (\mathrm{A} \cap \mathrm{B}) \cap C)$ is non-null soft set over V. It means $x \in supp(N, (\mathrm{A} \cap \mathrm{B}) \cap C)$ then, $\mathrm{N}(x) = \mathrm{M}(x) \cup \mathrm{H}(x) = \mathrm{F}(x) \cup G(x) \cup \mathrm{H}(x) \neq \emptyset$.

Therefore, $\mathrm{N}(x) = \mathrm{F}(x) \cup G(x) \cup \mathrm{H}(x)$.

Since W and N are the same mapping for all $x \in \mathrm{A} \cap (\mathrm{B} \cap C) = (\mathrm{A} \cap \mathrm{B}) \cap C$, the Proof is completed.

(ii) Since $\mathrm{A} \not\subseteq \mathrm{A} \cap \mathrm{B}$ without any extra condition being given, $(F,\mathrm{A}) \not\subseteq (F,\mathrm{A}) \cup_R (G,\mathrm{B})$ in general. Now assume that (F,A) is a soft sub vector space of $(G,\mathrm{B})\ and\ (F,\mathrm{A}) \cup_R (G,\mathrm{B}) = (\mathrm{H}, \mathrm{A} \cap \mathrm{B} = \mathrm{C})$, where $\mathrm{H}(x) = F(x) \cup G(x)\ for\ all\ x \in C$. Then, $(\mathrm{F},\mathrm{A}) \tilde{\subseteq} (G,B) \Leftrightarrow \mathrm{A} \subset \mathrm{A} \cap \mathrm{B} = \mathrm{A}$ and $F(x)\ and\ G(x)$ are the same approximations for all $x \in \mathrm{A} \Leftrightarrow \mathrm{H}(x) = F(x) \cup G(x) = F(x) \cup F(x) = F(x)$ for all $x \in \mathrm{A}$. Therefore, F and H are the same set-valued mapping for all $x \in \mathrm{A}$, Hence the proof is completed.

(iv) We first handle the left hand side of the equality. Suppose that $(G,\mathrm{B}) \Cap (\mathrm{H},C) = (T, B \cap C), where\ \mathrm{T}(x) = G(x) \cap \mathrm{H}(x)$ for all $x \in \mathrm{B} \cap C$. By hypothesis $(T, B \cap C)$ is a non-null soft set over V. If $x \in supp(T, B \cap C)$, then $\mathrm{T}(x) = G(x) \cap \mathrm{H}(x) \neq \emptyset$.

Let$(F,\mathrm{A}) \cup_R (\mathrm{T}, \mathrm{B} \cap \mathrm{C}) = \big(\mathrm{W}, \mathrm{A} \cap (\mathrm{B} \cap C)\big)$ where $\mathrm{W}(x) = F(x) \cup T(x) =$ $F(x) \cup \big(G(x) \cap \mathrm{H}(x)\big)\ for\ all\ x \in (\mathrm{A} \cap \mathrm{B}) \cap C$. Since, $\big(\mathrm{W}, \mathrm{A} \cap (\mathrm{B} \cap C)\big)$ is a non-null soft set over V. If $x \in supp\big(\mathrm{W}, \mathrm{A} \cap (\mathrm{B} \cap C)\big)$ then $\mathrm{W}(x) = F(x) \cup T(x) = F(x) \cup \big(G(x) \cap \mathrm{H}(x)\big) \neq \emptyset$.

Therefore, $\mathrm{W}(x) = F(x) \cup \big(G(x) \cap \mathrm{H}(x)\big)$

Now consider the right hand side of the equality. Assume that $(F,\mathrm{A}) \cup_R (G,\mathrm{B}) = (\mathrm{M}, \mathrm{A} \cap \mathrm{B})$, where $\mathrm{M}(x) = F(x) \cup G(x)\ for\ all\ x \in \mathrm{A} \cap \mathrm{B}$ And let $(F,\mathrm{A}) \cup_R (\mathrm{H},C) = (N, \mathrm{A} \cap C)$, where $\mathrm{N}(x) = F(x) \cup \mathrm{H}(x)$ for all $x \in \mathrm{A} \cap \mathrm{B} \neq \emptyset$.

Suppose that $(M, \mathrm{A} \cap \mathrm{B}) \Cap (N, \mathrm{B} \cap C) = \big(\mathrm{K}, (\mathrm{A} \cap \mathrm{B}) \cap (\mathrm{A} \cap C)\big) =$ $(\mathrm{K}, (\mathrm{A} \cap \mathrm{B}) \cap C)$, where $\mathrm{K}(x) = M(x) \cap N(x) = \big(F(x) \cup G(x)\big) \cap \big(F(x) \cup \mathrm{H}(x)\big) = F(x) \cup \big(G(x) \cap$ $\mathrm{H}x$ for all $x \in \mathrm{A} \cap \mathrm{B} \cap C$.

Since W and K are the same Set-valued mapping. The Proof is completed.

(vi) Suppose that $(G,B) \cap_\epsilon (\mathrm{H},\mathrm{C}) = (T, \mathrm{B} \cup C)$, where $T(x) = \begin{cases} F(x), & if\ x \in \mathrm{B} - \mathrm{C}, \\ \mathrm{H}(x), & if\ x \in \mathrm{B} - \mathrm{A}, \\ G(x) \cap \mathrm{H}(x), & if\ x \in \mathrm{B} \cap \mathrm{C}. \end{cases}$

For all $x \in (\mathrm{B} \cup C) \neq \emptyset$

Assume that$(F,\mathrm{A}) \cup_R (T, \mathrm{B} \cup \mathrm{C}) = (\mathrm{M}, \mathrm{A} \cap (\mathrm{B} \cup \mathrm{C})$, where

$\mathrm{M}(x) = F(x) \cup T(x)\ for\ all\ x \in \mathrm{A} \cap (\mathrm{B} \cup \mathrm{C}) \neq \emptyset$. By taking into account the properties of operations in set theory and the definitions of M along with T and considering that T is a piece wise function, we can write the below equalities for M:

$$\mathrm{M}(x) = \begin{cases} F(x) \cup G(x), & if\ x \in \mathrm{A} \cap (\mathrm{B} - \mathrm{C}), = (\mathrm{A} \cap \mathrm{B}) - (\mathrm{A} \cap \mathrm{C}), \\ \mathrm{F}(x) \cup \mathrm{H}(x), & if\ x \in \mathrm{A} \cap (\mathrm{C} - \mathrm{B}) = (\mathrm{A} \cap \mathrm{C}) - (\mathrm{A} \cap \mathrm{B}), \\ F(x) \cup \big(\mathrm{G}(x) \cap \mathrm{H}(x)\big), if\ x \in \mathrm{A} \cap (\mathrm{B} \cap \mathrm{C}). \end{cases}$$

for all $x \in \mathrm{A} \cap (\mathrm{B} \cup \mathrm{C})$.

Now, consider the right hand side of the equality. Suppose that $(F, \mathrm{A}) \cup_R (G, \mathrm{B}) = (\mathrm{Q}, \mathrm{A} \cap \mathrm{B})$, where $\mathrm{Q}(x) = F(x) \cup G(x)$ for all $x \in \mathrm{A} \cap \mathrm{B} \neq \emptyset$. Assume $(F, \mathrm{A}) \cup_R (\mathrm{H}, C) = (\mathrm{W}, \mathrm{A} \cap \mathrm{C})$, where $\mathrm{W}(x) = F(x) \cup H(x)$ for all $x \in \mathrm{A} \cap \mathrm{C} \neq \emptyset$. Let $(\mathrm{Q}, \mathrm{A} \cap \mathrm{B}) \cap_{\in} (\mathrm{W}, \mathrm{A} \cap \mathrm{C}) = \big(N, (\mathrm{A} \cap \mathrm{B}) \cup (\mathrm{A} \cap \mathrm{C})\big)$, where

$$\mathrm{N}(x) = \begin{cases} Q(x), & if\ x \in (\mathrm{A} \cap \mathrm{B}) - (\mathrm{A} \cap \mathrm{C}), \\ \mathrm{W}(x), & if\ x \in (\mathrm{A} \cap \mathrm{C}) - (\mathrm{A} \cap \mathrm{B}), \\ Q(x) \cap \mathrm{W}(x), if\ x \in (\mathrm{A} \cap \mathrm{B}) \cap (\mathrm{A} \cap \mathrm{C}) = (\mathrm{B} \cap \mathrm{C}). \end{cases}$$

for all $x \epsilon (\mathrm{A} \cap \mathrm{B}) \cup (\mathrm{A} \cap \mathrm{C})$. By taking into account the definitions of Q and W, we can rewrite N as follows:

$$\mathrm{N}(x) = \begin{cases} F(x) \cup G(x), & if\ x \in (\mathrm{A} \cap \mathrm{B}) - (\mathrm{A} \cap \mathrm{C}), \\ F(x) \cup \mathrm{H}(x), & if\ x \in (\mathrm{A} \cap \mathrm{C}) - (\mathrm{A} \cap \mathrm{B}), \\ \big(F(x) \cup G(x)\big) \cap \big(F(x) \cup \mathrm{H}(x)\big), if\ x \in \mathrm{A} \cap (\mathrm{B} \cap \mathrm{C}). \end{cases}$$

This follows that N and M are the same set-valued mapping when considering the properties of operations on set theory which completes the proof.

Theorem 4.2. If $(F,\ A)$, $(G,\ B)$ and $(H,\ C)$ are three soft vector spaces over V, and the left and right hand side of the equality are also soft vector spaces if it is non-null, then the following holds:

(i) $(\mathrm{F}, \mathrm{A}) \cap_{\in} \big((\mathrm{G}, \mathrm{B}) \cap_{\in} (\mathrm{H}, \mathrm{C})\big) = \big((\mathrm{F}, \mathrm{A}) \cap_{\in} (\mathrm{G}, \mathrm{B})\big) \cap_{\in} (\mathrm{H}, C)$

(ii) $(\mathrm{F}, \mathrm{A}) \cap_{\in} (\mathrm{G}, \mathrm{B}) \nsubseteq (\mathrm{G}, \mathrm{B})$, in general. But if $(\mathrm{F}, \mathrm{A}) \tilde{\subset} (\mathrm{G}, \mathrm{B})$, then $(\mathrm{F}, \mathrm{A}) \cap_{\in} (\mathrm{G}, \mathrm{B}) \tilde{\subset} (\mathrm{G}, \mathrm{B}), moreover\ (\mathrm{F}, \mathrm{A}) \cap_{\in} (\mathrm{G}, \mathrm{B}) = (\mathrm{G}, \mathrm{B})$.

(iii) $\big((\mathrm{F}, \mathrm{A}) \cap_{\in} (\mathrm{G}, \mathrm{B})\big) \cup_R (\mathrm{H}, C) = \big((\mathrm{F}, \mathrm{A}) \cap_{\in} (\mathrm{H}, C)\big) \cup_R \big((\mathrm{G}, \mathrm{B}) \cap_{\in} (\mathrm{H}, C)\big)$.

Proof:

(ii) Since $\mathrm{A} \cup \mathrm{B} \nsubseteq \mathrm{A}$ without any extra condition given
$(\mathrm{F}, \mathrm{A}) \cap_{\in} (\mathrm{G}, \mathrm{B}) \nsubseteq (\mathrm{F}, \mathrm{A})$, in general. Now assume that
$(\mathrm{F}, \mathrm{A}) \tilde{\subset} (\mathrm{G}, \mathrm{B})\ and\ (\mathrm{F}, \mathrm{A}) \cap_{\in} (\mathrm{G}, \mathrm{B}) = (\mathrm{H}, C)$, where

$$\mathrm{H}(e) = \begin{cases} \mathrm{F}(x), & if\ x \in \mathrm{A} - \mathrm{B}, \\ \mathrm{G}(x), & if\ x \in \mathrm{B} - \mathrm{A}, \\ \mathrm{F}(x) \cap \mathrm{G}(x), if\ x \in \mathrm{A} \cap \mathrm{B}. \end{cases}$$

for all $x \in C = \mathrm{A} \cup \mathrm{B} \neq \emptyset$. Since $\mathrm{A} \subset \mathrm{B}$, then it is obvious that $\mathrm{A} \cup \mathrm{B} = \mathrm{B} \subset \mathrm{B}$. Now we need to show that $\mathrm{H}(x)\ and\ \mathrm{G}(x)$ are the same approximations for all $x \in \mathrm{B}$. Let $x \in \mathrm{B}$, then either $x \in \mathrm{B} - \mathrm{A}\ or\ x \in \mathrm{A} \cap \mathrm{B} = \mathrm{A}$. If $x \in \mathrm{B} - \mathrm{A}$, then $\mathrm{H}(x)\ and\ \mathrm{G}(x)$, and if
$x \in \mathrm{A} \cap \mathrm{B} = \mathrm{A}$, then $\mathrm{H}(x) = \mathrm{F}(x) \cap \mathrm{G}(x) = \mathrm{G}(x) \cap \mathrm{G}(x) = \mathrm{G}(x)\ for\ all\ x \in \mathrm{A}$, Since $\mathrm{F}(x)\ and\ \mathrm{G}(x)$ are the same approximations for all $x \in supp(F, \mathrm{A})$. Thus $\mathrm{G}(x)\ and\ \mathrm{H}(x)$ are the identical approximations for all $x \in Supp(G, \mathrm{B})$, which completes the proof.

(iii) Suppose that $(\mathrm{G}, \mathrm{B}) \cup_R (\mathrm{H}, C) = (\mathrm{M}, \mathrm{B} \cap \mathrm{C})$, where
$\mathrm{M}(x) = \mathrm{G}(x) \cup \mathrm{H}(x)$ for all $x \in \mathrm{B} \cap C \neq \emptyset$. Assume that
$(\mathrm{F}, \mathrm{A}) \cap_{\in} \big(\mathrm{M}, (\mathrm{B} \cap C)\big) = \big(\mathrm{N}, \mathrm{A} \cup (\mathrm{B} \cap C)\big)$, where

$$\mathrm{N}(x) = \begin{cases} \mathrm{F}(x), & if\ x \in \mathrm{A} - (\mathrm{B} \cap \mathrm{C}), \\ \mathrm{M}(x), & if\ x \in (\mathrm{B} \cap \mathrm{C}) - \mathrm{A}, \\ \mathrm{F}(x) \cap \mathrm{M}(x), if\ x \in \mathrm{A} \cap (\mathrm{B} \cap \mathrm{C}), \end{cases}$$

for all $x \in \mathrm{A} \cup (\mathrm{B} \cap \mathrm{C})$.

By taking into account the properties of operations in set theory, it follows that,

$$\mathrm{N}(x)=\begin{cases}\mathrm{F}(x), & if\ \ x\in(\mathrm{A}-\mathrm{B})\cup(\mathrm{A}-\mathrm{C}),\\ \mathrm{G}(x)\cup\mathrm{H}(x), & if\ \ x\in(\mathrm{B}-\mathrm{A})\cap(\mathrm{C}-\mathrm{A}),\\ \mathrm{F}(x)\cap\big(\mathrm{G}(x)\cup\mathrm{H}(x)\big), & if\ x\in\mathrm{A}\cap(\mathrm{B}\cap\mathrm{C}).\end{cases}$$

Now, consider the right hand side of the equality. Suppose that $(\mathrm{F},\mathrm{A})\cap_{\epsilon}(\mathrm{G},\mathrm{B})=(\mathrm{T},\mathrm{A}\cup\mathrm{B})$, where

$$\mathrm{T}(x)=\begin{cases}\mathrm{F}(x), & if\ \ x\in\mathrm{A}-\mathrm{B}\\ \mathrm{G}(x), & if\ x\in\mathrm{B}-\mathrm{A}\\ \mathrm{F}(x)\cap\mathrm{G}(x), if\ x\in\mathrm{A}\cap\mathrm{B}\end{cases}$$

For all $x\in\mathrm{A}\cup\mathrm{B}\neq\emptyset$. And suppose $(\mathrm{F},\mathrm{A})\cap_{\epsilon}(\mathrm{H},\mathrm{C})=(\mathrm{W},\mathrm{A}\cup\mathrm{C})$, where

$$\mathrm{W}(x)=\begin{cases}\mathrm{F}(x), & if\ \ x\in\mathrm{A}-\mathrm{C}\\ \mathrm{H}(x), & if\ \ x\in C-\mathrm{A}\\ \mathrm{F}(x)\cap\mathrm{H}(x), if\ x\in\mathrm{A}\cap\mathrm{C}\end{cases}$$

for all $x\in\mathrm{A}\cup\mathrm{C}\neq\emptyset$.

Let $(\mathrm{T},\mathrm{A}\cup\mathrm{B})\cup_{R}(\mathrm{W},\mathrm{A}\cup\mathrm{C})=\big(\mathrm{P},(\mathrm{A}\cup\mathrm{B})\cap(\mathrm{A}\cup\mathrm{C})\big)$, where

$\mathrm{P}(x)=\mathrm{T}(x)\cup\mathrm{W}(x)\ for\ all\ x\in(\mathrm{A}\cup\mathrm{B})\cap(\mathrm{A}\cup C)\neq\emptyset.$ By considering the definitions of T and W along with P, we can write below the equalities.

$$\mathrm{P}(x)=\begin{cases}\mathrm{F}(x), & if\ \ x\in(\mathrm{A}-\mathrm{B})\cup(\mathrm{A}-\mathrm{C}),\\ \mathrm{G}(x)\cup\mathrm{H}(x), & if\ \ x\in(\mathrm{B}-\mathrm{A})\cap(\mathrm{C}-\mathrm{A}),\\ \big(\mathrm{F}(x)\cap\mathrm{G}(x)\big)\cup\big(\mathrm{F}(x)\cap\mathrm{H}(x)\big), & if\ x\in(\mathrm{A}\cap\mathrm{B})\cap(\mathrm{A}\cap\mathrm{C}).\end{cases}$$

for all $x\in(\mathrm{A}\cup\mathrm{B})\cap(\mathrm{A}\cup\mathrm{C})$. This follows that N and P are the same set-valued mapping . Therefore, the proof is completed.

Theorem 4.3. Let $(F,\ A)$, $(G,\ B)$ and $(H,\ C)$ be three soft vector spaces over V, if it is non-null, then the following hold:

(i) $(\mathrm{F},\mathrm{A})\Cap\big((\mathrm{G},\mathrm{B})\Cap(\mathrm{H},C)\big)=\big((\mathrm{F},\mathrm{A})\Cap(\mathrm{G},\mathrm{B})\big)\Cap(\mathrm{H},C)$

(ii) $(\mathrm{F},\mathrm{A})\Cap(\mathrm{G},\mathrm{B})\nsubseteq(\mathrm{G},\mathrm{B})$, in general. But if $(\mathrm{F},\mathrm{A})\mathrel{\tilde{\subseteq}}(\mathrm{G},\mathrm{B})$, then $(\mathrm{F},\mathrm{A})\Cap(\mathrm{G},\mathrm{B})\mathrel{\tilde{\subseteq}}(\mathrm{G},\mathrm{B}), moreover\ (\mathrm{F},\mathrm{A})\Cap(\mathrm{G},\mathrm{B})=(\mathrm{F},\mathrm{A})$.

(iii) $(\mathrm{F},\mathrm{A})\Cap\big((\mathrm{G},\mathrm{B})\cup_{R}(\mathrm{H},C)\big)=\big((\mathrm{F},\mathrm{A})\Cap(\mathrm{G},\mathrm{B})\big)\cup_{R}\big((\mathrm{F},\mathrm{A})\Cap(\mathrm{H},C)\big)$

(iv) $\big((\mathrm{F},\mathrm{A})\cup_{R}(\mathrm{G},\mathrm{B})\big)\Cap(\mathrm{H},C)=\big((\mathrm{F},\mathrm{A})\Cap(\mathrm{H},C)\big)\cup_{R}\big((\mathrm{G},\mathrm{B})\Cap(\mathrm{H},C)\big)$.

(v) $(\mathrm{F},\mathrm{A})\Cap\big((\mathrm{G},\mathrm{B})\mathbin{\tilde{\cup}}(\mathrm{H},C)\big)=\big((\mathrm{F},\mathrm{A})\Cap(\mathrm{G},\mathrm{B})\big)\mathbin{\tilde{\cup}}\big((\mathrm{F},\mathrm{A})\Cap(\mathrm{H},C)\big)$

(vi) $\big((\mathrm{F},\mathrm{A})\mathbin{\tilde{\cup}}(\mathrm{G},\mathrm{B})\big)\Cap(\mathrm{H},C)=\big((\mathrm{F},\mathrm{A})\Cap(\mathrm{H},C)\big)\mathbin{\tilde{\cup}}\big((\mathrm{G},\mathrm{B})\Cap(\mathrm{H},C)\big)$.

(vii) $(\mathrm{F},\mathrm{A})\Cap\big((\mathrm{G},\mathrm{B})\sim_{R}(\mathrm{H},C)\big)=\big((\mathrm{F},\mathrm{A})\Cap(\mathrm{G},\mathrm{B})\big)\sim_{R}\big((\mathrm{F},\mathrm{A})\Cap(\mathrm{H},C)\big)$.

(viii) $\big((\mathrm{F},\mathrm{A})\sim_{R}(\mathrm{G},\mathrm{B})\big)\Cap(\mathrm{H},C)=\big((\mathrm{F},\mathrm{A})\Cap(\mathrm{G},\mathrm{B})\big)\sim_{R}\big((\mathrm{G},\mathrm{B})\Cap(\mathrm{H},C)\big)$.

Proof:

(ii) Let $(\mathrm{F},\mathrm{A})\Cap(\mathrm{G},\mathrm{B})=(\mathrm{H},C), and\ C=\mathrm{A}\cap\mathrm{B}\neq\emptyset\ and\ \mathrm{H}(x)=\mathrm{F}(x)\cap\mathrm{G}(x)$ for all $x\in C$. Since H and F do not need to be the same set-valued mapping for all $x\in\mathrm{A}\cap\mathrm{B},(\mathrm{F},\mathrm{A})\Cap(\mathrm{G},\mathrm{B})\nsubseteq(\mathrm{F},\mathrm{A})$, in general.

Now assume that $(\mathrm{F},\mathrm{A})\mathrel{\tilde{\subseteq}}(\mathrm{G},\mathrm{B})$, then it is obvious that $\mathrm{A}\cap\mathrm{B}=\mathrm{A}\subset\mathrm{A}$. We need to show that $\mathrm{H}(x)\ and\ \mathrm{F}(x)$ are the same approximations for all $x\in\mathrm{A}\cap\mathrm{B}=\mathrm{A}$. Since$(\mathrm{F},\mathrm{A})\mathrel{\tilde{\subseteq}}(\mathrm{G},\mathrm{B})$ and $\mathrm{F}(x)$ and $\mathrm{G}(x)$ are the same approximation for all $x\in supp(F,\mathrm{A})$, it follows that $\mathrm{H}(x)=\mathrm{F}(x)\cap\mathrm{G}(x)=\mathrm{F}(x)\cap\mathrm{F}(x)=\mathrm{F}(x)$ for all $x\in supp(F,\mathrm{A})$, which completes the proof.

(iii) First, we investigate the left hand side of the equality.

Suppose that $(\mathrm{G},\mathrm{B})\cup_{R}(\mathrm{H},C)=(\mathrm{T},\mathrm{B}\cap\mathrm{C})\ where\ \mathrm{T}(x)=\mathrm{G}(x)\cup\mathrm{H}(x)$ for all $x\in\mathrm{B}\cap\mathrm{C}\neq\emptyset$. Then by the hypothesis, $(\mathrm{T},\mathrm{B}\cap\mathrm{C})$ is a non-null soft set over V. If $x\in supp(\mathrm{T},\mathrm{B}\cap\mathrm{C})$, then $\mathrm{T}(x)=\mathrm{G}(x)\cup\mathrm{H}(x)\neq\emptyset$. Assume that $(\mathrm{F},\mathrm{A})\Cap(\mathrm{T},\mathrm{B}\cap\mathrm{C})=\big(\mathrm{W},\mathrm{A}\cap(\mathrm{B}\cap\mathrm{C})\big)$, where $\mathrm{W}(x)=\mathrm{F}(x)\cap\mathrm{T}(x)=\mathrm{F}(x)\cap\big(\mathrm{G}(x)\cup\mathrm{H}(x)\big)$ for all $x\in\mathrm{A}\cap(\mathrm{B}\cap\mathrm{C})\neq\emptyset.$ Suppose that $\big(\mathrm{W},\mathrm{A}\cap(\mathrm{B}\cap\mathrm{C})\big)$ is a non-null soft set over V. If $x\in supp\big(\mathrm{W},\mathrm{A}\cap$ B∩C, then Wx=Fx∩Tx=Fx∩Gx∪Hx≠∅ for all x∈(A∩B∩C).

Let us consider the right hand side of the equality.
Assume that $(\mathrm{F},\mathrm{A}) \Cap (\mathrm{G},\mathrm{B}) = (\mathrm{M},\mathrm{A}\cap\mathrm{B})$, where $\mathrm{M}(x) = \mathrm{F}(x)\cap\mathrm{G}(x)$ for all $x\in\mathrm{A}\cap\mathrm{B}\neq\emptyset$. Suppose that $(\mathrm{M},\mathrm{A}\cap\mathrm{B})$ is a non-null soft set over V. If $x\in supp(\mathrm{M},\mathrm{A}\cap\mathrm{B})$, then $\mathrm{M}(x)=\mathrm{F}(x)\cap\mathrm{G}(x)\neq\emptyset$ for all $x\in supp(\mathrm{M},\mathrm{A}\cap\mathrm{B})$, and let $(\mathrm{F},\mathrm{A}) \Cap (\mathrm{H},\mathrm{C}) = (\mathrm{N},\mathrm{A}\cap\mathrm{C})$, where $\mathrm{N}(x)=\mathrm{F}(x)\cap\mathrm{H}(x)$ for all $x\in\mathrm{A}\cap\mathrm{C}\neq\emptyset$. Suppose that $(\mathrm{M},\mathrm{A}\cap\mathrm{B})\cup(\mathrm{N},\mathrm{A}\cap\mathrm{C}) = \big(\mathrm{K},(\mathrm{A}\cap\mathrm{B})\cap(\mathrm{A}\cap\mathrm{C})\big) = (\mathrm{K},(\mathrm{A}\cap\mathrm{B})\cap C)$, where $\mathrm{K}(x)=\mathrm{M}(x)\cup\mathrm{N}(x) = \big(\mathrm{F}(x)\cap\mathrm{G}(x)\big)\cup\big(\mathrm{F}(x)\cap\mathrm{H}(x)\big) = \mathrm{F}(x)\cap(\mathrm{G}(x)\cup\mathrm{H}(x))$ for all $x\in(\mathrm{A}\cap\mathrm{B})\cap(\mathrm{A}\cap\mathrm{C})$. Since W and K are the same set-valued mapping for all $x\in(\mathrm{A}\cap\mathrm{B})\cap(\mathrm{A}\cap\mathrm{C}) = \mathrm{A}\cap(\mathrm{B}\cap\mathrm{C})$.
The proof of (iv) – (vi) follows similar analysis.

(vii) First of all, we look at the left hand side of the equality. Suppose that $(\mathrm{G},\mathrm{B})\sim_R(\mathrm{H},C) = (\mathrm{T},\mathrm{B}\cap\mathrm{C})$, $\mathrm{T}(x)=\mathrm{G}(x)\setminus\mathrm{H}(x)$ for all $x\in\mathrm{A}\cap(\mathrm{B}\cap\mathrm{C})\neq\emptyset$.
Now, consider the right hand side of the equality. Assume that $(\mathrm{F},\mathrm{A}) \Cap (\mathrm{G},\mathrm{B}) = (\mathrm{M},\mathrm{A}\cap\mathrm{B})$, where $\mathrm{M}(x) = \mathrm{F}(x)\cap\mathrm{G}(x)$ for all $x\in\mathrm{A}\cap\mathrm{B}\neq\emptyset$. And let
$(\mathrm{F},\mathrm{A}) \Cap (\mathrm{H},\mathrm{C}) = (\mathrm{N},\mathrm{A}\cap\mathrm{C})$, where $\mathrm{N}(x)=\mathrm{F}(x)\cap\mathrm{H}(x)$ for all $x\in\mathrm{A}\cap\mathrm{C}$.
Suppose that $(\mathrm{M},\mathrm{A}\cap\mathrm{B})\sim_R(\mathrm{N},\mathrm{A}\cap\mathrm{C}) = \big(\mathrm{K},(\mathrm{A}\cap\mathrm{B})\cap(\mathrm{B}\cap\mathrm{C})\big) = \big(\mathrm{K},\mathrm{A}\cap(\mathrm{B}\cap\mathrm{C})\big)$, where $\mathrm{K}(x)=\mathrm{M}(x)\setminus\mathrm{N}(x) = \big(\mathrm{F}(x)\cap\mathrm{G}(x)\big)\setminus\big(\mathrm{F}(x)\cap\mathrm{H}(x)\big)$
for all $x\in(\mathrm{A}\cap\mathrm{B})\cap(\mathrm{A}\cap\mathrm{C})$. Since W and K are the same set-valued mapping for all $x\in(\mathrm{A}\cap\mathrm{B})\cap(\mathrm{A}\cap\mathrm{C}) = \mathrm{A}\cap(\mathrm{B}\cap\mathrm{C})$, this ends the Proof.

By using similar techniques used to prove (vi), (vii) can be proved also. Therefore, we skip the proof.

Proposition 4.3. If (F, A), (G, B) and (H, C) are three soft vector spaces over V, if it is non-null, then the following hold:

(i) $(F,A)\widetilde{\wedge}\big((G,B)\widetilde{\wedge}(H,C)\big) = \big((F,A)\widetilde{\wedge}(G,B)\big)\widetilde{\wedge}(H,C)$

(ii) $(F, A)\widetilde{\vee}\big((G,B)\widetilde{\vee}(H,C)\big) = \big((F,A)\widetilde{\vee}(G,B)\big)\widetilde{\vee}(H,C)$,

(iii) $(F, A)\widetilde{\vee}\big((G,B)\widetilde{\wedge}(H,C)\big) = \big((F,A)\widetilde{\vee}(G,B)\big)\widetilde{\wedge}\big((F,A)\widetilde{\vee}(H,C)\big)$,

(iv) $(F, A)\widetilde{\wedge}\big((G,B)\widetilde{\vee}(H,C)\big) = \big((F,A)\widetilde{\wedge}(G,B)\big)\widetilde{\vee}\big((F,A)\widetilde{\wedge}(H,C)\big)$.

Proof: We first investigate the **l**eft hand side equalities:

(i) Let $(G,B)\widetilde{\wedge}(H,C) = (R, B\times C)$, where $R(y,z) = G(y)\cap H(z)$ for all $(y,z)\in B\times C$. Then by hypothesis, $(R,B\times C)$ is a non-null soft set over V. If $(y,z)\in supp(R,B\times C)$, then $R(y,z) = G(y)\cap H(z)\neq\emptyset$. Assume that $(F,A)\widetilde{\wedge}(R,B\times C) = (T, A\times(B\times C))$, where $T(x,y,z) = F(x)\cap G(y)\cap H(z)$ for all $(x,y,z)\in(A\times B\times C)$. By the same hypothesis $(T,A\times(B\times C))$ is a non-null soft set over V. If $(x,y,z)\in supp(T,A\times(B\times C))$, then $T(x,y,z) = F(x)\cap G(y)\cap H(z)\neq\emptyset$.Therefore, $T(x,y,z) = F(x)\cap R(y,z) = F(x)\cap G(y)\cap H(z)$.
Next we investigate the right Hand side:
$(F,A)\widetilde{\wedge}(G,B) = (M, A\times B)$, where $M(x,y) = F(x)\cap G(y)$ for all $(x,y)\in A\times B$. Then by the hypothesis, $(M,A\times B)$ is a non-null soft set over V.
If $(x,y)\in supp(M,A\times B)$, then $M(x,y) = F(x)\cap G(y)\neq\emptyset$.
Assume that $(M,A\times B)\widetilde{\wedge}(H,C) = (N, (A\times B)\times C)$, where $N(x,y,z) = F(x)\cap G(y)\cap H(z)$ for all $(x,y,z)\in(A\times B\times C)$. By hypothesis, $(N, (A\times B)\times C)$ is a non-null soft set over V. If $(x,y,z)\in supp(N,(A\times B)\times C$, then $N(x,y,z) = F(x)\cap G(y)\cap H(z)\neq\emptyset$.
Therefore, $N(x,y,z) = M(x,y)\cap H(z) = F(x)\cap G(y)\cap H(z)$. Since T and N are the same mapping for all $(x,y,z)\in(A\times B\times C)$. This completes the proof.

Proof of (ii)-(iv) follows from the proof of (i).

4. Conclusion

A soft set is a mapping from parameter set to the crisp subset of the universe. Since soft set theory is an effective method for solving problems of uncertainty. In this paper, we further studied soft vector space and introduced some new definitions such as normal soft sub vector space, normalistic soft vector space, trivial normalistic soft vector space, whole normalistic soft vector space and equal soft vector space. We extensively discussed on soft vector space operations and prove several results on them.

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