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## Theoretical Analysis of Sound Insulation Performance through Multi-Layered Structures

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**Abstract** Based on the continuum mechanics, a theoretical model is proposed to simulation and investigate the sound insulation through multi-layered panels separated by air gap. In the study, the influences of parameters of multi-layered structures, such as the panel thickness, the air medium between plates and the air space, on the sound transmission loss are simulated and discussed based on the proposed method. The result shows that the sound transmission loss through multi-layered panels can be improved by separated air space thickness or decreasing air pressure due to a change in wave resonance frequencies of mass-air-mass.

**Keywords** Sound insulation, multi-layered panels, acoustics, numerical simulation

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### 1. Introduction

Sandwich panels and multi-layered structural composites with lightweight and high-strength have been widely used in application fields including sound insulation, environment protection and energy conservation [1, 2]. The dominant physical parameters such as material constants, surface mass and structural dimensions, affects largely on the sound insulation properties of panels [3-5]. In the basic design stage of products, the determination of design parameters and the investigation are very important before the products are developed and manufactured. Theoretical studies of the sound insulation are based on the physical consideration of sound propagation. The sound absorption properties of multi-walled plates could be well understood by explicit formulations derived according to the effective medium theory. For perforated plates or porous materials, the sound transmission loss of micro-perforated panel system was reported by using transfer matrix method and distributed models [6-8]. Based on the classical thin-plate theory, Takahashi and Tanaka developed an analytical model by introducing flow continuity at the panel surface in a spatially mean sense and air-solid interaction within the panel material [9]. Moreover, some fundamental acoustic problems were analyzed and discussed in relation to the interactive effect of flexural vibration and plate permeability. The sound insulation for high frequency range was largely affected by the damping material and its property [10]. The investigation of sound transmission loss revealed that the skin damping alone did not reduce wave coincident peaks significantly. The damping materials in core layers were more important than those in skin layers, resulting in an increase of sound transmission loss with increasing damping property.

In this study, a mathematical model and solution were derived to predict the sound transmission loss through multi-layer panels separated by air spaces. Based on the proposed model and derived theoretical formulation, the effects of various parameters on the sound transmission loss of panels were investigated in detail. The analytical model can be used to explain the effects of various system parameters on the performance characteristics of sound insulation characteristics.



**2. Theoretical Approach**

**2.1 Governing Equation**

To analyse the sound insulation performance, schematic diagram of three-layer panels divided by air spaces is shown in Fig. 1. The size of panels in the  $x$ -direction is taken as infinite, and the  $z$ -coordinate is taken along the direction of panel thickness.  $p_i$ ,  $p_r$  and  $p_t$  are the incident, the reflected and the transmitted acoustic pressures, respectively.  $u_j$  denotes the transverse displacement of different panels with  $j = 1, 2, 3$ .  $\rho_0$  and  $c_0$  are the air density and the speed of sound at atmosphere, respectively.  $\rho'$  and  $c'$  are the decompressed density and the sound speed under the decompressed air, respectively.

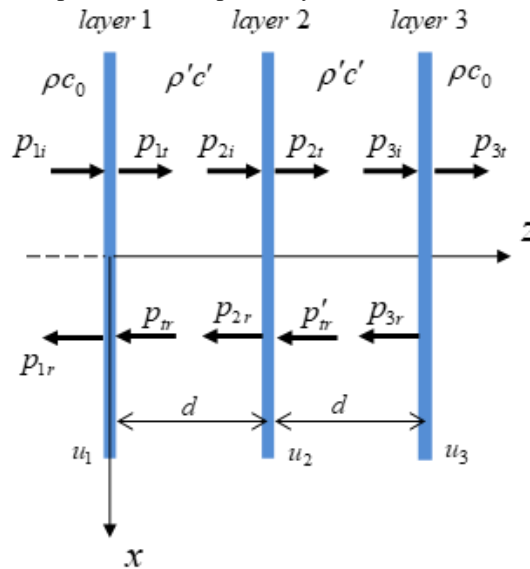


Figure 1: Schematic illustration showing three-layer wall construction

The sound transmission is caused by the wave propagation in panels. Based on Newton’s second law of motion, as shown in Figure 1, the governing equations of motion are given by

$$i\omega m_1 u_1 = p_{1i} + p_{1r} - p_{1t} - p_{1r} \tag{1}$$

$$i\omega m_2 \ddot{u}_2 = p_{2i} + p_{2r} - p_{2t} - p'_{tr} \tag{2}$$

$$i\omega m_3 \ddot{u}_3 = p_{3i} + p_{3r} - p_{3t} \tag{3}$$

where  $m_j$ ,  $j = 1, 2, 3$  is the mass of the wall, and  $\omega$  is the angular velocity of the wave.

**2.2. Solutions of Governing Equation**

According to the continuum condition of wave velocity on the plate surface and the contacted air, the mechanism is expressed as follows:

$$p_{1t} - p_{1r} = \rho'cu_1 \quad p_{2i} - p_{2r} = \rho'cu_2 \tag{4}$$

$$p_{2t} - p'_{tr} = \rho'cu_2 \quad p_{3i} - p_{3r} = \rho'cu_3 \tag{5}$$

$$p_{1i} - p_{1r} = \rho cu_1 \quad p_{3i} = \rho cu_3 \tag{7}$$

Moreover, the relationship of sound pressure between the panels with air space layer thickness of  $d$  can be given by

$$p_{1t} = e^{ikd} p_{2i} \quad p_{1r} = e^{-ikd} p_{2r} \tag{8}$$

$$p_{2t} = e^{ikd} p_{3i} \quad p'_{tr} = e^{-ikd} p_{3r} \tag{9}$$

where  $k$  ( $k = \omega/c$ ), is the wave number of the sound velocity in air.



Substituting Eq. (8) into Eq. (4), and Eq. (9) into Eq. (5), we obtain

$$p_{2i} = Z' \cdot \frac{u_2 e^{-ikd} - u_1}{e^{-ikd} - e^{ikd}}, \quad P_{2r} = Z' \cdot \frac{W_2 e^{ikd} - W_1}{e^{-ikd} - e^{ikd}} \quad (10)$$

$$p_{3i} = Z' \cdot \frac{u_3 e^{-ikd} - u_2}{e^{-ikd} - e^{ikd}}, \quad P_{3r} = Z' \cdot \frac{u_3 e^{ikd} - u_2}{e^{-ikd} - e^{ikd}} \quad (11)$$

where  $Z'$  ( $Z' = \rho'c$ ), is the impedance of air layers between the plates.

Furthermore, substituting Eqs. (10) and (11) into Eqs. (1)-(3), the governing equations of motion can be expressed as matrix form

$$\begin{bmatrix} s_{11} & s_{12} & 0 \\ s_{21} & s_{22} & s_{23} \\ 0 & s_{32} & s_{33} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 2p_{1i} \\ 0 \\ 0 \end{Bmatrix} \quad (12)$$

where

$$s_{11} = i\omega m_1 + Z - Z' \cdot \frac{1 + e^{i2kd}}{1 - e^{i2kd}} \quad s_{12} = 2Z'\omega \cdot \frac{e^{ikd}}{1 - e^{i2kd}} \quad s_{13} = 0 \quad (13)$$

$$s_{21} = s_{12} \quad s_{22} = i\omega m_2 - 2Z'\omega \cdot \frac{1 + e^{i2kd}}{1 - e^{i2kd}} \quad s_{23} = s_{12} \quad (14)$$

$$s_{31} = 0 \quad s_{32} = s_{23} \quad s_{33} = i\omega m_3 + Z - Z' \cdot \frac{1 + e^{i2kd}}{1 - e^{i2kd}} \quad (15)$$

According to Eq. (7) and Eq. (12), we obtain the transmission coefficient  $\tau(f)$ , which is defined as the ratio between the transmitted sound power and the incident sound power, given as

$$\tau(f) = \frac{|p_{3r}|^2}{|p_{1i}|^2} = \frac{|2Zs_{21}s_{32}|^2}{|s_{11}s_{22}s_{33} - s_{11}s_{32}s_{23} - s_{33}s_{12}s_{21}|^2} \quad (16)$$

Finally, the sound transmission loss (STL) through panels, defined as the inverse of the transmission coefficient in decibel scale, written as

$$STL = 10 \log_{10} \left| \frac{1}{\tau(f)} \right| \quad (17)$$

### 3. Analytical Results and Discussion

In this simulation, the panels are considered to be two- or three-layered glass plates with air layers there between. The Young's modulus of a glass plate is 72 GPa, the density is 2200 kg/m<sup>3</sup>. The density ( $\rho$ ) and the sound velocity ( $c$ ) of decompressed air layers were calculated by

$$\rho = \frac{p}{RT}, \quad c = \sqrt{\frac{\kappa p}{\rho}} \quad (18)$$



where  $R = 287 \text{ J/kg} \cdot \text{K}$  is the specific gas constant for air, and  $\kappa = 1.403$  is the heat capacity ratio for air.  $T$  is the absolute temperature, and  $p$  is the atmospheric pressure in air.

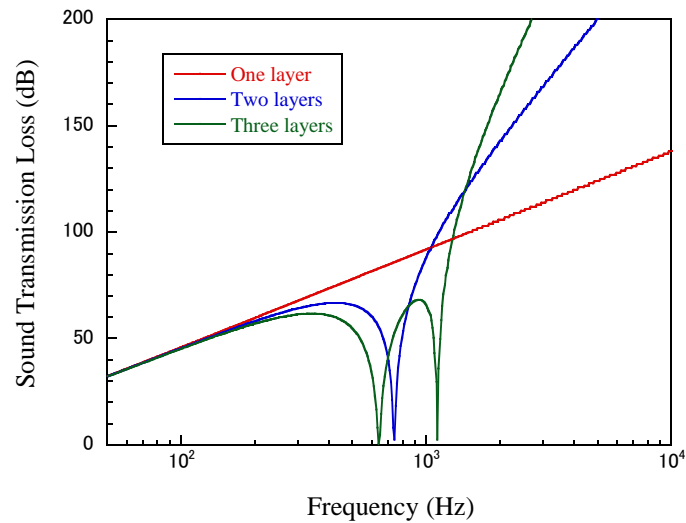


Figure 2: Relationship between STL and vibration frequency for different layer number

Figure 2 shows the sound transmission loss through multi-layered panels, which have the same total thickness, but different layer numbers. The plate thickness is 6 mm for a single layer plate. The two layers of glass plate are each 3 mm thick, and the three layers are 2 mm thick. The air layers between glass plates are the same interval with 2 mm. It is found that the effects of layer numbers on the sound insulation are quite small when the frequency range is below 200 Hz. However, the effects on the sound insulation due to air space becomes better when the frequency is over 1000 Hz. The sound transmission loss at high frequency increases with increasing layer number. The dips in the Figure 2 indicate the minimum of mass-air-mass resonance frequencies for multi-layered panels with air layers, which is called coincidence critical frequency. The number of critical frequencies increases with increasing air layer thickness, and the value moves to lower frequency. As shown in Figure 3, we also simulate the sound transmission loss of panels with different layer thickness ratios but the same total thickness. The total panels are 6.0 mm thick separated by an air space of 2.0 mm, and the thickness ratio of two separated layers ( $t_1 : t_2$ ) are 1:1, 1:1.5 and 1:2, respectively. It is observed that the sound transmission loss is hardly affected by the thickness ratio of double-walled panels.

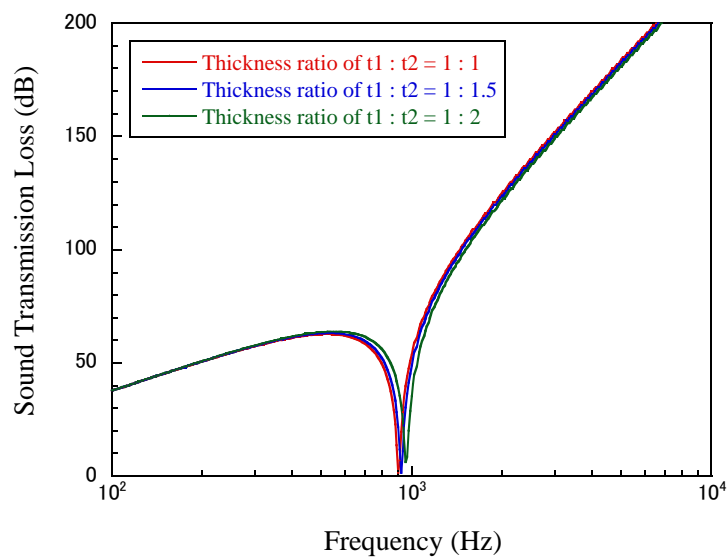


Figure 3: STL of two layered panels with different thickness ratios and an air space of 2 mm thick



Figure 4 shows the influences of air layer thickness on the sound transmission loss through double-layered glass panels with 2 mm thick. In this simulation, the air-layer thickness is given by 2 mm, 5 mm, and 10 mm. The sound transmission loss is observed to increase as the thickness of air layer increases at the frequency range more than 500 Hz. The coincidence critical frequency of glass panels moves to lower frequency when air-space layers become larger. This suggests that the decrease of the critical frequency results in advantages of the sound insulation performance at high-frequency range.

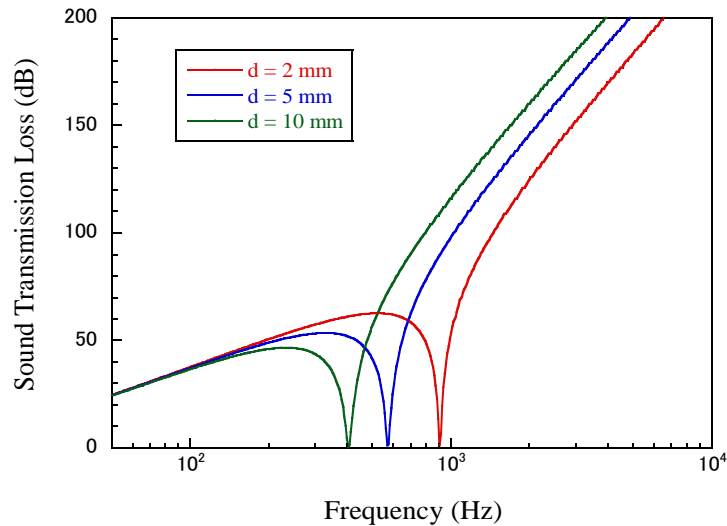


Figure 4: STL of the double wall panels with different air layer thickness and the panel thickness of 2 mm.

The influence of the decompressed air pressure on the sound transmission loss of double-walled panels with air space is shown in Figure 5 as a function of frequency. The simulations are carried out for the double panel with each 2.0 mm thick separated by an air space of 2.0 mm. The pressure values of internal air layers are taken as the standard atmospheric pressure (1.0 atm), 1/5 and 1/10 atmospheric pressure with decompressed air. It is seen that the air pressure influences significantly the mass-air-mass resonance frequency and the sound transmission loss. The coincidence critical frequencies decrease with decreasing air pressure, resulting in the improvement of the sound insulation ability at high frequency range. For the frequency range between 1000-5000 Hz, the sound transmission loss can be increased by about 5 dB when the atmospheric air pressure decreases by 10% of atmosphere under conditions of unchanged other parameters.

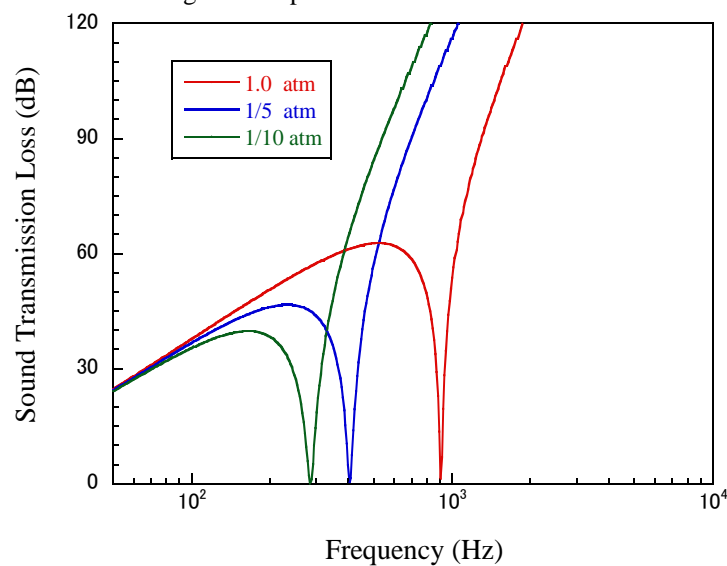


Figure 5: Influence of the decompressed air layer on STL of double-walled panels with each 3 mm thick and separated by 2 mm air layer



#### 4. Conclusions

A theoretical model and solution procedure are developed to investigate the sound transmission characteristics of multi-layered structures with air space. The governing equations of motion for the multi-layered panels is presented based on Newton's second law, and the solution of the sound transmission loss is derived by using *acoustic pressure method*. In the simulation, the influences of various parameters on the sound insulation characteristics of multi-layered panels are investigated and discussed in detail. The result shows that the sound transmission loss through panels with air space is can be improved by wave resonance frequencies of mass-air-mass. The sound transmission loss increases with decreasing air pressure or increasing air thickness under the higher frequency.

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