



The Effect of Magnetic Field on Superconductive Properties of Electrical Devices

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Abstract The research work was carried out to determine the possible way of overcoming the effect of magnetic field on superconducting materials. London and Maxwell equations were employed and modified, to reduce the external magnetic field effect on superconductors. We obtained a relationship between the permeability and penetration depth of superconducting materials. A specimen of aluminum was used to carry out the simulation of the equation obtained. It was discovered that both increase and decreases of the penetration depth of the aluminum shows a parabolic movement on the graph. It was concluded that materials of high permeability can be used to coat a superconducting material; when it is to be applied in any magnetic medium.

Keywords Magnetic field, London's equation, Permeability and Penetration depth

Introduction

When the temperature of metals and alloys are cooled, their electrical resistivity decreases. The thermal vibrations of the atoms decrease and the conduction electrons are less frequently scattered when the temperature is lowered. Electron motion is impeded by the thermal vibrations of lattice for a perfectly pure metal, and the resistivity approaches zero as the temperature reduces towards zero degree kelvin (0K). It is known that every real specimen of metal cannot be perfectly pure; they will contain some impurities which causes residual resistivity (ρ_0) at the lowest temperature [1]. Hence, the more impure the metal, the larger will be its residual resistivity.

However, when certain metals are cooled they show a remarkable behaviour; their electrical resistance decreases in the usual way but, on reaching a temperature few degrees above absolute zero, they lose all trace of electrical resistance. At this stage the metal is said to have passed into superconductivity state [2].

In 1933, Meissner and Ochsenfeld using type 1 superconductor showed that, magnetic field has a great effect on superconductors. When the superconductor is applied in a steady-state magnetic field, the field lines are expelled, and the superconductor starts behaving like a perfect diamagnetic material. These occur at the transition temperature T_c of the superconducting material [3]. Also, if an external magnetic field H (exceeding the critical field H_c) is brought close to the superconducting material, it destroys the superconductivity of that material [4]. This has posed great challenge to the use of superconducting materials in electrical devices.

This research work will profane the possible solution that will help in reducing the power losses witnessed in our present day electrical devices due to internal resistance. Thus, the research work started with London's equations, penetration depth of superconductors, relationship between the permeability and penetration depth of superconductors, simulation and discussion of result, and the conclusion.

London Equation

In 1935, F and H London gave the first phenomenological description of superconductor [5]. They added two conditions $E=0$ (from the absence of resistivity) and $B=0$ (from Meissner effect) to Maxwell's electromagnetic equations.



The first one state that:

$$\frac{\partial \mathbf{J}_s}{\partial t} = \frac{n_s e^2}{m} \mathbf{E} \quad (1)$$

Where, \mathbf{J}_s is the current density, n_s is the number density, e is the electron charge and \mathbf{E} is the electric field. Under the action of an electric field, equation (1) expresses the free, collision less acceleration of the superconductivity charge carrier [6].

Taking the curl of both side of equation (3.1)

$$\frac{\partial}{\partial t} (\nabla \times \mathbf{J}_s) = \frac{n_s e^2}{m} \nabla \times \mathbf{E}$$

From Maxwell's equations, according to Faraday's law of induction; $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ [7].

$$\frac{\partial}{\partial t} (\nabla \times \mathbf{J}_s) = -\frac{\partial}{\partial t} \left(\frac{n_s e^2}{m} \mathbf{B} \right)$$

$$\text{Or} \quad \nabla \times \mathbf{J}_s = -\frac{n_s e^2}{m} \mathbf{B} \quad (2)$$

The above equation is the London's second equation that leads to the Meissner effect (when superconductors expel magnetic field from their interior) [5]. Let $\mathbf{B}_c = \mu \mathbf{H}_c$,

$$\nabla \times \mathbf{J}_s = -\frac{n_s e^2}{m} \mu \mathbf{H}_c \quad (3)$$

Taking the curl of both side of equation (3), we have:

$$\nabla \times \nabla \times \mathbf{J}_s = -\frac{n_s \mu e^2}{m} (\nabla \times \mathbf{H}) \quad (4)$$

But we know that $\nabla \times \mathbf{H} = \frac{\partial \mathbf{E}}{\partial t}$. And also from vector identities $\nabla \times \nabla \times \mathbf{J}_s = \nabla(\nabla \cdot \mathbf{J}_s) - \nabla^2 \mathbf{J}_s$, where $\nabla(\nabla \cdot \mathbf{J}_s) = \mathbf{0}$ [8].

Then we get,

$$\nabla^2 \mathbf{J}_s = \frac{n_s \mu e^2}{m} \left(\frac{\partial \mathbf{E}}{\partial t} \right) \quad (5)$$

Since the $\mathbf{J}_s = \sigma \mathbf{E}$, where σ is the electrical conductivity of the material [9]. We have;

$$\nabla^2 \mathbf{E} = \frac{n_s \mu e^2}{\sigma m} \left(\frac{\partial \mathbf{E}}{\partial t} \right) \quad (6)$$

From vector identity, the divergence of a vector field is zero that is:

$$\nabla^2 \mathbf{E} - \frac{n_s \mu e^2}{\sigma m} \left(\frac{\partial \mathbf{E}}{\partial t} \right) = 0 \quad (7)$$

Penetration Depth of Superconductor

From Maxwell equation, magnetic monopole does not exist that is $\nabla \cdot \mathbf{B} = 0$ [9]. Therefore,

$$\nabla^2 \mathbf{B} = -\mu_0 \nabla \times \mathbf{J}_s \quad (8)$$

Substituting the equation (2) into the R.H.S of equation (8), we get

$$\begin{aligned} \nabla^2 \mathbf{B} &= -\mu_0 \left(-\frac{n_s e^2}{m} \mu \mathbf{H}_c \right) \\ \therefore \nabla^2 \mathbf{H}_c &= \frac{\mu_0 n_s e^2}{m} \mathbf{H}_c \end{aligned} \quad (9)$$

Let $\lambda = \sqrt{\frac{m}{\mu_0 n_s e^2}}$, be the penetration depth. Then we can rewrite equation (3.18) as given below:

$$\nabla^2 \mathbf{H}_c = \frac{1}{\lambda^2} \mathbf{H}_c \quad (10)$$

According to Prozorov and Glanetta [10] the superconductors add a new dimension to the penetration depth measurement, when an external magnetic field is applied. Also the London brothers state that the applied field decays exponentially, instead of dropping suddenly to zero. That is equation (3.19) becomes,

$$\mathbf{H}_c = \mathbf{H}_0 \exp\left(\frac{-x}{\lambda}\right) \quad (11)$$



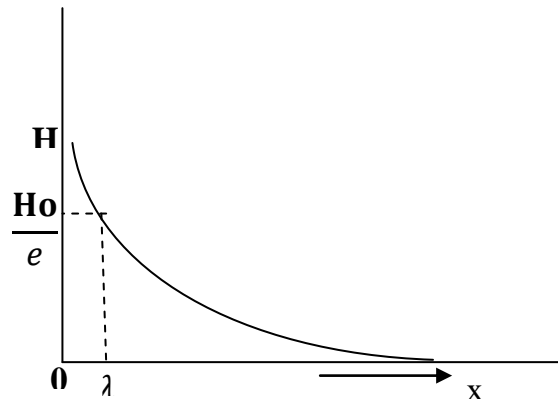


Figure 1: Decay of field

Where H_0 is the value of the magnetic field at the surface and λ is the penetration depth (the distance for H to fall from H_0 to H_0/e , which reduce the magnetic field) [4].

Hence, we are going to write equation (11) in terms of electric field only. In electromagnetic field theory, when the magnetic field B is perpendicular to the electric field $E = cB$ or $B = \frac{E}{c}$ or $H = \frac{E}{\mu c}$ (Dawar, 2014).

$$E = \frac{E}{\mu^2 c^2} \exp\left(\frac{-x}{\lambda}\right) \tag{12}$$

But $c^2 = \frac{1}{\mu_0 \epsilon_0}$ [7], substituting into equation (12) we have;

$$E = \frac{\epsilon_0}{\mu} E_0 \exp\left(\frac{-x}{\lambda}\right) \tag{13}$$

We can also write the equation as;

$$\mu = \epsilon \exp\left(\frac{-x}{\lambda}\right) \tag{14}$$

Where μ is the permeability and ϵ is the permittivity of the superconducting materials.

Results and Discussion

Equation (14) is been apply to Aluminum specimen, whose value of penetration depth is given as 50nm. At a constant permittivity, we decrease the value of the penetration depth and thus obtained:

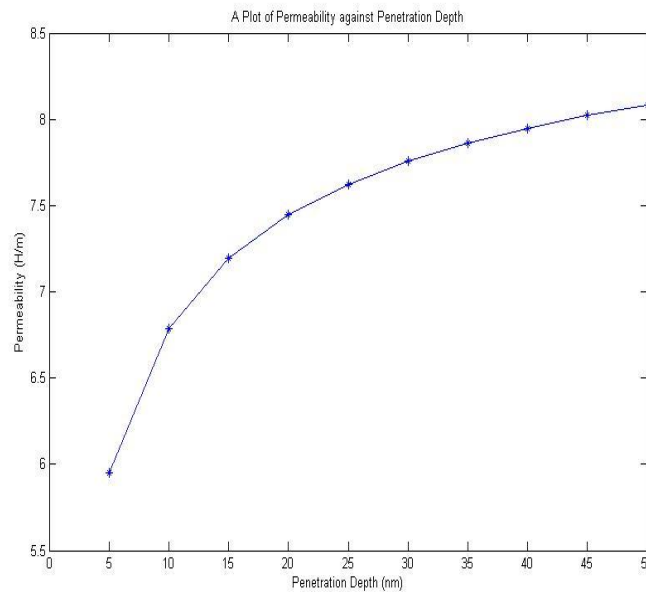


Figure 2: Decrease Permeability and Penetration depth

When the penetration depth of aluminum is decrease to zero, we observe a parabolic decrease. And at zero, there is no corresponding value of the permeability.

In similar manner, the permeability was also increase to observe the relationship between the two parameters.

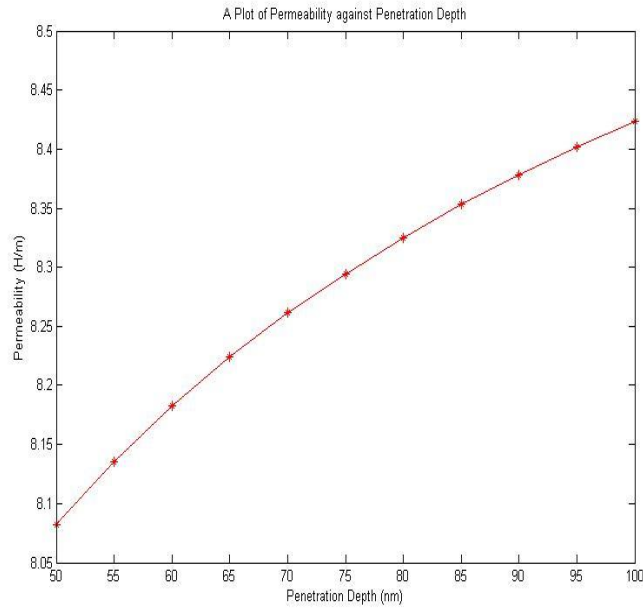


Figure 3: Increased permeability with penetration depth

The above graph indicate parabolic pattern, when the penetration depth is been increase above the normal penetration depth of 50nm.

It therefore implies that materials with high permeability can be used to shield off external magnetism from penetrating into our superconductors.

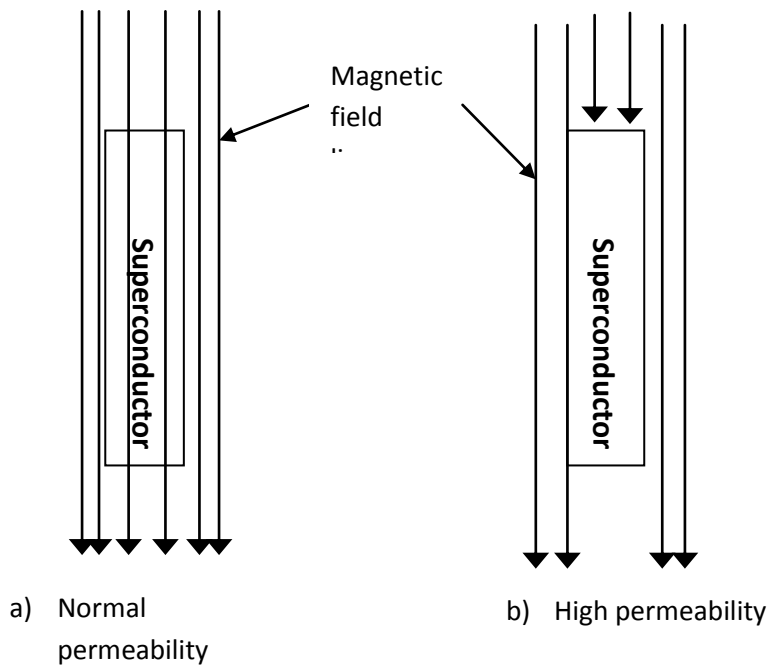


Figure 4: Comparison between normal and high permeability of a superconducting material in magnetic field

Conclusion

The final result obtained shows a parabolic increase in permeability due to the corresponding increase of the penetration depth; which indicate that, high permeability can be achievable for our superconducting material. Based on this result and discovery in this research work, the researcher therefore noted that materials with high permeability can be used to shield off external magnetic field from destroying our superconducting materials. Similarly, this result will also enable the magnetic flux within the superconductor to be maintained, and not expel from within. If the magnetic flux within the superconductor can be kept stable, then our superconductors will then be made possible for the transmission of electricity, and other applications.

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