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Research Article

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Absorption Spectrum for a Multi-photon Ξ-type Three-level Atom Driven by a Single-Mode Field with Nonlinearities

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Abstract A treatment of a multi-photon Ξ-type three-level atom interacting with a single mode field in a cavity, taking explicitly the existence of forms of nonlinearities of both the field and the intensity-dependent atom-field coupling into account. Analytical expressions of the absorption spectrum is presented using the dressed states of the system. The characteristics of the absorption spectrum considering the field to be initially in a squeezed coherent state is exhibited. The effects of the photon multiplicities, mean number of photons, detuning and the nonlinearities on the spectrum are investigated.
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Keywords Absorption spectrum, Multi-photon, Nonlinearities

1. Introduction

The system of a three-level atom in the Λ-configuration interacting with the electromagnetic field has been extensively studied [1, 2] because of its applications in a number of different contexts such as laser cooling [3] lasing without inversion [4] and electromagnetically induced transparency [5, 6]. Furthermore, the spectra for the three-level atom one-mode[7-10] and two-mode[11, 12] model has been studied.

On the other hand, recently a great deal of activity has centered on the analysis of the physical properties of nonlinear interaction models describing a localized center coupled to the modes of a quantized bosonic field [13, 14]. There are many physical situations where such models may find applications [15, 16]. For example it may be of interest in the context of the effective Hamiltonian approach to the two-mode two-photon micromaser [17, 18]. Moreover it is worthwhile to remark that investigating such models goes beyond an intrinsic theoretical interest in condensed matter systems too, because the development of new and improved materials is expected to lead to the fabrication of three dimensional photonic band gap systems possessing few isolated high-Q resonant field modes [19-21]. The physical origin of these nonlinear interactions may be traced back to the existence of a strong coupling between few levels of the material center and some selected modes of the quantized elastic or electromagnetic field. Very often the problem of interest is investigating the effects of nonlinearity on the quantum dynamics of the system starting from an appropriately chosen initial conditions. The starting point is the construction of an effective basic Hamiltonian model which contains the essential ingredients of the microscopic physical situation, at the same time providing us with an exactly solvable model. The introduction of such Hamiltonian is considered in this paper.

In recent years, there has been tremendous progress in the ability to generate states of the electromagnetic field with manifestly quantum or nonclassical characteristics experimentally [22-24]. Squeezed states of light are nonclassical states for which the fluctuations in one of two quadrature phase amplitudes of the electromagnetic field drop below the level of fluctuations associated with the vacuum state of the field. Squeezed states therefore

provide a field which is in some sense quieter than the vacuum state and hence can be employed to improve measurement precision beyond the standard quantum limits.

The goal of this paper is to shed some light on the absorption spectrum for a multi-photon three-level system. The model we shall consider is consisting of a single atom interacting with one mode field in a perfect cavity, including acceptable kinds of nonlinearities of both the field and the intensity-dependent atom-field coupling via multi-photon processes. To reach our goal it will be more convenient to use exact expression for the unitary operator $U(t)$ in the frame of the dressed state formalism. This will be considered in section **2**. In section **3** we employ the analytical results obtained and by using the finite double-Fourier transform of the two-time field correlation function, we find an analytical expression for the absorption spectrum.We devote section **4** to numerical results and discussion. Finally the conclusions are summarized in Section **5**.

2. Formulation of the Problem

The Hamiltonian of the system in the rotating-wave approximation is of the form $(\hbar = 1)$

$$H = H_0 + H_{in} \tag{1}$$

$$H_0 = \sum_{j=1}^{3} \omega_j \sigma_{j,j} + \Omega \hat{a}^{+} \hat{a} \tag{2}$$

where ω_1, ω_2 and ω_3 are the atomic levels energies $(\omega_1 > \omega_2 > \omega_3)$ and Ω is the field frequency, with the detuning parameters Δ_1 and Δ_2 given by

$$\Delta_1 = -k\Omega + (\omega_1 - \omega_2)$$
$$\Delta_2 = -k\Omega + (\omega_2 - \omega_3) \tag{3}$$

The operators $\hat{a}$ and $\hat{a}^{+}$ are the boson operators for the field satisfying $[\hat{a}, \hat{a}^{+}] = 1$, while k refers to the photon multiplicity number.

The interaction part of the Hamiltonian for the multi-photon processes and in the presence of an arbitrary nonlinear medium can be written as:

$$H_{in} = \Re(\hat{a}^{\dagger}\hat{a}) + \lambda_1(\sigma_{12} f_1(\hat{a}^{\dagger}\hat{a})\hat{a}^{k} + \hat{a}^{+k} f_1(\hat{a}^{\dagger}\hat{a})\sigma_{21})$$
$$+ \lambda_2(\sigma_{23} f_2(\hat{a}^{\dagger}\hat{a})\hat{a}^{k} + \hat{a}^{+k} f_2(\hat{a}^{\dagger}\hat{a})\sigma_{32}). \tag{4}$$

$\Re(\hat{a}^{\dagger}\hat{a})$ and $f(\hat{a}^{\dagger}\hat{a})$ are Hermitian operator functions of photon number operators, such that $\lambda_1 f(\hat{a}^{\dagger}\hat{a})$ and $\lambda_2 f(\hat{a}^{\dagger}\hat{a})$ represent arbitrary intensity-dependent atom-field couplings, while $\Re(\hat{a}^{\dagger}\hat{a})$ denotes the one-mode field nonlinearity which can model Kerr-like medium nonlinearity as will be discussed later. The operators σ_{ij} satisfy the following commutation relations $[\sigma_{ij}, \sigma_{kl}] = \sigma_{il}\delta_{jk} - \sigma_{kj}\delta_{il}$, $[\hat{a}, \sigma_{ij}] = 0$.

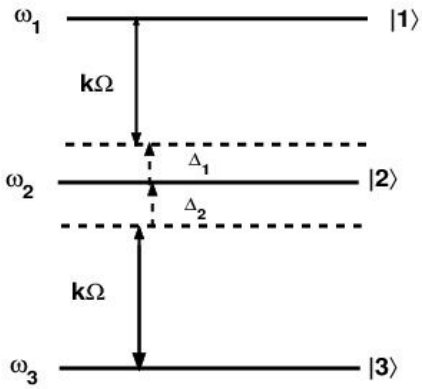

Figure 1: Energy level diagram for a Ξ -type three-level atom with multi-photon detuning Δ_1, Δ_2

The initial state $|\Psi(0)_{AF}\rangle$ of the combined atom-field system may be written as

$$|\Psi(0)_{AF}\rangle = |\Psi(0)_A\rangle \otimes |\Psi(0)_F\rangle \tag{5}$$

where $|\Psi(0)_A\rangle = |1\rangle\langle 1|$ the initial state of the atom and $|\Psi(0)_F\rangle = |\Theta\rangle\langle\Theta|$ is the initial state of the field.

The initial state $|\Theta\rangle = \sum p^{(n)} |n\rangle$ where the probability amplitude $p^{(n)}$ is defined in the usual manner as $p^{(n)} = \langle n|\Theta\rangle$.

The time evolution between the atom and the field is defined by the unitary evolution operator generated by H. Thus $U(t)$ is given by:

$$U(t) \equiv \exp(-iHt).$$

This unitary operator $U(t)$ is written as

$$U(t) = \sum_{s=0}^{k-1} \exp(-iE_{03}^{(s)}t)\,|\vartheta^{(s)}\rangle\langle\vartheta^{(s)}| + \sum_{s=0}^{k-1}\sum_{\kappa=1}^{2} \exp(-i\mu_{\kappa}^{(s)}t)\,|\Phi_{\kappa}^{(s)}\rangle\langle\Phi_{\kappa}^{(s)}|$$
$$+ \sum_{n=0}^{\infty}\sum_{j=1}^{3} \exp(-iE_j^{(n)}t)\,|\Psi_j^{(n)}\rangle\langle\Psi_j^{(n)}| \tag{6}$$

where the eigenvalues

$$E_{03}^{(s)} = \omega_3 + \Omega s + \Re(s), \qquad (s = 0,1,...,k-1)$$
$$\mu_{1,2}^{(s)} = \frac{R_1 + R_2}{2} \pm \frac{1}{2}\sqrt{(R_1+R_2)^2 - 4[R_1R_2 - Q]}$$
$$E_j^{(n)} = -\frac{X_1}{3} + \frac{2}{3}\left(\sqrt{X_1^2 - 3X_2}\right)\cos(\theta_j) \tag{7}$$

where j=1,2,3 and

$$\theta_j = \left(\frac{1}{3}\cos^{-1}\left[\frac{9X_1X_2 - 2X_1^3 - 27X_3}{2(X_1^2 - 3X_2)^{\frac{3}{2}}}\right] + (j-1)\frac{2\pi}{3}\right), \tag{8}$$

with

$$R_1 = \omega_2 + \Omega s + \Re(s)$$
$$R_2 = \omega_3 + \Omega(s+k) + \Re(s+k)$$
$$Q = \lambda_2 f_2(s)\sqrt{\frac{(s+k)!}{s!}}$$
$$X_1 = -(r_1 + r_2 + r_3)$$
$$X_2 = -\left[V_1^2 + V_2^2 - r_1r_2 - r_1r_3 - r_2r_3\right]$$
$$X_3 = r_3V_1^2 + r_1V_2^2 - r_1r_2r_3$$
$$r_1 = \omega_1 + \Omega n + \Re(n) \tag{9}$$
$$r_2 = \omega_2 + \Omega(n+k) + \Re(n+k)$$
$$r_3 = \omega_3 + \Omega(n+2k) + \Re(n+2k)$$
$$V_1 = \lambda_1 f_1(n)\sqrt{\frac{(n+k)!}{n!}}$$
$$V_2 = \lambda_2 f_2(n+k)\sqrt{\frac{(n+2k)!}{(n+k)!}}$$

and $|\vartheta^{(s)}\rangle$, $|\Phi_{\kappa}^{(s)}\rangle$, $|\Psi_{j}^{(n)}\rangle$ are the dressed states of the system associated with the eigenvalues $E_{03}^{(s)}$, $\mu_{\kappa}^{(s)}$ and $E_{j}^{(n)}, (\kappa = 1,2,\ j = 1,2,3)$ where

$$\begin{aligned} |\vartheta^{(s)}\rangle &= |s,3\rangle \qquad\qquad ,(s = 0,1,...,k-1) \\ |\Phi_{\kappa}^{(s)}\rangle &= L_{\kappa}^{(s)}\,|s,2\rangle + q_{\kappa}^{(s)}\,|s+k,3\rangle \\ \left|\Psi_{j}^{(n)}\right\rangle &= \alpha_{j}^{(n)}\left|n,1\right\rangle + \beta_{j}^{(n)}\left|n+k,2\right\rangle + \gamma_{j}^{(n)}\left|n+2k,3\right\rangle \end{aligned} \tag{10}$$

and

$$L_{\kappa}^{(s)} = \frac{-Q}{\sqrt{(R_1 - \mu_{\kappa}^{(s)})^2 + Q^2}}$$

$$q_{\kappa}^{(s)} = \frac{R_1 - \mu_{\kappa}^{(s)}}{\sqrt{(R_1 - \mu_{\kappa}^{(s)})^2 + Q^2}} \tag{11}$$

$$\begin{pmatrix} \alpha_{j}^{(n)} \\ \beta_{j}^{(n)} \\ \gamma_{j}^{(n)} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} -V_1(r_3 - E_{j}^{(n)}) \\ (r_1 - E_{j}^{(n)})(r_3 - E_{j}^{(n)}) \\ -V_2(r_1 - E_{j}^{(n)}) \end{pmatrix} \tag{12}$$

where

$$M^2 = (r_1 - E_{j}^{(n)})^2 (r_3 - E_{j}^{(n)})^2 + V_1^2 (r_3 - E_{j}^{(n)})^2 + V_2^2 (r_1 - E_{j}^{(n)})^2 \tag{13}$$

Having obtained the explicit form of the unitary operator $U(t)$, the eigenvalues and the eigenfunctions for the system under consideration, we are therefore in a position to discuss any property related to the atom or the field.

3. Absorption Spectrum

The absorption spectrum $A(\nu)$ of radiation field is given by the expression [8].

$$\begin{aligned} A(\nu) &= \Gamma \int_0^T dt_1 \int_0^T dt_2 \times \exp[-(\Gamma - i\nu)(T - t_1) - (\Gamma + i\nu)(T - t_2)] \\ &\times [\langle (\sigma_{12}(t_1) + \sigma_{23}(t_1)), (\sigma_{21}(t_2) + \sigma_{32}(t_2))\rangle]. \end{aligned} \tag{14}$$

Where T is the interaction time and Γ is the bandwidth of the filter. The Fourier transform of the two time commutator averaged dipole-dipole correlation, is directly related to the absorption spectrum. Where

$$\begin{aligned} &\langle [(\sigma_{12}(t_1) + \sigma_{23}(t_1)), (\sigma_{21}(t_2) + \sigma_{32}(t_2))]\rangle \\ &= \langle (\sigma_{12}(t_1) + \sigma_{23}(t_1))(\sigma_{21}(t_2) + \sigma_{32}(t_2))\rangle \\ &- \langle (\sigma_{21}(t_2) + \sigma_{32}(t_2))(\sigma_{12}(t_1) + \sigma_{23}(t_1))\rangle \end{aligned} \tag{15}$$

The first term $\langle (\sigma_{12}(t_1) + \sigma_{23}(t_1))(\sigma_{21}(t_2) + \sigma_{32}(t_2))\rangle$ is associated with emission processes while the second term $\langle (\sigma_{21}(t_2) + \sigma_{32}(t_2))(\sigma_{12}(t_1) + \sigma_{23}(t_1))\rangle$ which has an opposite sign, corresponds to stimulated absorption. In order to calculate the absorption spectrum, we need to calculate the two-time commutator of Eq.(14). The probe absorption coefficient is given by the difference between the stimulated absorption function and the emission function. However in what follows we analyze the case when the atom is initially prepared in the most excited state. While the field is being initially in a squeezed coherent states. After carrying out the various operations we obtain the absorption spectrum in the form

$$A(\nu) = \Gamma \sum_{s=0}^{k-1} \sum_{j=1}^{3} \sum_{\kappa=1}^{2} |p^{(s)}|^2 \quad |\alpha_j^{(s)}|^2 \quad \Upsilon(\mu_\kappa^{(s)}, E_j^{(s)})$$
$$\times \left[|\alpha_j^{(s)}|^2 |L_\kappa^{(s)}|^2 + \alpha_j^{*(s)} \beta_j^{(s)} L_\kappa^{(s)} q_\kappa^{*(s)} \right.$$
$$\left. + \alpha_j^{(s)} \beta_j^{*(s)} L_\kappa^{*(s)} q_\kappa^{(s)} + |\beta_j^{(s)}|^2 |q_\kappa^{(s)}|^2 \right]$$
$$+ \Gamma \sum_{n=k}^{\infty} \sum_{i,j=1}^{3} |p^{(n)}|^2 \quad |\alpha_j^{(n)}|^2 \quad \Upsilon(E_i^{(n-k)}, E_j^{(n)})$$
$$\times \left[\alpha_j^{(n)}|^2 |\beta_i^{(n-k)}|^2 + \alpha_j^{*(n)} \beta_j^{(n)} \beta_i^{(n-k)} \gamma_i^{*(n-k)} \right.$$
$$\left. + \alpha_j^{(n)} \beta_j^{*(n)} \gamma_i^{(n-k)} \beta_i^{*(n-k)} + |\beta_j^{(n)}|^2 |\gamma_i^{(n-k)}|^2 \right]$$
$$- \Gamma \sum_{n=0}^{\infty} \sum_{i=1}^{3} \sum_{j=1}^{3} |p^{(n)}|^2 |\alpha_j^{(n)}|^2 |\alpha_i^{(n+1)}|^2$$
$$\times \left[|\beta_j^{(n)}|^2 + \beta_j^{(n)} \gamma_j^{*(n)} + \gamma_j^{(n)} \beta_j^{*(n)} + |\gamma_j^{(n)}|^2 \right]$$
$$\times \Upsilon(E_j^{(n)}, E_i^{(n+1)}) \qquad (16)$$

Where

$$\Upsilon(x, y) = \left[\frac{1 + \exp(-2\Gamma T) - 2\exp(-\Gamma T)\cos(\nu + x - y)T}{\Gamma^2 + (\nu + x - y)^2} \right] \qquad (17)$$

Thus the time averaged spectrum consists of resonant structures which arise from transitions among different dressed states. The final structure of the time averaged spectrum will depend on the form of the input photon distribution $p^{(n)}$. As the cavity field starts to interact with the atom the initial photon number distribution $p^{(n)}$ starts to change. Due to the quantum interference between component states the oscillations in the cavity field become composed of different component states.

4. Results and Discussion

On the basis of the analytical solution presented in above, we shall study numerically the absorption spectrum in squeezed coherent states initial field. The photon number distribution for a squeezed coherent state [25] can be written as

$$|P_n|^2 = \frac{(\tanh r)^n}{2^n n! \cosh r} \left| H_n\left(\frac{\varepsilon}{\sqrt{2\cosh r \sinh r}} \right) \right|^2 \times \exp\left[-|\varepsilon|^2 + \tanh r \quad Re(\varepsilon)^2 \right], \qquad (18)$$

where, $\varepsilon = \alpha \cosh r + \alpha^* \sinh r, \quad \alpha = |\alpha| \exp(i\varsigma)$ and H_n is the Hermite polynomial. We suppose here the minor axis of the ellipse, representing the direction of squeezing, parallel to the coordinate of the field oscillator. The initial phase ς of α is the angle between the direction of coherent excitation and the direction of squeezing. The mean photon number of this field is equal to $\bar{n} = |\alpha|^2 + \sinh^2 r.$ Putting $r = 0$ we get the photon distribution for an initial coherent state with $\bar{n} = |\alpha|^2$ whereas for $\alpha = 0$ the photon distribution for an initial squeezed vacuum state with $\bar{n} = \sinh^2 r$ is recovered. The latter distribution is oscillatory with zeros for odd n.

In general, the spectrum attains negative and positive values: a negative value represents amplification and a positive value represents absorption. Since the expression in Eq.(16) depends on the difference in population between the upper and the lower energies of the dressed states, one can easily predicts that at high intensities the absorption spectrum exhibits an equal number of absorbing and amplifying components. The total components appearing spectrum are proportional to the difference between the absorption and emission processes occurring during the transit time T. Also,the various components of absorption spectra have the same positions and widths as in the emission spectrum. so, one can predicts that the components of the spectrum are symmetric around a

central line. In the case of weak field the vacuum and the few photon states play the dominant role. The rates depend on $p(n)$ and $p(n\pm k)$ which differ slightly and consequently they would give non-zero contributions to the central peak. Transitions to atomic levels and the dressed states exhibit a structure in absorption spectra whose components being either absorbing or amplifying. Also, the peak height is related to the photon number $\bar{n}$ and atom-field coupling constant $\lambda_{1(2)}$. While, the peak position is associated with not only the photon number multiplicity and the intensity-dependent atom-field coupling constant $\lambda_i f_i$, but also the frequency Ω of the cavity field distribution of the photons in the field and their characteristics . These aspects would be presented in the discussion that follow.

4.1. Effect of multiplicity and mean photon number

For $k=1$ and small values of the mean photon number we observe that there are a central peak surrounded by two hole-burning and many symmetric numbers of peaks and bulges distributed in both sidebands due to the quantum beats see Fig. $(2.1a)$. As the values of the mean photon number and hence the variance increases we note that the absorption of probe field reduces gradually and the depths of burning holes become shallow. Also, the sideband peaks move away from the central line and the numbers of peaks distributed in both sidebands are gradually increased (compare frames a and b in Fig.$(2).1$). The central absorption peak nearly disappear for a large mean photon number. Due to the equal difference in population between the upper and the lower energies of the dressed states this explains why the central component disappears for a large mean photon number with $k=1$ and for k greater than one see Fig.(2)). Also, for $k>1$ we observe absorption and amplification spikes group interchange in the sidebands. As the mean photon number increase all side spikes move away from the central line and becomes shorter, so that some of these spikes may be disappear at large values of mean photon number.

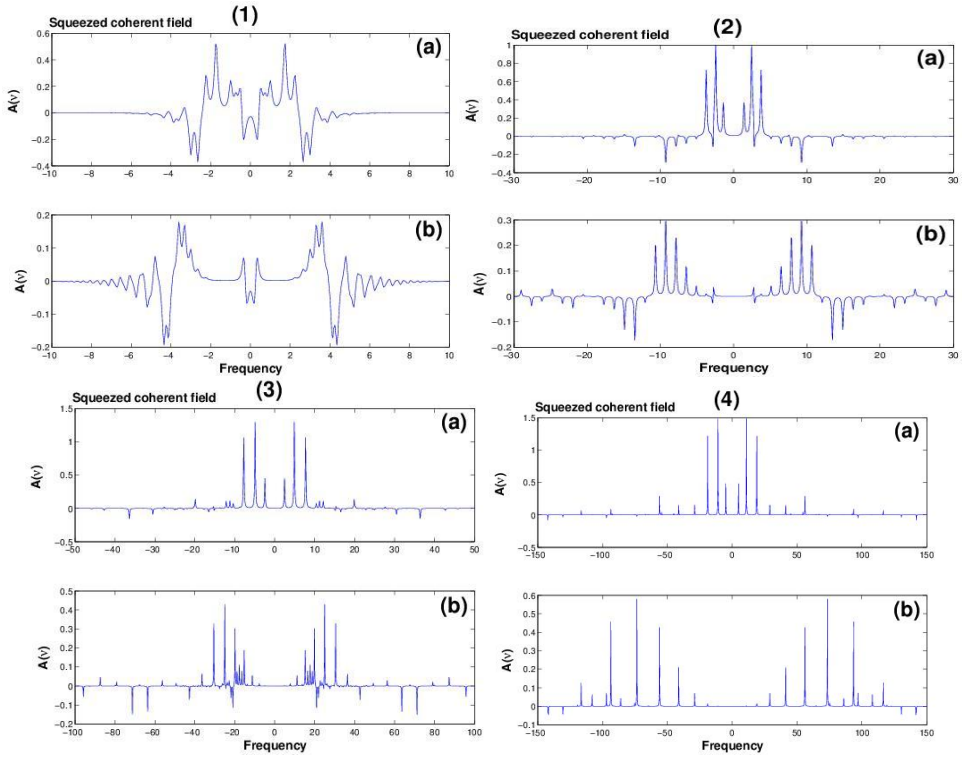

Figure 2: The evolution of the function $A(\nu)$ in a perfect cavity as a function of $(\nu-k\Omega)/\sqrt{\lambda_1\lambda_2}$ with $\lambda_{1,2}=1,\Delta_{1,2}=0,\chi=0,\Gamma=0.1,\varsigma=0,T=100,f_{1,2}=1$ and $(a)r=1.1,\alpha=\sqrt{1}$, $(b)r=1.7,\alpha=\sqrt{5}$, for all $(1)k=1,(2)k=2,(3)k=3,(4)k=4$

Finally, it is interesting to note, weaker amplification and absorption on increasing mean photon number, while weaker amplification and strong absorption on increasing the photon multiplicity k (compare frames in Fig. 2).

4.2. Effect of Detuning

Now, we would like to shed some light on the spectrum behavior when the detuning parameters (Δ_1, Δ_2) differ from zero. In Fig.(3) we examine the absorption spectrum $A(\nu)$ for various values of the detuning parameters Δ_1, Δ_2 for different multi-photon processes $(k = 1,2,3,4)$. Generally, detuning adds asymmetry to the spectrum. For $k = 1$ and small values of the detuning parameters (Δ_1, Δ_2) we note that the peaks are redistributed on both sides of the center giving strong absorption with minimal amplification see Fig. $(3.1a)$. As the values of the detuning parameters increase the right side band shows strong absorption with almost no amplification. Furthermore, the absorption on the left side band disappear, while amplification reduces gradually until it disappears for large detuning parameters. Hence the spectrum tends to only absorption elements in the right hand side as shown in Fig. $(3.1b)$. But, the situation changed for $k = 2,3$, where we note a stronger amplification and absorption of the spectrum in the right side band, while, a weaker amplification and absorption of the spectrum in the left side band as the values of detuning parameters increases. So that, for large detuning parameters the left side band shows almost no absorption or amplification. in the left side band disappear (see Fig. $(3.2b,3b)$.

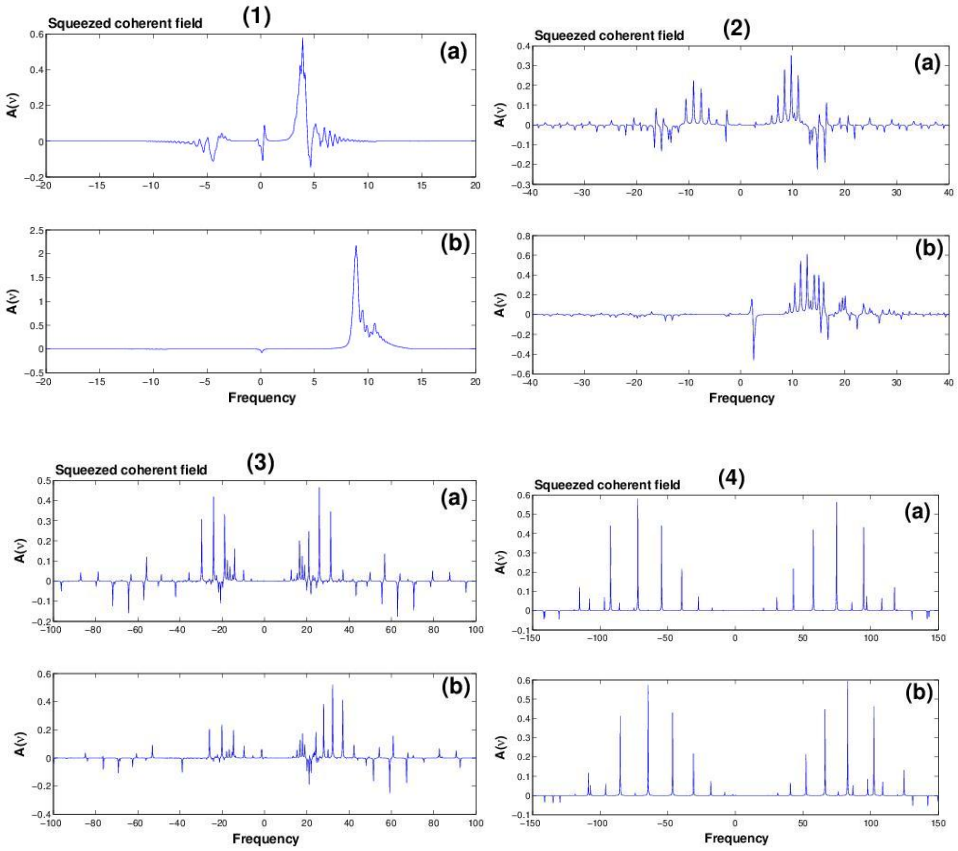

Figure 3: The evolution of the function $A(\nu)$ in a perfect cavity as a function of $(\nu - k\Omega)/\sqrt{\lambda_1 \lambda_2}$ with

$$\lambda_{1,2} = 1, \chi = 0, \Gamma = 0.1, \varsigma = 0, f_{1,2} = 1, T = 100, \alpha = \sqrt{5}, r = 1.7 \text{ and}$$

$$(1)k = 1, (2)k = 2, (3)k = 3, (4)k = 4, \text{ for all } (a)\Delta_1 = 1, \Delta_2 = 2, (b)\Delta_1 = 8, \Delta_2 = 10$$

It is interesting to note that this phenomena takes place in a $k = 2$ case faster than in the $k = 3$ case. But, this phenomena disappear for large $k = 4$, where we have situation similar to that when no detuning added

(compare Fig.(3.4) with Fig.(2.4b)). Finally, we can conclude that detuning adds asymmetry to the spectrum and increase of the heights of the sidebands at the right hand side as we compare the frames of the present figure and Fig. $(2.b)$ where the case of absence of detuning is considered. The sideband on the left hand-side is suppressed, while the sideband on the right hand side gains height and becomes narrower. Also, the shape of the spectrum is changed on both sides of the central line until the one on the left almost disappear while the one on the right gains strong. Furthermore, addition detuning not affect the spectrum for large values of the photon multiplicity (*i.e. k*>3).

4.3. Effect of Kerr medium

Now we will turn our attention to the effect on the absorption spectrum $A(\nu)$ of the nonlinearity of the field with a Kerr-type medium due to the term $\Re(n)$ being taken in the form $\chi n(n-1)$, where χ is related to the third-order nonlinear susceptibility. In fact the optical Kerr effect is one of the most extensively studied phenomenon in the field of nonlinear optics because of its applications [26-28]. In general, addition of the Kerr-like medium parameter to the problem adds asymmetry to the absorbtion spectrum and changes the shape of the spectrum. For small values χ the absorption and amplification peaks are redistributed on both side of the spectrum. Also, the right hand sideband is suppressed while the left hand sideband gains height and becomes narrower gradually as the values of χ increased. So, for large values of χ the spectrum components in the right side band almost disappear. It is interesting to note that, this phenomena appearing in fast way as k decrease (compare frames b in Fig.(4)).

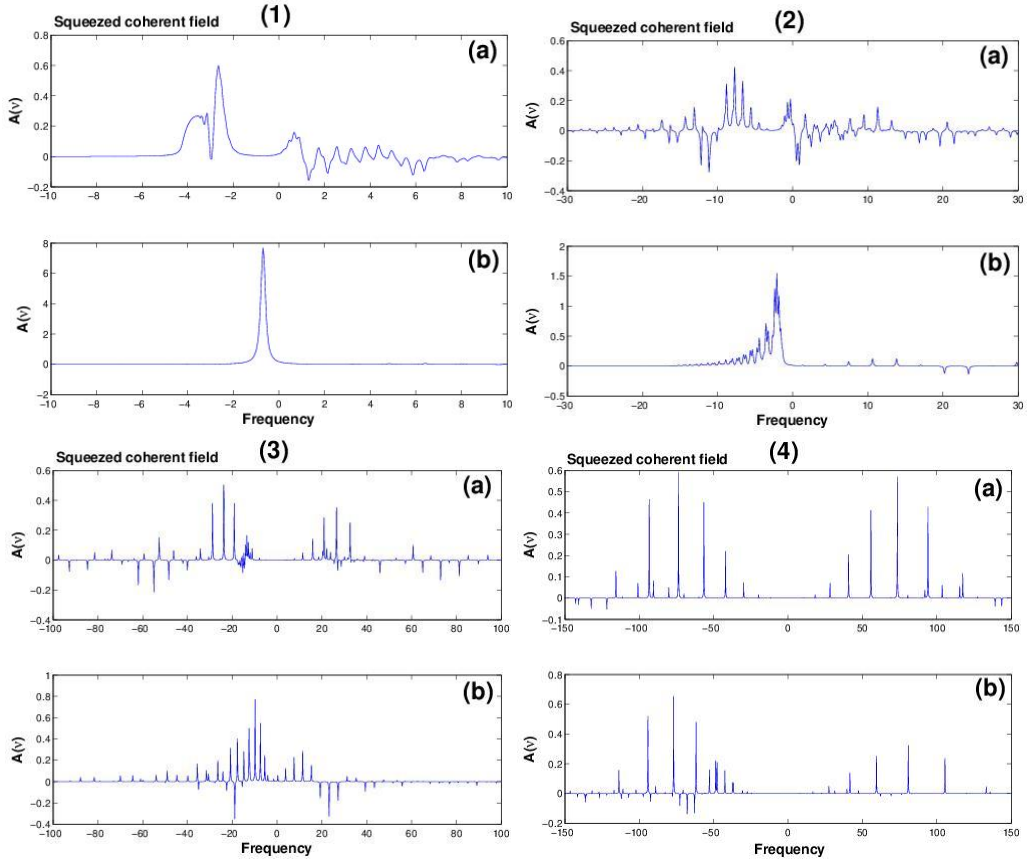

Figure 4: The evolution of the function $A(\nu)$ in a perfect cavity as a function of $(\nu-k\Omega)/\sqrt{\lambda_1\lambda_2}$ with $\lambda_{1,2}=1,\Delta_{1,2}=0,\Gamma=0.1,\varsigma=0,\ f_{1,2}=1,T=100,\alpha=\sqrt{5},r=1.7,$ and for all (a) $\chi=0.1$, (b) $\chi=0.8$ with $(1)k=1,(2)k=2,(3)k=3,(4)k=4$

4.4. Effect of intensity dependent coupling functional

In Fig. (5) we study the effect of different functionals of the intensity dependent coupling $f_1(n), f_2(n+k)$ on the absorption spectrum $A(\nu)$ for different multi-photon processes k. Generally we can observe that the structure shown in Figs. (2.b) is no longer evident here. Due to the larger Rabi-frequencies when we take $f_1(n)=\sqrt{n}, f_2(n+k)=\sqrt{n+k}$ than that for the case $f_1(n)=f_2(n+k)=1$ the stepwise excitation become larger. So, the range of the spectra becomes larger and the spectrum elements divide into groups one absorbtion and other amplification beside a number of small outer elements emerge due to quantum beats which are nearly nonvisible see Fig. $(5a)$. As k increase the absorption peaks becomes stronger while the amplification weaker, so for large $k(i.e., k=4)$ we see maximum absorption and no amplification see Fig. $(5.4a)$.

But for $f_1(n)=\frac{1}{\sqrt{n+k}}, f_2(n+k)=\frac{1}{\sqrt{n+k}}$ as in Fig.(5.b) the spectrum behavior varies greatly from the previous case. In this case Rabi frequencies becomes smaller than that at $f_1(n)=f_2(n+k)=1$, hence the stepwise excitation are very small. So that the spectrum components become closer to each other, and the range of the spectrum decreases (compare Fig.(5.b) with Fig.(2.b)). It is interesting to be noted that, we still have a stronger absorption and weaker amplification on increasing k. Finally, the absorption spectrum can be controlled by choosing the right intensity dependent coupling functional.

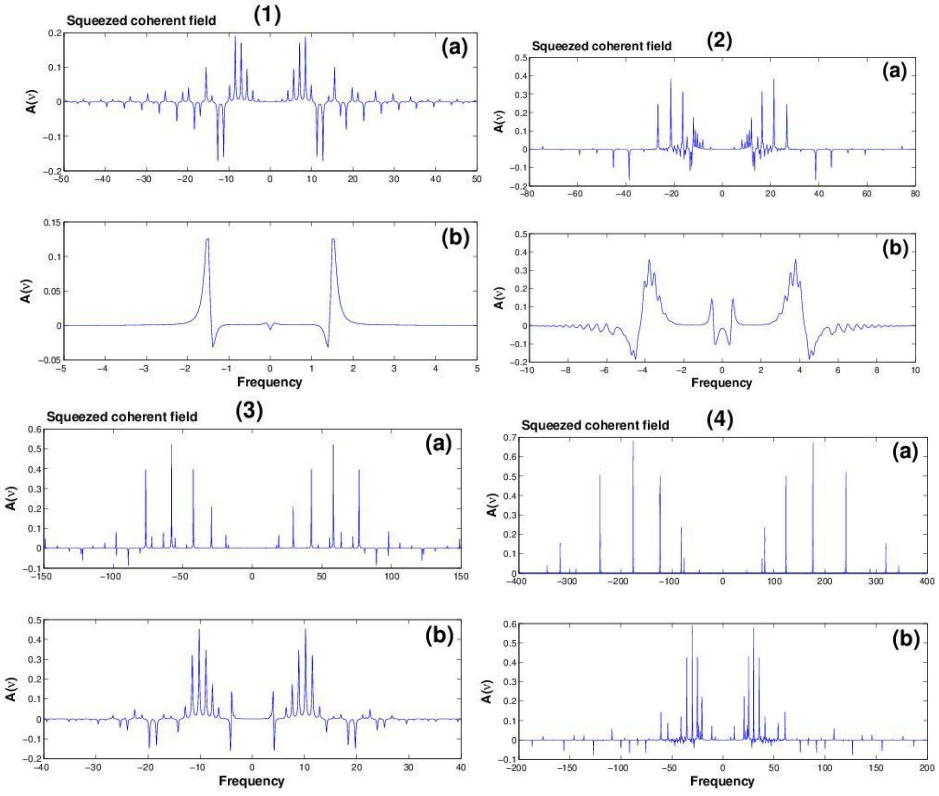

Figure 5: The evolution of the function $A(\nu)$ in a perfect cavity as a function of $(\nu-k\Omega)/\sqrt{\lambda_1\lambda_2}$ with $\lambda_{1,2}=1, \Delta_{1,2}=0, \chi=0, \Gamma=0.1, \varsigma=0, T=100, \alpha=\sqrt{5}, r=1.7, (1)k=1, (2)k=2, (3)k=3, (4)k=4$, and for all $(a)\, f_1(n)=\sqrt{n}, f_2(n+k)=\sqrt{n+k}$, $(b)\, f_1(n)=\frac{1}{\sqrt{n+k}}, f_2(n+k)=\frac{1}{\sqrt{n+k}}$

5. Conclusion

In summary, We have investigated the absorption spectrum for a general formalism for a multi-photon Ξ -type three-level atom, taking into account arbitrary forms of nonlinearities of both the field and the intensity-dependent atom-field coupling. We have explored the influence of various parameters of the system on the absorption spectrum. We have used the finite double-Fourier transform of the two-time field correlation function to find an analytical expression for the spectra. The absorption spectrum in the cavity is calculated for the field to be initially in a squeezed coherent state.It is observed that:

- The absorption and amplification peaks position is associated with not only the photon number $(\bar{n})$ and the photon multiplicity number k but also the intensity-dependent atom-field coupling constant $\lambda_i f_i(n)$.
- The absorption and amplification components becomes weaker and the distances between them is larger as the mean photon number increased.
- The symmetry shown in the standard three-level atom model for the absorption spectrum is no longer present once Kerr effect or detuning is considered.
- The effect of detuning on the absorption spectrum is twofold. The first effect is the shift of the spectrum components to the right hand side. The second effect is the dependence of the amplitudes and heights of the peaks on Δ_i.
- The Kerr medium has an effect opposite to the effect of the detuning , where the earlier has shorter elements. Also, the heights and widths of the peaks not only depend on the photon multiplicity but also on the value of χ. Consequently, changes in the detuning and the Kerr medium parameters can show in the spectra, and hence the heights of the peaks, their shifts and widths are altered compared the case of resonance.
- The strong field effects can be produced by choosing the right parameters for these nonlinearities.

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