



Optimization Approach to Power Generation and Scheduling

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Abstract The proposed optimization model to power generation and scheduling is considered in this paper. The value or price cut-off for buying or selling electricity generation is determined. The data obtained from the bureau of statistics were modelled as a non Linear Programming problem (polynomial of second degree). The Matlab software is used to generate an optimal solution from which a fair price λ is obtain, sometimes called shadow price. Specifically, the value of the Lagrange multiplier λ is the rate at which the optimal value of the objective function f changes if you change the constraints (Crow 2003) [1]. It is found that in Nigeria, the true cost of electricity production is not reflected in the consumer tariff, to be cost-reflective and provide financial incentives for urgently-needed increased investments in the industry. These investments, in turn, lead to a significant and continuous improvement in the quantity of energy and quality of service enjoyed by the consumer.

Keywords Optimization, Lagrange Multiplier, Constraint, Price Cut-Off, Optimal Value Price

1. Introduction

Electricity supply in Nigeria dates back to 1886 when two small generating sets were installed to serve the then Colony of Lagos. By an Act of Parliament in 1951, the Electricity Corporation of Nigeria (ECN) was established, and in 1962, the Niger Dams Authority (NDA) was also established for the development of Hydro Electric Power. However, a merger of the two (2) was made in 1972 to form the National Electric power Authority (NEPA 1995) [2]. As a result of unbundling and the power reform process, was renamed Power holding Company of Nigeria (PHCN) in 2005. The Nigerian power sector is controlled by state-owned Power Holding Company of Nigeria (PHCN), formerly known as the National Electric Power Authority (NEPA). In March 2005, the government signed the Power Sector Reform Bill into law, enabling private companies to participate in electricity generation, transmission, and distribution. The government has separated PHCN into eleven distribution firms, six generating companies, and a transmission company, all of which will be privatized. Several problems, including union opposition, have delayed the privatization, which was later rescheduled for 2006. In February 2005, the World Bank agreed to provide PHCN with \$100 million to assist in its privatization efforts. The Nigerian government has made an effort to increase foreign participation in the electric power sector by commissioning independent power projects (IPPs) to generate electricity and sell it to PHCN. In April 2005, Agips 450MW plant came online in Kwale in Delta State. The NNPC and Joint Venture (JV) partners, ConocoPhillips and Agip, provided the \$480 million to construct the plant. IPPs currently under construction include the 276MW Siemens station in Afam, Exxon Mobils 388MW plant in Bonny, ABBs 450MW plant in Abuja, and Eskoms 388MW plant in Enugu. Several state governments have also commissioned Oil majors to increase generation including Rivers State, which contracted Shell to expand the 700MW Afam station. The Nigerian government also approved the construction of four thermal power plants (Geregu, Alaoji, Papalanto, and Omotosho), with a combined capacity of 1,234MW to meet its generating goal of 6,500MW in 2006. In



addition fourteen hydroelectric and Natural Gas plants were planned for kick-up but yet to commence since then. Chinas EXIM Bank Su Zhong and Sino Hydro have committed to funding the Mambilla (3,900MW) and Zungeru (950MW) hydroelectric projects. In addition, Sino Hydro proposed that it should construct the two power projects. Also, NNPC with Chevron are to construct a 780MW gas-fired thermal plant in Ijede, Lagos State. The project was expected to be constructed in three phases, with the first two phases expected to have capacity of 256MW each. The plant is expected to be operational in 2007 but yet to commence construction. The Total Installed Capacity of the currently generating plants is 8,417MW but the Installed available Capacity is less than 4,000MW as at December 2010. Seven of the twenty one generation stations are over 20 years old and the average daily power generation is below 3,000MW which is far below the peak load forecast of 8,900MW for the currently existing infrastructure. As a result, the nation experiences massive load shedding Oyelami (2013) [3]. Through the planned generation capacity projects for a brighter future, the current status of power generation in Nigeria presents the following challenges:

- i. Inadequate generation availability;
- ii. Inadequate and delayed maintenance of facilities;
- iii. Insufficient funding of power stations;
- iv. Under-pricing of electricity;
- v. Inadequate and obsolete communication equipment
- vi. Lack of exploration to tap all sources of energy form the available resources; and
- vii. Low staff morale.

2. Model Formulation of Problem:

Let,

$$f_1 = a_1x_1 + b_1x_1^2 + c_1 \quad (1)$$

$$f_2 = a_2x_2 + b_2x_2^2 + c_2 \quad (2)$$

Where, x_i is the output power of the i^{th} generator; f_i is the cost per hour of the i^{th} generator. The cost function f_i with respect to x_i is generated by polynomial curve fitting based on the generator operation data. x_i has the unit Mw. Hence, the cost f_i has the unit naira per hour, $i=1, 2, \dots, n$

The first step in determining the optimal scheduling of the generators is to express the problem in mathematical form. Crow (2003) [1]. Thus the optimization statement is:

$$\min: f = f_1 + f_2 = a_1x_1 + b_1x_1^2 + c_1 + a_2x_2 + b_2x_2^2 + c_2 \quad (3)$$

Subject to:

$$G = x_1 + x_2 - load = 0 \quad (4)$$

The corresponding lagrange function is:

$$F = a_1x_1 + b_1x_1^2 + c_1 + a_2x_2 + b_2x_2^2 + c_2 - \lambda(x_1 + x_2 - load) \quad (5)$$

Setting $\nabla F = 0$ yield the linear equations.

3. Solution of the problem

The Lagrange multipliers method for nonlinear optimization problems only with equality constraints is discussed. Then the method is extended to cover the inequality constraints. Without the inequality constraints, the standard form of the nonlinear optimization problems can be formulated Arfken and Weber (2005) [4].

The constrained optimization problems can be formulated into the standard form Bertsekas (1982) [5]:

$$\min f(x_1, \dots, x_n)$$

$$\text{subject to } G(x_1, \dots, x_n) = 0$$

Where, $G = [G_1(x_1, \dots, x_n) = 0, \dots, G_k(x_1, \dots, x_n) = 0]^T$, The constraints functions vector.

The Lagrange function F is constructed as

$$F(X, Y) = F(X) - \lambda G(X)$$

$$\lambda_k \geq 0, \quad \frac{\partial L}{\partial \lambda_k} \geq 0, \quad \lambda_k \cdot \frac{\partial L}{\partial \lambda_k} = 0, \quad k = 1 \dots m$$



Where, $X=[x_1, \dots, x_n]$, The variable vector, $\lambda=[\lambda_1, \dots, \lambda_k]$, $[\lambda_1, \dots, \lambda_k]$ are called Lagrange multipliers (Shofield 2003) [6].

Where, $X=[x_1, \dots, x_n]$,

The extreme points of the f and the Lagrange multipliers λ Satisfy:

$$\nabla F = 0 \quad (6)$$

Implies

$$\frac{\partial f}{\partial x_i} - \sum_{m=1}^k \lambda_m \frac{\partial G_m}{\partial x_i} = 0, i = 1, \dots, n$$

$$\text{And } G(x_1, \dots, x_n) = 0$$

Let H be the bordered hessian of L , then $H(\lambda, x)$ is positive definite at (λ, x) .

[This is so if last $n - m$ leading principle minor of H at (λ, x) all have the same as $(-1)^m$]

Theorem

The second order condition for a function $y = f(x) \in \mathbb{R}^2$ to have a local minimum or local maximum at a turning point $x = a$ i.e. the quadratic form

$x^T H(f(a)) x$ to be negative definite or positive definite otherwise the turning point is called saddle point.

Using Matlab to Solve Linear Algebraic Equations

Many of the applications of linear algebra such as matrices, determinants, systems of equations and the eigenvalue problem can all be easily handled using MATLAB. You should already have a basic idea of how to create and manipulate matrices (arrays) in MATLAB.

This system of equations could also be solved using Cramer's Rule using a little more programming and incorporating determinants. The code below does the job

```
A= [a11,a12,a13;a21,a22,a23;a31,a32,a33];
```

```
b= [b1,b2,b3];
```

```
X= inv (A)*b
```

Matrix A contains the coefficients of the unknown variables. The coefficients of x_1 are in column 1, x_2 in column 2, and λ in column 3.

Vector X will contain the solution to the unknown variables.

Vector b contains the constraints for each equation, stored in a column vector.

Note that the two methods of solution (Inverse and Cramer's Rule) are not computationally efficient for large systems that would have say more than 102 equations. For such cases, techniques of Gauss elimination, Gauss Seidel iteration, and methods of Doolittle, Crout and Cholesky are often used.

Matlab Code for Polynomial Curve Fitting

Polyfit is going to create a polynomial regression line of best fit, the first argument x variable is the output power of the i th generating plant. The second variable f is the cost value of the i th generating plant from the DATA and the third argument which is 2 is the degree of the polynomial.

```
fx>>polyfit(x,f,2)
```

The coefficients of x_1, x_2 and c_1, c_2 are determined

4. Result and Discussion

$$a_2 = 15$$

$$b_2 = 0.03$$

$$c_2 = 400$$

$$x_1 = 3538.2$$

$$x_2 = 841.8$$

$$\lambda = 90.8$$



The coefficients $a_1, b_1, c_1, a_2, b_2, c_2$ of x_1, x_2 and c_1, c_2 are determined

x_1 is the output power of the thermal generating plant

x_2 is the output power of the hydro generating plant

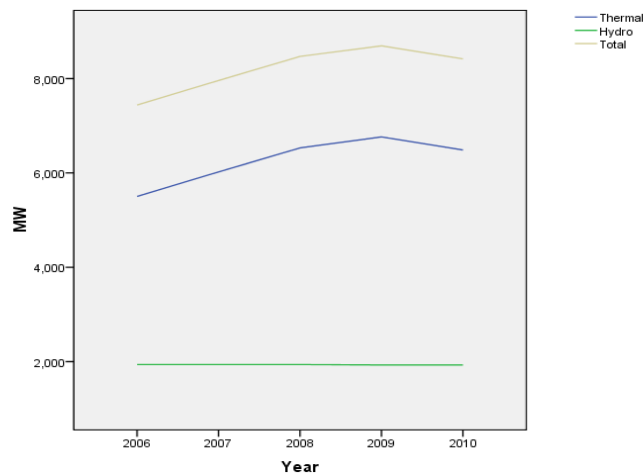
This is the generation scheduling that minimizes the hourly cost of production. The value of λ is the incremental of break-even cost of production. This gives a company a price cut-off for buying or selling generation:

If they can purchase generation for less than λ , then their overall costs will decrease. Likewise, if generation can be sold for greater than λ , their overall costs will decrease.

5. Conclusion

Electricity differs from other products in that it cannot be economically stored as it is produced the implication of instantaneous supply and consumption is that price has to be sufficient to cover the cost of production, otherwise supply will be jeopardized. It is imperative that electricity should be priced properly such that it covers its supply costs, in order for adequate and reliable electricity is to be produced to meet demand. As with any other product, its price must, at the minimum, cover the operating and capital costs. If the price is at a sufficient level to ensure a reasonable return on investment, it will keep the current producers, and also attract new producers.

In Nigeria, the true cost of electricity production is not reflected in the consumer tariff, to be cost-reflective and provide financial incentives for urgently-needed increased investments in the industry. These investments, in turn, lead to a significant and continuous improvement in the quantity of energy and quality of service enjoyed by the consumer.



BARChart FOR THERMAL POWER GENERATION

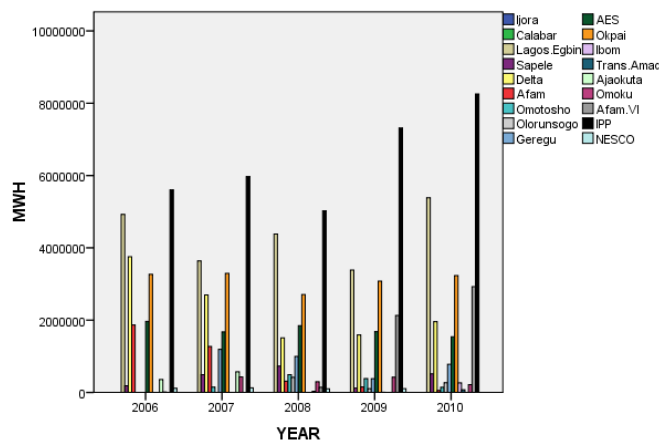


Figure 1: The generated power from 2006 - 2010

Source: National Bureau of Statistics (NBS), Annual Abstract of Statistics 2011

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