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Properties of Operations on Soft Ideals and Idealistic Soft Ring

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Abstract Soft set theory initiated by Molodtsov is an important mathematical tool that deals with uncertainties about imprecision and vagueness. Research on soft ideals of ring has been carried out by other researchers. In this paper, we discuss on some properties of soft ring ideals and idealistic soft ring. We state and prove some important theorems and propositions on soft ring ideals and idealistic of soft ring that has not been studied by other researchers.

Keywords Soft set, operations, soft ring, soft ring ideal, idealistic soft ring

Introduction

In real life situation, there are serious complicated problems in many areas such as economics, engineering, environment, medicine, social and management sciences. They are characterized with uncertainties, imprecision and vagueness. We cannot successfully utilize the classical methods to handle these problems because there are various types of uncertainties involved in these problems. Moreover, though there are many theories, such as theory of probability, theory of fuzzy sets, theory of interval mathematics and theory of rough sets to be considered as mathematical tools to deal with uncertainties, Molodtsov [1] proposed a new mathematical tool named soft set theory to deal with uncertainty, imprecision and vagueness. This theory has been demonstrated to be useful tool in many applications such as decision making, measurement theory and game theory. Maji *et al.* [2] worked on theoretical study of soft sets in detail and [3] presented an application of soft set in decision making problems using the reduction of soft set.

Aktas and Cagman [4] introduce a definition of soft groups and their basic properties. Park *et al.* [5] worked on the notion of WS-algebras, soft subalgebras and soft deductive systems. Jun [6] dealt with the algebraic structure of BCK/BCI-algebras by applying soft set theory. Jun and Park [7] presented the notion of soft ideals, idealistic soft and idealistic soft BCK/BCI-algebra. Jun *et al.* [8] applied soft set theory to commutative ideals in BCK-algebras.

In this study we discussed on soft ring ideal and idealistic soft ring with relevant examples and investigate some basic properties of idealistic soft ring.

The rest of the paper is organized as follows: In section 2, we present some basic definitions in soft sets and its operations. Section 3, we define soft ideals and idealistic soft ring. we also state and prove important theorems and propositions. Section 4, summarizes the entire work.

2. Preliminaries

In this section we recall the concept of soft set theory and some basic definitions. Let U be an initial universe set, E be a set of parameters or attributes with respect to U, P(U) be the power set of U and A⊆E.

Definition 2.1.[1]

A pair (Γ, A) is called a **soft set** over U, where Γ is a mapping given by $\Gamma: A \longrightarrow P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $x \in A$, $\Gamma(x)$ may be considered as the set of x-elements or as the set of x-approximate elements of the soft set (Γ, A). Thus, (Γ, A) is defined as:

$(\Gamma, A) = \{\Gamma(x) \in P(U), x \in A\}$

The soft set (Γ, A) can be represented as a set of ordered pairs as follows:

$(\Gamma, A) = \{(x, \Gamma(x)), x \in A, \Gamma(x) \in P(U)\}$

Example 2.1.

Let $U = \{c_1, c_2, c_3, c_4, c_5\}$ be a universal set consisting of five cars and $E = \{e_1, e_2, e_3, e_4\}$ be the set of parameters under consideration, where each parameter e_i $i = 1, 2, 3, 4$ stands for, Expensive , Fuel economy, Fast, Modern, respectively.

Let $A = \{e_1, e_3, e_4\} \subset E$ such that $\Gamma(e_1) = \{c_1, c_3\}$, $\Gamma(e_3) = \{c_1, c_2, c_4\}$ and $\Gamma(e_4) = \{c_4\}$. Then the soft set (Γ, A) over U is given by $(\Gamma, A) = \{(e_1, \{c_1, c_3\}), (e_3, \{c_1, c_2, c_4\}), (e_4, \{c_4\})\}$.

Definition 2.2.[9]

Let (Γ, A) and (G, B) be two soft sets over U. Then

(Γ, A) is said to be a **soft subset** of (G, B), denoted by $(\Gamma, A) \tilde{\subseteq} (G, B)$, if $A \subseteq B$ and $\Gamma(x) \subseteq G(x), \forall x \in A$

Definition 2.3.[10]

Let (Γ, A) be a soft set over U. Then the support of (Γ, A) written **supp** (Γ, A) is the set defined as; $supp(\Gamma, A) = \{x \in A: \Gamma(x) \neq \emptyset\}$

(i) (Γ, A) is called a **non-null** soft set if supp $(\Gamma, A) \neq \emptyset$.

(ii) (Γ, A) is called a **relative null** soft set denoted by $\emptyset_A$ if $\Gamma(x) = \emptyset, \forall x \in A$

(iii) (Γ, A) is called a **relative whole** soft set, denoted by U_A if $\Gamma(x) = U, \forall x \in A$

Definition 2.4.[10]

Let (Γ, A) and (G, B) be two soft sets over U. Then the **union** of (Γ, A) and (G, B), denoted by $(\Gamma, A) \tilde{\cup} (G, B)$ is a soft set defined as: $(\Gamma, A) \tilde{\cup} (G, B) = (H, C), where\ C = A \cup B\ and\ \forall x \in C$

$$H(x) = \begin{cases} \Gamma(x), & if\ x \in A - B \\ G(x), & if x \in B - A \\ \Gamma(x) \cup G(x), & if\ x \in A \cap B \end{cases}$$

Definition 2.5.[10]

Let (Γ, A) and (G, B) be two soft sets over U. Then the **restricted union** of (Γ, A) and (G, B), denoted by (Γ, A) $\tilde{\cup}_R$ (G, B) is a soft set defined as;

$(\Gamma, A) \tilde{\cup}_R (G, B) = (H, C)$, where $C = A \cap B \neq \emptyset$ and $\forall x \in C$

$H(x) = \Gamma(x) \cup G(x)$.

Definition 2.6.[10]

Let (Γ, A) and (G, B) be two soft sets over U. Then the **extended intersection** of (Γ, A) and (G, B), denoted by $(\Gamma, A) \tilde{\cap}_E (G, B) = (H, C)$, where $C = A \cup B$ and $\forall x \in C$

$$H(x) = \begin{cases} \Gamma(x), & if\ x \in A - B \\ G(x), & if x \in B - A \\ \Gamma(x) \cap G(x), & if\ x \in A \cap B \end{cases}$$

Definition 2.7. [10]

Let (Γ, A) and (G, B) be two soft sets over U. Then the **restricted intersection** of (Γ, A) and (G, B) denoted by $(\Gamma, A) \Cap (G, B) = (H, C)$, where $C = A \cap B \neq \emptyset$ and $\forall x \in C$, $H(x) = \Gamma(x) \cap G(x)$.

Definition 2.8. [10]

Let (Γ, A) and (G, B) be two soft sets over U. Then the **AND-produc**t or **AND-intersection** of (Γ, A) and (G, B) denoted by $(\Gamma, A) \tilde{\wedge} (G, B)$ is a soft set defined as $(\Gamma, A) \tilde{\wedge} (G, B) = (H, C), where\ C = A \times B\ and\ \forall (x, y) \in A \times B$

$H(x, y) = \Gamma(x) \cap G(y)$.

Definition 2.9.[10]

Let (Γ, A) and (G, B) be two soft sets over U. Then the **OR-product** or **OR-union** of (Γ, A) and (G, B), denoted by $(\Gamma, A)\widetilde{\vee}(G, B)$ is a soft set defined as $(\Gamma, A)\widetilde{\vee}(G,B) = (H, C), where\ \ C = A \times B\ and\ \forall\ (x, y) \in A \times B$ $H(x, y) = \Gamma(x) \cup G(y)$.

Definition 2.10.[10]

Let (Γ, A) and (G, B) be two soft sets over the universes U_1 and U_2 respectively. Then the **Cartesian product** of (Γ, A) and (G, B), denoted by

$(\Gamma, A)\ \widetilde{\times}\ (G, B)$ is a soft define as: $(\Gamma, A)\ \widetilde{\times}\ (G, B) = (\mathrm{H}, \mathrm{C})$, where $\mathrm{C} = \mathrm{A} \times \mathrm{B}$ and $\forall\ (x, y) \in \mathrm{A} \times \mathrm{B}$ $H(x, y) = \ \Gamma(x) \times \ \mathrm{G}(y)$.

Definition 2.11.

Let (Γ, A) and (G, B) be two nonempty soft sets over U. The **sum** $(\Gamma, A)\widetilde{+}(G, B)$ is define as the soft set $(H, C) = (\Gamma, A)\widetilde{+}(G, B)$, where $C = A \times B$ and $H(x, y) = \ \Gamma(x) + G(y), \forall\ (x, y) \in C$.

Definition 2.12.[9]

Let $(\Gamma_i, A_i)_{i\in I}$ be a nonempty family of soft sets over a common universe U. The union of these soft sets is defined to be the soft set (G, B) such that $B = \cup_{i\in I} A_i$ and for all $x \in B$, $G(x) = \cup_{i\in I(x)} \Gamma_i(x)\ where\ I(x) = \{i \in I: x \in A_i\}$. In this case we write $\widetilde{\cup}_{\,i\in I}\ (\Gamma_i,\ A_i) = \ (G, B)$.

Definition 2.13. [9]

Let $(\Gamma_i, A_i)_{i\in I}$ be a nonempty family of soft sets over a common universe U. The AND-soft set $\widetilde{\wedge}_{\boldsymbol{i\in I}}(\Gamma_i,\ A_i)$ of these soft sets is defined to be the soft set (H, B) such that $B = \prod_{i\in I} A_i$ and $H(x) = \cap_{i\in I} \Gamma_i(x_i)\ for\ all\ \ x = (x_i)_{i\in I} \in B$.

Definition 2.14. [13]

Let $(\Gamma_i, A_i)_{i\in I}$ be a nonempty family of soft sets over a common universe set U. The OR-soft set $\widetilde{\vee}_{\,i\in I}(\Gamma_i,\ A_i)$ of these soft sets is defined to be the soft set (H, B) such that $B = \prod_{i\in I} A_i$ and $H(x) = \cup_{i\in I} \Gamma_i(x_i)$ for all $x = (x_i)_{i\in I} \in B$.

Note that if $A_i = A$ and $\Gamma_i = \ \Gamma$ for all $i \in I$, then $\widetilde{\wedge}_{\boldsymbol{i\in I}}(\Gamma_i,\ A_i)$ (res. $\widetilde{\vee}_{\,i\in I}(\Gamma_i,\ A_i)$) is denoted by $\widetilde{\wedge}_{\boldsymbol{i\in I}}(\Gamma, A)$ **(**res. $\widetilde{\vee}_{\,i\in I}(\Gamma, A)$**).** In this case, $\prod_{i\in I} A_i = \prod_{i\in I} A$ means the direct power A^I.

Definition 2.15.[13]

The restricted union of a nonempty family of soft sets $(\Gamma_i, A_i)_{i\in\Delta}$ over a common universe U is defined as the soft set $(H, B) = \widetilde{\cup}_{R\ i\in\Delta}\ (\Gamma_i,\ A_i)$, where $B = \cap_{i\in\Delta}\ A_i \neq \emptyset$ and $H(x) = \cup_{i\in\Delta} \Gamma_i(x)$ for all $x \in B$.

Definition 2.16.[13]

The extended intersection of a nonempty family of soft sets $(\Gamma_i,\ A_i)_{i\in\Delta}$ over a common universe set U is defined as the soft set $(H, B) = \widetilde{\cap}_{E\ \ i\in\Delta}\ (\Gamma_i, A_i)$ such that $B = \cup_{i\in\Delta} A_i$ and $H(x) = \cap_{i\in\Delta} \Gamma_i(x)$ where $\Delta(x) = \{i \in \Delta: x \in A_i\}$ for all $x \in B$.

Definition 2.17.[13]

Let $(\Gamma_i, A_i)_{i\in\Delta}$ be a nonempty family of soft sets over a common universe set U. The restricted intersection of these soft sets is defined to be the soft set (G, B) such that $B = \cap_{i\in\Delta} A_i \neq \emptyset$ and for all $x \in B, G(x) = \cap_{i\in\Delta} \Gamma_i(x)$. In this case we write $\Cap_{\boldsymbol{i}\in\Delta}\ (\Gamma_i,\ A_i) = (G, B)$.

Definition 2.18. [13]

Let $(\Gamma_i, A_i)_{i\in\Delta}$ be a nonempty family of soft sets over $U_i, i \in \Delta$. The Cartesian product of $(\Gamma_i, A_i)_{i\in\Delta}$ over U_i is defined as the soft set $(H, B) = \widetilde{\prod}_{i\in\Delta}\ (\Gamma_i, A_i)\ where\ B = \prod_{i\in\Delta}\ A_i$ and $H(x) = \ \prod_{i\in\Delta} \Gamma_i(x_i)\ \ for\ all\ \ x = (x_i)_{i\in\Delta} \in B$. It is worth noting that if $A_i = A$ and $\Gamma_i = \ \Gamma$ for all $i \in \Delta$, then $\widetilde{\prod}_{i\in\Delta}\ (\Gamma_i, A_i)$ is denoted by $\prod_{i\in\Delta}(\Gamma, A)$. In this case $\prod_{i\in\Delta}(\ A_i) = \ \prod_{i\in\Delta} A$ means the direct power A^{Δ}.

Definition 2.19.[13]

Let $(\Gamma_i, A_i)_{i\in\Delta}$ be a nonempty family of soft sets over $U_i, i \in \Delta$. The sum of $(\Gamma_i, A_i)_{i\in\Delta}$ over U_i is defined as the soft set $(H, B) = \ \widetilde{\Sigma}_{i\in\Delta}\ (\Gamma_i, A_i)\ where\ B = \prod_{i\in\Delta}\ A_i$ and $H(x) = \Sigma_{i\in\Delta}\Gamma_i(x_i)\ \ for\ all\ \ x = \ (x_i)_{i\in\Delta} \in B$.

Definition 2.4 [13]. Let (Γ, A) be a soft set over U. If $\Gamma(x) \neq \emptyset$ for all $x \in A$, then (Γ, A) is called a non-empty soft set.

3. Soft Ideals

Definition 4.1[14] Soft Ring

Let $\mathbb{R}$ be a ring and let (Γ, A) be a non-null soft set over $\mathbb{R}$. Then (Γ, A) is called a **soft ring** over $\mathbb{R}$ if $\Gamma(x)$ is a sub ring of $\mathbb{R}$, denoted by $\Gamma(x) <_r \mathbb{R}$, $\forall$ all $x \in A$.

Example 3.1

Let $\mathbb{R}$ =A=$\mathbb{Z}_6$={0,1,2,3,4,5}. Consider the soft set (Γ, A) over $\mathbb{R}$, where $\Gamma: A \to P(\mathbb{R})$ is a set valued function defined by $\Gamma(x)$ ={y∈ $\mathbb{R}$: x Ry ⇔ x.y=0}. Then Γ (0)= $\mathbb{R}$, Γ (1)={0}, Γ (2)={0,3}, Γ (3)={0,2,4}, Γ (4)={0,3} and Γ (5)={0} which are all sub rings of $\mathbb{R}$. Hence, (Γ, A) is a soft ring over $\mathbb{R}$.

3.1. Soft Ideal of a Soft Ring

In classical algebra, the notion of ideals is very important. For this reason, we present the definition of soft ideals of a soft ring.

Note that if I is an ideal of a ring $\mathbb{R}$, we write I $\tilde{\lhd}$ $\mathbb{R}$.

Definition 3.2

Let (Γ, A) be a soft ring over $\mathbb{R}$. A non-null soft set (I, B) over $\mathbb{R}$ is called a **soft ring ideal** of (Γ, A) denoted by $(I, B) \tilde{\lhd}_R (\Gamma, A)$, if it satisfies the following conditions:

(1) B⊂A

(2) $I(x)$ is an ideal of $\Gamma(x)$ for all $x \in$ supp (I, B).

Example 3.2

Let $\mathbb{R}$ =A=$\mathbb{Z}_4$={0,1,2,3} and B={0,1,2}.

Consider the set-valued function Γ:A→P($\mathbb{R}$), given by $\Gamma(x)$={y ∈ $\mathbb{R}$: x Ry ⇔ x.y ∈{0,2}}. Then Γ (0)= $\mathbb{R}$, Γ (1)={0,2}, Γ (2)={0,1,2,3} and Γ (3)={0,2} are sub rings of $\mathbb{R}$.Thus (Γ, A) is a soft ring over $\mathbb{R}$.

Consider the function I:B→P($\mathbb{R}$) given by I(x)={y ∈ $\mathbb{R}$ | x Ry ⇔ x.y=0}⇒I(0)= $\mathbb{R}$ ⊲ Γ (0), I(1)={0} ⊲ Γ (1)={0,2} and I(2)={0,2} ⊲ Γ (2)=$\mathbb{Z}_4$. Hence $(I, B) \tilde{\lhd} (\Gamma, A)$.

Definition 3.3

Let (Γ, A) be a non-null soft set over $\mathbb{R}$. Then (Γ, A) is called an **idealistic soft ring** over $\mathbb{R}$ if $\Gamma(x)$ is an ideal of $\mathbb{R}$ for all $x \in$ supp (Γ, A).

Example 3.3

In Examples (3.1 &3.2), (Γ, A) are idealistic soft ring over $\mathbb{R}$ for all $x \in A$.

Theorem 3.1. Let (Γ, A) and (G, B) be idealistic soft ring over R. Then

(i) $(\Gamma, A) \tilde{\cup} (G, B)$ is an idealistic soft ring over R, if $A \cap B = \emptyset$.

(ii) $(\Gamma, A) \tilde{\wedge} (G, B)$ is an idealistic soft ring over R, if it is non-null.

(iii) $(\Gamma, A) \Cap (G, B)$ is an idealistic soft ring over$R$, if it is non-null and $A \cap B \neq \emptyset$.

(iv) $(\Gamma, A) \tilde{\vee} (G, B)$ is an idealistic soft ring over R, if $\Gamma(x)$ and $G(y)$ are ordered by the set inclusion for all $(x, y) \in supp((\Gamma, A) \tilde{\vee} (G, B))$.

(v) $(\Gamma, A) \tilde{\cap} (K, A)$ is an idealistic soft ring over$R$, if it is non-null.

(vi) $(\Gamma, A) \tilde{\cap}_E (G, B)$ is an idealistic soft ring over R , if it is non-null.

(vii) $(\Gamma, A) \tilde{\cup}_R (G, B)$ is an idealistic soft ring over R , if $C = A \cap B \neq \emptyset$ and $\Gamma(x)$ and $G(x)$ are ordered by the set inclusion for all $x \in supp((\Gamma, A) \tilde{\cup}_R (G, B))$.

(viii) $(\Gamma, A) \tilde{\mp} (G, B)$ is an idealistic soft ring over R, if it is non-null.

(i) Let $(\Gamma, A) \tilde{\cup} (G, B) = (V, C)$ where

$$V(x) = \begin{cases} \Gamma(x), & if\ x \in A - B, \\ G(x), & if x \in B - A, \\ \Gamma(x) \cup G(x), & if\ x \in A \cap B, \end{cases}$$

For all $x \in A \cup B$. By the hypothesis $A \cap B = \emptyset$, it follows that either $x \in A - B$ or $x \in B - A$, for all $x \in A \cup B$. If $x \in A - B$, then $V(x) = \Gamma(x)$ is an ideal of R and if $x \in B - A$, then $V(x) = G(x)$ is an ideal of R. Therefore, (V, C) is an idealistic soft ring over F.

(ii) Let $(\Gamma, A)\tilde{\wedge}(G, B) = (H, C)$, where $C = A \times B$ and $H(x, y) = \Gamma(x) \cap G(y)$, for all $(x, y) \in C$. Then by hypothesis, (H, C) is non-null soft set over R. If $(x, y) \in supp(H, C)$, then $H(x, y) = \Gamma(x) \cap G(y) \neq \emptyset$. It follows that $\Gamma(x) \neq \emptyset$ and $G(y) \neq \emptyset$ are both ideals of R. Hence, $H(x, y)$ is an ideal of R for all $(x, y) \in supp(H, C)$. Therefore, (H, C) is an idealistic soft ring over R.

(iii) Let $(\Gamma, A) \Cap (G, B) = (J, C)$ where $C = A \cap B$ and $J(x) = \Gamma(x) \cap G(x)$ for all $x \in C = A \cap B \neq \emptyset$. By the hypothesis (J, C) is a non-null soft set over R. If $x \in supp(J, C)$, then $J(x) = \Gamma(x) \cap G(x) \neq \emptyset$. It means that $\Gamma(x)$ *and* $G(x)$ are both ideals of R. Hence, $J(x)$ is an ideal of R, for all $x \in supp(J, C)$. Thus, (J, C) is an idealistic soft ring over R.

(iv) Let $(\Gamma, A)\tilde{\vee}(G, B) = (N, C)$, where $C = A \times B$ and $N(x, y) = \Gamma(x) \cup G(y)$, for all $(x, y) \in C$. Then by the hypothesis, (N, C) is a non-null soft set over R. If $(x, y) \in supp(N, C)$, then $N(x, y) = \Gamma(x) \cup G(y) \neq \emptyset$. Since $\Gamma(x)$ and $G(y)$ are ordered by inclusion relation for all $(x, y) \in supp(N, C)$, $\Gamma(x) \cup G(y) = \Gamma(x)$ or $\Gamma(x) \cup G(y) = G(y)$. Since, $\Gamma(x) \neq \emptyset$ and $G(y) \neq \emptyset$ are both ideals of R for all $(x, y) \in supp(N, C)$. Therefore, (N, C) is an idealistic soft ring over R.

(v) Let $(\Gamma, A) \tilde{\cap} (K, A) = (T, A)$ where $T(x) = \Gamma(x) \cap K(x)$, for all $x \in A$. Suppose that (T, A) is a non-null soft set over R. If $x \in supp(T, A)$, then $T(x) = \Gamma(x) \cap K(x) \neq \emptyset$. Therefore, $\Gamma(x) \neq \emptyset$ and $K(x) \neq \emptyset$ are both ideals of R. Hence, $T(x)$ is an ideal of R for all $x \in supp(T, A)$. Therefore, (T, A) is an idealistic soft ring over R.

(vi) Let $(\Gamma, A) \tilde{\cap}_E (G, B) = (M, C)$, where $C = A \cup B$

$$M(x) = \begin{cases} \Gamma(x), & if\ x \in A - B \\ G(x), & if x \in B - A \\ \Gamma(x) \cap G(x), & if\ x \in A \cap B \end{cases}$$

For all $x \in C$.

Suppose that (M, C) is a non-null soft set over R. Let $x \in supp(M, C)$. If $x \in A - B$, then $\emptyset \neq M(x) = \Gamma(x) \leq R$. If $x \in B - A$, then $\emptyset \neq M(x) = G(x) \leq R$, and if $x \in A \cap B$, then $M(x) = \Gamma(x) \cap G(x) \neq \emptyset$. Since, $\emptyset \neq \Gamma(x) \leq R$ and $\emptyset \neq G(x) \leq R$, it implies that $M(x) \leq R$ for all $x \in supp(M, C)$. Therefore, $(\Gamma, A) \tilde{\cap}_E (G, B) = (M, C)$ is an idealistic soft ring over R.

(vii) Let $(\Gamma, A) \tilde{\cup}_R (G, B) = (Q, C)$ where $C = A \cap B$ and $Q(x) = \Gamma(x) \cup G(x)$ for all $x \in C = A \cap B \neq \emptyset$. Then by the hypothesis (Q, C) is a non-null soft set over R. if $x \in supp(Q, C)$, $Q(x) = \Gamma(x) \cup G(x) \neq \emptyset$. Since, $\Gamma(x)$ and $G(x)$ are ordered by inclusion relation for all $x \in supp(Q, C)$, $\Gamma(x) \cup G(x) = \Gamma(x)$ or $\Gamma(x) \cup G(x) = G(x)$. Since, $\Gamma(x) \neq \emptyset$ and $G(x) \neq \emptyset$ are both ideals of R, $Q(x)$ is an ideal of F for all $x \in supp(R, C)$. Therefore, (Q, C) is an idealistic soft ring over R.

(viii) Let $(\Gamma, A)\tilde{+}(G, B) = (H, C)$, where $C = A \times B$ and $H(x, y) = \Gamma(x) + G(y)$, for all $(x, y) \in C$. Then by the hypothesis, (H, C) is a non-null soft set over R. Suppose $(x, y) \in supp(H, C)$, then $H(x, y) = \Gamma(x) + G(y) \neq \emptyset$. It means that $\Gamma(x) \neq \emptyset$ and $G(y) \neq \emptyset$ are both ideals of R. Hence, $H(x, y)$ is an ideal of R for all $(x, y) \in supp(H, C)$. Therefore, $(\Gamma, A)\tilde{+}(G, B)$ is an idealistic soft ring over R.

Definition 3.4. Let (Γ, A) be an idealistic soft ring over R. Then,

(i) (Γ, A) is called **trivial idealistic soft ring** if $\Gamma(x) = \{0_R\}$ $(the\ zero\ elements\ of\ R)$ for all $x \in A$.

(ii) (Γ, A) is said to be an **improper (whole) idealistic soft ring** if $\Gamma(x) = R$ for all $x \in A$.

Theorem 3.2. Let (Γ, A) and (G, B) be two idealistic soft rings over R. Then,

(i) If (Γ, A) and (G, B) are trivial idealistic soft rings over R, then $(\Gamma, A) \Cap (G, B)$ is a trivial idealistic soft ring over R.

(ii) If (Γ, A) and (G, B) are improper idealistic soft rings over R, then $(\Gamma, A) \Cap (G, B)$ is a improper idealistic soft ring over R.

(iii) If (Γ, A) is a trivial idealistic soft ring over R and (G, B) is an improper idealistic soft ring over R, then $(\Gamma, A) \Cap (G, B)$ is a trivial idealistic soft ring over R.

(iv) If (Γ, A) and (G, B) are trivial idealistic soft rings over R, then $(\Gamma, A)\tilde{\mp}\,(G, B)$ is a trivial idealistic soft ring over R.

(v) If (Γ, A) and (G, B) are improper idealistic soft ring over R, then $(\Gamma, A)\tilde{\mp}\,(G, B)$ is a improper idealistic soft ring over R.

(vi) If (Γ, A) is a trivial idealistic soft ring over R and (G, B) is an improper idealistic soft ring over R, then $(\Gamma, A)\tilde{\mp}\,(G, B)$ is an improper idealistic soft ring over R.

Proof:

(i) Suppose that, $(\Gamma, A) \Cap (G, B) = (H, C)$, where $C = A \cap B$ and for all $x \in C$, $H(x) = \Gamma(x) \cap G(x)$. Since (Γ, A) and (G, B) are trivial idealistic soft rings over R, it implies that $\Gamma(x) = \{0_R\}$ for all $x \in A$ and $G(x) = \{0_R\}$ for all $x \in B$. Thus, $H(x) = \Gamma(x) \cap G(x) = \{0_R\} \cap \{0_R\} = \{0_R\}$ is trivial, $\forall x \in (A \cap B)$. Therefore, $(\Gamma, A) \Cap (G, B)$ is a trivial idealistic soft ring over R.

(ii) Suppose, $(\Gamma, A) \Cap (G, B) = (K, C)$, where $C = A \cap B$, for all $x \in C$, $K(x) = \Gamma(x) \cap G(x)$. Since (Γ, A) and (G, B) are improper idealistic soft rings over R, it follows that $\Gamma(x) = R$, for all $x \in A$ and $G(x) = R$, for all $x \in B$. This implies that $K(x) = \Gamma(x) \cap G(x) = R \cap R = R$, for all $\forall x \in (A \cap B)$. Therefore, $(\Gamma, A) \Cap (G, B)$ is a improper idealistic soft ring over R.

(iii) Let $(\Gamma, A) \Cap (G, B) = (T, C)$, where $C = A \cap B$, for all $x \in C$, $T(x) = \Gamma(x) \cap G(x)$. Since (Γ, A) is a trivial idealistic soft ring over R, it implies that $\Gamma(x) = \{0_R\}, \forall x \in A$ and (G, B) is an improper idealistic soft ring over R, then $G(x) = F$, for all $\forall x \in B$. It means that $T(x) = \Gamma(x) \cap G(x) = \{0_R\} \cap R = \{0_R\}$. Hence, $(\Gamma, A) \Cap (G, B)$ is a trivial idealistic soft ring over R.

(iv) Let $(\Gamma, A)\tilde{\mp}\,(G, B) = (W, A \times B)$, where $W(x, y) = \Gamma(x) + G(y)$, for all $(x, y) \in A \times B$. Since (Γ, A) is a trivial idealistic soft ring, It means that, $\Gamma(x) = \{0_R\}$ is a trivial idealistic soft ring over R, for all $x \in A$ and (G, B) is also a trivial idealistic soft ring over R for all $x \in B$, it means $G(y) = \{0_R\}$ is a trivial idealistic soft ring over R, for all $x \in B$ then $W(x, y) = \Gamma(x) + G(y) = \{0_R\} + \{0_R\} = \{0_R\}$, for all $(x, y) \in (A \times B)$. Therefore, $(\Gamma, A)\tilde{\mp}\,(G, B)$ is a trivial idealistic soft ring over R.

(v) Let $(\Gamma, A)\tilde{\mp}\,(G, B) = (M, A \times B)$, where $M(x, y) = \Gamma(x) + G(y)$, for all $(x, y) \in A \times B$. Since, (Γ, A) and (G, B) are improper idealistic soft rings, it means that $\Gamma(x) = R$ is an improper idealistic soft ring over R for all $x \in A$ and $G(y) = R$ is an improper idealistic soft ring over R for all $x \in B$. It implies that, $M(x, y) = \Gamma(x) + G(y) = R + R$ is an improper idealistic soft ring over R, for all $(x, y) \in A \times B$. Therefore, $(\Gamma, A)\tilde{\mp}\,(G, B)$ is an improper idealistic soft ring over R.

Let $(\Gamma, A)\tilde{\mp}\,(G, B) = (R, A \times B)$, where $R(x, y) = \Gamma(x) + G(y)$, for all $(x, y) \in A \times B$. Since, (Γ, A) is a trivial idealistic soft ring over R for all $x \in A$, it means $\Gamma(x) = \{0_R\}$ and since (G, B) is an improper idealistic soft ring over R for all $x \in B$, it means $G(y) = R$, it implies that $R(x, y) = \Gamma(x) + G(y) = \{0_R\} + R = R$ is an improper idealistic soft ring over R, for all $(x, y) \in A \times B$. Hence, $(\Gamma, A)\tilde{\mp}\,(G, B)$ is an improper idealistic soft ring over R.

Proposition 3.1

Let (Γ, A) be a soft ring over R and $(\Gamma_i, A_i)_{i\in I}$ be a nonempty family of soft ideals of (Γ, A). Then the following results hold:

(i) $\tilde{\bigwedge}_{i\in I}(\Gamma_i,\ A_i)$ is a soft ideal of (Γ, A), if it is non-null.

(ii) $\Cap_{i\in I}\,(\Gamma_i,\ A_i)$ is a soft ideal of (Γ, A), if it is non-null.

(iii) $\tilde{\cap}_{E\ i\in I}\,(\Gamma_i,\ A_i)$ is a soft ideal of (Γ, A), if it is non-null.

(iv) If $\{A_i : i \in I\}$ are pairwise disjoint, that is, $i \neq j$ implies $A_i \cap A_j = \emptyset$, then $\tilde{\cup}_{i\in I}\,(\Gamma_i,\ A_i)$ is a soft ideal of (Γ, A) .

(v) Let $\Gamma_i(x_i) \subseteq \Gamma_j(x_j)$ or $\Gamma_j(x_j) \subseteq \Gamma_i(x_i)$, for all $i, j \in I$ and $x_i \in A_i$, then $\tilde{\cup}_{R\ i\in I}\,(\Gamma_i,\ A_i)$ is a soft ideal of (Γ, A), whenever it is non-null.

(vi) Let $\Gamma_i(x_i) \subseteq \Gamma_j(x_j)$ or $\Gamma_j(x_j) \subseteq \Gamma_i(x_i)$, for all $i, j \in I$ and $x_i \in A_i$, then $\tilde{\bigvee}_{i\in I}(\Gamma_i,\ A_i)$ is a soft ideal of (Γ, A), whenever it is non-null.

(vii) $\tilde{\Sigma}_{i\in I}(\Gamma_i,\ A_i)$ is a soft ideal of (Γ, A), whenever it is non-null.

(viii) $\tilde{\prod}_{i\in I}\,(\Gamma_i, A_i)$ is a soft ideal of (Γ, A), whenever it is non-null.

Proof:

(i) Let $\widetilde{\wedge}_{i\in I}(\Gamma_i, A_i) = (K, C)$, where $C = \prod_{i\in I} A_i$ and $K(x) = \cap_{i\in I} \Gamma_i(x_i)$ for all $x = (x_i)_{i\in I} \in C$. Suppose that (K, C) is a non-null soft ideal over R. If $x \in supp(K, C)$, then $K(x) = \cap_{i\in I} \Gamma_i(x) \neq \emptyset$. Thus, we have $\Gamma_i(x) \neq \emptyset$ for all $i \in I$. Since (Γ_i, A_i) is a family of soft ideals over R for all $i \in I$. It follows that, $C = \prod_{i\in I} A_i \neq \emptyset$, It means that C is nonempty and $C \subseteq A$, meaning every $x \in C$ implies $x \in A$. Also, $K(x) = \cap_{i\in I} \Gamma_i(x_i) <_R \Gamma(x)$, which means $K(x)$ is an ideal of $\Gamma(x)$ .Therefore, $\widetilde{\wedge}_{i\in I}(\Gamma_i, A_i)$ is a soft ideal of (Γ, A).

(ii) Let $\Cap_{i\in I} (\Gamma_i, A_i) = (H, C)$, where $C = \cap_{i\in\Delta} A_i \neq \emptyset$ for all $x = (x_i)_{i\in\Delta} \in C$, $H(x) = \cap_{i\in\Delta} \Gamma_i(x)$. By the hypothesis (H, C) is a non-null soft ideal over R. If $x \in supp(H, C)$, then $H(x) = \cap_{i\in\Delta} \Gamma_i(x) \neq \emptyset$, so we have $\Gamma_i(x) \neq \emptyset$ for all $i \in \Delta$. Since (Γ_i, A_i) is a family of soft ideals over R for all $i \in \Delta$. It follows that, $C = \cap_{i\in\Delta} A_i \neq \emptyset$, it implies that C is nonempty and $C \subseteq A$, means every $x \in C$ implies $x \in A$. Also $H(x) = \cap_{i\in\Delta} \Gamma_i(x) <_R \Gamma(x)$,which means $H(x)$ is an ideal of $\Gamma(x)$ for all $x \in supp(H, C)$. Therefore, $\Cap_{i\in I} (\Gamma_i, A_i)$ is a soft ideal of (Γ, A).

(iii) Let $\widetilde{\cap}_{E\, i\in I} (\Gamma_i, A_i) = (G, B)$, where $B = \cup_{i\in\Delta} A_i$ and $G(x) = \cap_{i\in\Delta} \Gamma_i(x)$ where $\Delta(x) = \{i \in \Delta : x \in A_i\}$ for all $x \in B = \cup_{i\in\Delta} A_i$. By the hypothesis (G, B) is a non-null soft ideal over R. If $x \in supp(G, B)$, then $G(x) = \cap_{i\in\Delta} \Gamma_i(x) \neq \emptyset$. Since, $\{A_i : i \in I\}$ are pairwise disjoint, that is, $i \neq j$ implies $A_i \cap A_j = \emptyset$, so we have $\Gamma_i(x) \neq \emptyset$ for all $i \in \Delta$, since (Γ_i, A_i) is a family of soft ideals over R for all $i \in \Delta$. It follows that, $B = \cup_{i\in\Delta} A_i \neq \emptyset$, it means B is nonempty and a subset of A, it means $x \in B \longrightarrow x \in A$ for all $x \in B$. Also, $G(x) = \cap_{i\in\Delta} \Gamma_i(x) <_R \Gamma(x)$, which means $G(x)$ is an ideal of $\Gamma(x)$ for all $x \in supp(G, B)$. Therefore, $\widetilde{\cap}_{E\, i\in I} (\Gamma_i, A_i)$ is a soft ideal of (Γ, A).

(iv) Let $\widetilde{\cup}_{i\in I} (\Gamma_i, A_i) = (K, C)$, where $C = \cup_{i\in I} A_i$ and for all $x \in C$, $K(x) = \cup_{i\in I(x)} \Gamma_i(x)$ where $I(x) = \{i \in I : x \in A_i\}$. By the hypothesis, (K, C) is non-null soft ideal over R. If $x \in supp(K, C)$, then $K(x) = \cup_{i\in I(x)} \Gamma_i(x) \neq \emptyset$, so we have $\Gamma_i(x) \neq \emptyset$ for all $i \in I(x)$. It follows that, $C = \cup_{i\in I} A_i \neq \emptyset$. It means C is nonempty and a subset of A, this means $x \in B \longrightarrow x \in A$ for all $x \in C$. Also, $K(x) = \cup_{i\in I(x)} \Gamma_i(x) <_R \Gamma(x)$, which means $K(x)$ is an ideal of $\Gamma(x)$ for all $x \in supp(K, C)$. Therefore, $\widetilde{\cup}_{i\in I} (\Gamma_i, A_i)$ is a soft ideal of (Γ, A).

(v) Let $\widetilde{\cup}_{R\, i\in I} (\Gamma_i, A_i) = (H, C)$, where $C = \cap_{i\in I} A_i$ and $H(x) = \cup_{i\in I(x)} \Gamma_i(x)$ for all $x \in C$. Then by the hypothesis (H, C) is a non-null soft set over R. If $x \in supp(H, C)$, then $H(x) = \cup_{i\in I(x)} \Gamma_i(x) \neq \emptyset$. Since, $\Gamma_i(x_i) \subseteq \Gamma_j(x_j)$ or $\Gamma_j(x_j) \subseteq \Gamma_i(x_i)$, for all $i, j \in I$ and $x_i \in A_i$, so we have $\Gamma_i(x) \neq \emptyset$ for all $i \in I$. It follows that $C = \cap_{i\in I} A_i \neq \emptyset$, this means C is nonempty and a subset of A, it means that $x \in C \longrightarrow x \in A$ for all $x \in C$. Also, $H(x) = \cup_{i\in I(x)} \Gamma_i(x) <_R \Gamma(x)$, it means that $H(x)$ is an ideal of $\Gamma(x)$ for all $x \in supp(H, C)$. Therefore, $\widetilde{\cup}_{R\, i\in I} (\Gamma_i, A_i)$ is a soft ideal of (Γ, A).

(vi) Let $\widetilde{\vee}_{i\in I}(\Gamma_i, A_i) = (M, D)$, where $D = \prod_{i\in I} A_i$ and $M(x) = \cup_{i\in I} \Gamma_i(x_i)$ for all $x = (x_i) \in D$. Then by the hypothesis (M, D) is a non-null soft ideal over R. If $x \in supp(M, D)$, then $M(x) = \cup_{i\in I} \Gamma_i(x_i) \neq \emptyset$. Since, $\Gamma_i(x_i) \subseteq \Gamma_j(x_j)$ or $\Gamma_j(x_j) \subseteq \Gamma_i(x_i)$, for all $i, j \in I$ and $x_i \in A_i$, so we have $\Gamma_i(x_i) \neq \emptyset$ for all $i \in I$. Since (Γ_i, A_i) is a family of soft ideals over R for all $i \in I$. It follows that, $D = \prod_{i\in I} A_i \neq \emptyset$, it means D is nonempty and a subset of A, this implies that for $x \in D \longrightarrow x \in A$ for all $x \in D$. Also, $M(x) = \cup_{i\in I} \Gamma_i(x_i) <_R \Gamma(x)$, which means that $M(x)$ is an ideal of $\Gamma(x)$ for all $x \in supp(M, D)$. Therefore, $\widetilde{\vee}_{i\in I}(\Gamma_i, A_i)$ is a soft ideal of (Γ, A).

(vii) Let $\widetilde{\Sigma}_{i\in I}(\Gamma_i, A_i) = (W, C)$, where $C = \prod_{i\in I} A_i$ and $W(x) = \Sigma_{i\in I}\Gamma_i(x_i)$ for all $x = (x_i)_{i\in I} \in C$. Then by the hypothesis (W, C) is a non-null soft ideal over R. If $x \in supp(W, C)$, then $W(x) = \Sigma_{i\in I}\Gamma_i(x_i) \neq \emptyset$ for all $i \in I$. Since (Γ_i, A_i) is a family of soft ideals over R for all $i \in I$. It follows that, $C = \prod_{i\in I} A_i \neq \emptyset$. It means C is nonempty and $C \subseteq A$. It means $x \in C \longrightarrow x \in A$ for all $x \in C$. Also, $W(x) = \Sigma_{i\in I}\Gamma_i(x_i) <_R \Gamma(x)$, which means that $W(x)$ is a ideal of $\Gamma(x)$ for all $x \in supp(W, C)$. Therefore, $\widetilde{\Sigma}_{i\in I}(\Gamma_i, A_i)$ is a soft ideal of (Γ, A).

(ix) Let $\widetilde{\prod}_{i\in I} (\Gamma_i, A_i) = (H, B)$, where $B = \prod_{i\in\Delta} A_i$ and $H(x) = \prod_{i\in\Delta} \Gamma_i(x_i)$ for all $x = (x_i)_{i\in\Delta} \in B$. Then by the hypothesis (H, B) is a non-null soft ideal over R. If $x \in supp(H, B)$, then $H(x) = \prod_{i\in\Delta} \Gamma_i(x_i) \neq \emptyset$, so we have $\Gamma_i(x_i) \neq \emptyset$ for all $i \in \Delta$. Since (Γ_i, A_i) is a family of soft ideals over R for all $i \in \Delta$. It follows that, $B = \prod_{i\in\Delta} A_i \neq \emptyset$, it means that B is nonempty and $B \subseteq A$. Also, $H(x) = \prod_{i\in\Delta} \Gamma_i(x_i) <_R \Gamma(x)$, which means $H(x)$ is an ideal of $\Gamma(x)$ for all $x \in supp(H, B)$. Therefore, $\widetilde{\prod}_{i\in I} (\Gamma_i, A_i)$ is a soft ideal of (Γ, A).

4. Conclusion

Throughout this paper, we deal with soft ring ideals and idealistic soft ring. Some important theorems and propositions on soft ring ideal and idealistic soft rings were stated and proved particularly on the operations of family of soft ring ideal and idealistic soft ring and on summation operations on improper and trivial idealistic soft rings.

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