*Available online www.jsaer.com*

*Journal of Scientific and Engineering Research*, 2017, 4(9):244-253

Research Article

ISSN: 2394-2630
CODEN(USA): JSERBR

# Unsteady State Approach for Estimation of Sediment Transport

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**Abstract** Computation of sediment transport plays vital role in project planning, design, construction, operation and maintenance of hydraulic structures such as river (bridges and training dikes), reservoirs (dams and barrages), lakes and coastal (jetties, berths, breakwaters, dikes, wave absorbers, revetment, seawalls and bulkheads). This review paper describes sediment transport theory in brief, empirical equations proposed by different investigators such as U S army corps of engineers, Bagnold, E-H method, Nielsen, Bijker, A-W method, Brownlie, Y-G-R method, Laursen and Garde-Albertson method. Macroscopic concept based on single sediment size has been used for computations. Present research paper is compilation of work carried out by various investigators for steady state and unsteady state flow condition. Performance of empirical equations for Indian sub continent is evaluated and found that the results obtained by unsteady state equation proposed by authors match with observed quantum of sediment. This equation precisely predicts sedimentation well within 1.7% bandwidth. The variation between predicted by proposed equation and observed sedimentation was accurate up to 2500 concentration, beyond within ±5%.

**Keywords** diffusion, convection, bed load, suspended load, shear velocity

## Introduction

Evaluation of sedimentation will allow the economically optimum plan to be clearly evident and readily substantiated. Project safety and efficiency should receive primary consideration before the cost-effectiveness of the project is determined. Planning for the project will require the anticipation of any possible development and operational problems and evaluation of alternative solutions.

$$S_t(x,t) = \int_0^{h(x)+\eta(x,t)} C(x,y,t)u(x,y,t)dy \qquad (1)$$

where,

| | | | |
|---|---|---|---|
| $S_t(x, t)$ | = | local instantaneous sediment transport rate per unit width | (kg/ms) |
| C (x, y, t) | = | local instantaneous sediment concentration | (kg/m$^3$) |
| u (x, y, t) | = | local instantaneous x component of fluid velocity | (m/s) |
| x | = | horizontal coordinate | (m) |
| y | = | height above mean bed level | (m) |
| t | = | time | (s) |
| η | = | wave surface elevation | (m) |
| h | = | mean water depth | (m) |

$$C(x,y,t) = C(y) + C'(x,y,t)$$

and

$$u(x,y,t) = U(y) + u'(x,y,t)$$

where

| | | |
|---|---|---|
| C(y) | = | time and bed averaged component of the local instantaneous sediment concentration @ height (y) |

$C'(x, y, t)$ = fluctuating component of the instantaneous sediment concentration
$u(y)$ = time and bed averaged local instantaneous y component of fluid velocity
$U(y)$ = combined averaged of fluid velocity
$u'(x, y, t)$ = fluctuating component of the instantaneous fluid velocity

Mean fluctuating velocity and mean fluctuating concentration with reference to mean datum are zero. Averaging over time and bed; after substituting in equation 1 and rearranging one can get following equation.

$$S_t = S_t(x,t) = Convective\ transport + Diffusive\ transport = S_{con} + S_{diff} \quad (2)$$

$$S_t = \int_0^h C(y)U(y)dy + \int_0^h C'(x,y,t)u'(x,y,t)dy \quad (3)$$

The total sediment transport consists of steady part due to current (convective sediment transport) and unsteady part due to wave (diffusive sediment transport).

**Sediment Load**

The sediment transport on an undulated bed is equals to the total of suspended sediment load and the bed load which can be computed per unit width by the following equation:

$$S_t = Bed\ load + Suspended\ load = S_b + S_s \quad (4)$$

$$S_t = \int_0^h C(y)dy + \int_0^{\frac{r}{2}} C(y)dy + \int_{\frac{r}{2}}^h C(y)dy \quad (5)$$

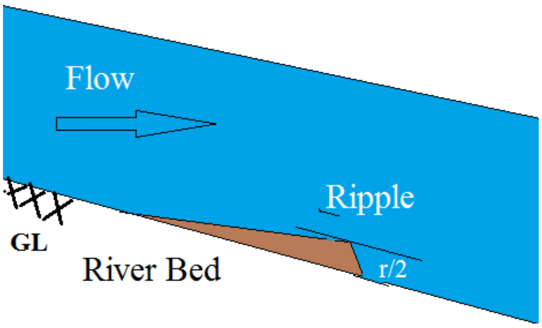


*Figure 1: Representative Ripple Formation*

where r is mean ripple height as shown in fig. 1. From equation 5, it can be seen that the total sediment transport consists of two parts:

a) determined by time and bed averaging velocity U(y) for steady current is called current related sediment transport and
b) determined by orbital movement of sediment u'(x, y, t) affected by irregular wave, thus this part is called as wave related sediment transport.

**Long Shore and Cross Shore Sediment Transport**

Waves approaching a coast will reach the coast with a small angle caused by refraction. The radiation stress generated by the waves under small angle and bottom friction results in long shore current. Fig. 2 shows the resulting long shore and offshore- on shore current responsible for the change in coastline.

Long-shore sediment transport is stirring up of sediment by waves and current transported by steady long-shore current is shown in fig 2a. A cross sediment transport is due to velocity oscillations $u'(x, y, t)$ introduced by orbital movement which influence the sediment transport during the wave.

$$S_{current} = \int_0^h C(y)U(y)dy \quad (6)$$

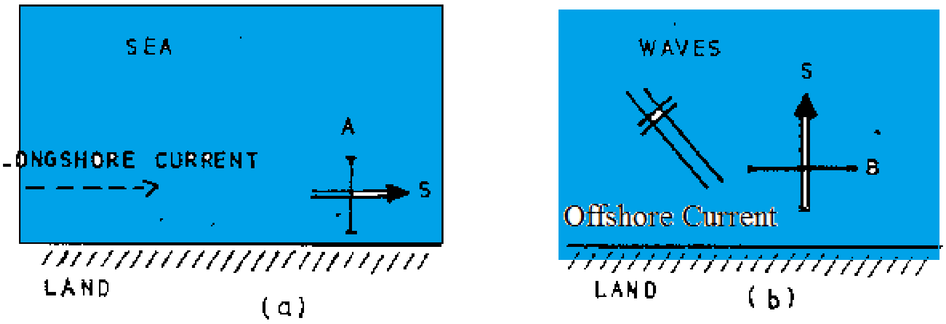


*Figure 2: Impoundment of Long shore sediment Transport*

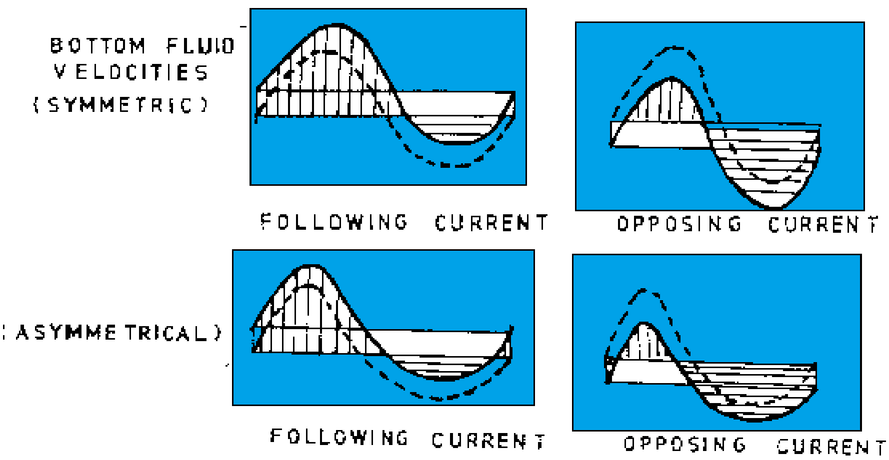


*Figure 3: Bottom Fluid Velocities*

To investigate the importance of the average wave crest and wave trough distribution, symmetrical sinusoidal waves are shown in Fig. 3 which indicates that wave related sediment transport for the following currents are larger than in the opposing one for symmetrical and asymmetrical waves. The crest is higher than the trough in case of asymmetrical waves whereas trough periods are larger than the crest period. Thus velocities caused by wave motion are not similar. The asymmetrical wave distribution causes larger sediment concentration when wave crest passes in the direction of flow whereas smaller sediment concentration occurs when wave trough passes in opposite direction. The wave will influence the current velocity profile by introducing extra roughness near the bed due to pressure force. Because of this effect the outer current profile is shifted as shown Fig 3. Waves superimpose upon a current introduce and apparent roughness increase by 1-7 percent for 200 micron sand grain but 100 micron such trend was not seen.

**Sediment Transport Mechanism**

Sediment transport mechanism is called the pickup and transport model which can be divided into two steps. Pickup sediment by eddies generated behind the ripple crest and transport of sediment by fluid velocity in opposite direction. Fig 4 shows the Sediment transport mechanism and various forces acted upon.

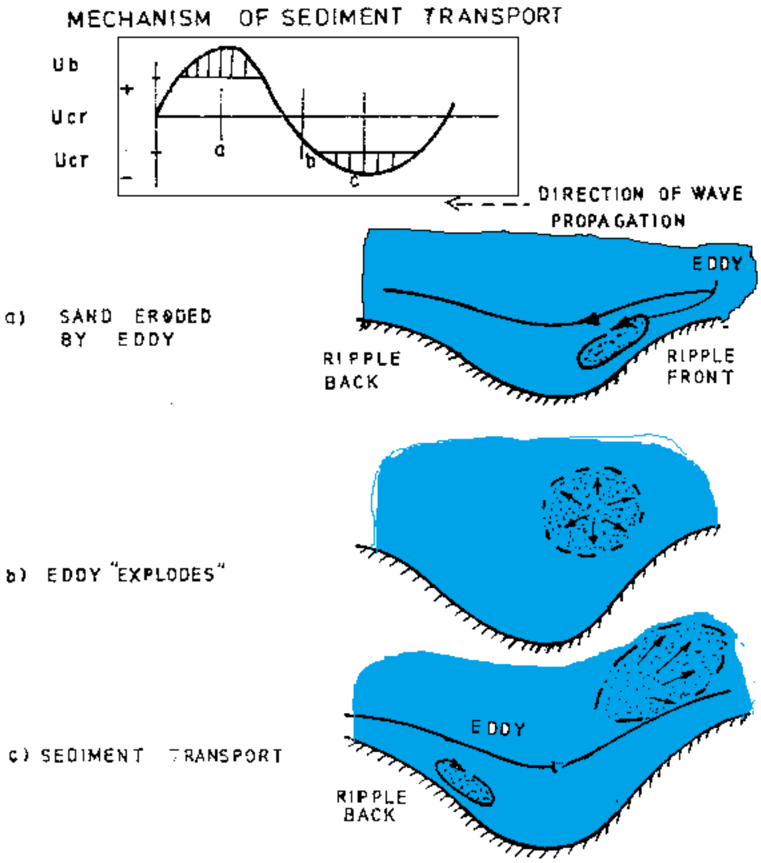


*Figure 4: Sediment Transport Mechanism*

**Computations of Sediment Transport**

Various methods are available for computations of sediment transport. These methods are theoretical, semi empirical and empirical. Out of these below mentioned are described in detail.

a. **Shore Protection Formula for Long-shore Current:**

The long-shore sediment transport formula (Energy Flux Method) in the shore protection manual probably (U. S. Army Corps of Engineers,1984)[1] the most widely use one. Energy Flux Method is based on an assumption that long shore sediment transport rate depends long-shore component of energy component energy flux in surf zone and it is approximated by assuming the conservation of energy relation in shoaling wave, small amplitude evaluating energy flux relation at the breaker position.

$$S = K_1 P_{ls} \qquad (7)$$

where

$$P_{ls} = \frac{\gamma H_{sb}^2 C_{gb} Sin\ 2\alpha}{16}$$

$P_{ls}$ is energy flux

$\gamma$ is specific weight of sea water

$H_{sb}$ is significant wave height at breaking line

$C_{gb}$ is wave group velocity

$\alpha$ is breaking wave angle with the shore

The energy flux for computing long-shore transport rate is based on the empirical relationship between the long-shore components of the wave energy flux entering the surf zone and immerse weight of the sand moved.

$K_1$=1290 (SI)

$K_1$=7500 (FPS)

Swart (1976)[2] replaced $K_1$ as 1876 log(0.00146/d).

Kamphuis and Readsaw (1978)[3] considered the bed slope and celerity.

Watts (1953) [4] empirically related the long-shore current as S = 2223 $P_{ls}^{0.9}$

and similarly Caldwell (1956)[5]gave the relation S = 2505 $P_{ls}^{0.8}$

**Limitations:**

- SPM equation is an empirical formula in which only significant wave height has been considered.
- Bathymetry of the sea, on a bed slope and morphological changes have not been considered.
- Importance to the turbulence is not given.

**b. Bagnold-Bailard:**

Bagnold (1966)[6] related the sediment transport to the dissipated energy from the steady current. The dissipated energy is partly used for stirring up of the sediment grains and transport of the same. Bailard modified Bagnold concept by introducing the oscillating fluid velocity component for sediment transport in the coastal area with the assumption that the instantaneous transport rates are related to instantaneous velocities. Bailard also introduced slope in the equation so that cross-shore sediment transport can be computed.

When the orbital fluid velocity above the bed u(t) and the depth averaged fluid velocity U is known, the sediment transport can be computed. It is possible to describe the fluid velocity $u_t$ under asymmetrical wave (Fig. 3). Combined average velocity (U) can be given by

$$U = u_c + u_t \tag{8}$$

where $u_c$ = fluid velocity due to current effect (m/s)

$u_t$ = Orbital fluid velocity above the bed at time t (m/s)

assume $u_c$ = U, then bed load transport $S_b$ is related to $IUI^2U$

$$S_b(t) = \frac{\tau_0}{\left[1-\frac{\rho f}{\rho s}\right]}\left\{\frac{e_b}{tan\alpha} + e_s(1-e_b)\frac{\bar{u}}{\omega_0}\right\} \tag{9}$$

the suspended load transport Ss is related to $IUI^3U$ and computed from

$$S_s(t) = \frac{\tau_0 U}{\left[1-\frac{\rho f}{\rho s}\right]}\left\{\frac{e_s}{tan\alpha} + e_s(1-e_s)\frac{\bar{u}}{\omega_0}\right\} \tag{10}$$

where εb = 0.10 and εs = 0.2

$f = e^{[-6+5.2(A_b/ks)-0.19]}$

Δρs is (ρs – ρ) = 1.65 as w is fall velocity of bed material

The total sediment transport can be computed as

$$S_t = \sum_0^n (S_b + S_s) \tag{11}$$

where n = the number of steps within one wave period.

Limitations

- The formula over estimated sediment transport by 100 times the sediment transport computed from the experiments for the median size of 0.107 mm.
- The bed load computed from this formula is 20% more than the observed.
- When root mean square wave height is increased the total computed sediment transport also increases.
- Wave following current gives consequently large sediment transports, than for waves opposing current.

**c. The Modified Engelund and Hansen Formula:**

The well known original E-H formula was developed for developed for predicting total load transport in rivers. Ven de Graff and Van Overeem (1979)[7] modified the formula by increasing the bed shear stress, when waves are present. Only five parameters namely the wave height H, the depth averaged fluid velocity Um the bed roughness Ks, median grain diameter of sediment d and the water depth are required to compute sediment transport with E-H formula{8}

$$S_t = \frac{0.05\ U\ C\tau_c^2\left[1+0.5\left(\xi\frac{U_b}{U}\right)^2\right]^2}{\rho^2 g^{\frac{5}{2}}\Delta\rho_s^2 d} \qquad (12)$$

where C = Chezy's coefficient

ξ = a coefficient

τc = bed shear stress due to current N/m$^2$

Ub = amplitude of orbital velocity at the bed according to linear wave theory.

D = grain diameter

Limitations

- The result shows poor tendency for all Ks = 7r and rather good tendency for Ks= 3r for both H = Hrms and H = Hsig.
- Larger sediment transport rate was obtained by computation when compared with experimental results.
- E-H modified formula does not seem to give adequate importance to sediment size.

**d. The Nielsen Formula:**

As discussed earlier the sediment transported by current is called convective sediment transport and the sediment transported by orbital velocity or waves is called diffusive sediment size.

**1. Convective Sediment Transport**

The convective sediment transport is computed as

$$S_{con} = \int_0^h C(y)U(y)dy \qquad (13)$$

To calculate the convective sediment transports the velocities are approximated by modified logarithmic law for velocity distribution and the zero velocity level was considered as 3% of roughness height.

The ripple calculation were based on the significant wave height. This is assumed to be justified because the characteristics ripple pattern does not adjust to each individual wave. To calculate the convective transport, Nielsen(2000)[11] used an exponential concentration distribution as given below.

$$C(y) = C_b e^{\left(\frac{-y}{L_c}\right)} \qquad (14)$$

where C(y) = Time average concentration at height (y)

$C_b$ = Reference concentration = 0.005 $\theta r^3$

$L_c$ = Mixing length for wave alone

θr = Shift parameter near ripple crest = θ'/(1-πRS)

Nielsen calculated the mixing the length Lc with the use of maximum (forward) and minimum (backward) combined velocity.

$$u_{max} = \frac{u_*}{\sqrt{0.5 f_w}} + U_b \qquad (15)$$

$$u_{min} = \frac{u_*}{\sqrt{0.5 f_w}} - U_b \qquad (16)$$

where $u_*$ is current shear velocity

and $U_b$ is amplitude of orbital velocity at the bed

Further the average mixing the length can be calculated from the following set of equations

$$L_{cf} = \gamma\left[0.2 + 1.24\ e^{\left\{-40\left(\frac{u_{max}}{\omega}\right)^2\right\}}\right] \qquad (17)$$

$$L_{cb} = \gamma\left[0.2 + 1.24\ e^{\left\{-40\left(\frac{u_{min}}{\omega}\right)^2\right\}}\right] \qquad (18)$$

$$L_c = \frac{L_{cf} + L_{cb}}{2} \qquad (19)$$

Nielsen replaced the formula by $C(y) = C_b e^{\left(\frac{-y}{L_c}\right)}$ because former formula takes no account for extra mixing and resulting upward stretching of the concentration profile due to steady current wave $L_{cy}$' is mixing length for wave and currents.

$$L'_{cy} = L_c + \left(\frac{K|u_*|y'}{\omega}\right) = L_c + \chi y' \quad (20)$$

where $\chi = \frac{K|u_*|}{\omega}$

$$C(y) = C_b \left(\frac{L_c}{L_c + \chi y'}\right)^{\frac{1}{x}} \quad (21)$$

$$S_{con} = \frac{L_{cf}^2 C_b A_f u_*}{KA_1\left[1 - e^{\left\{-1.9\left(\frac{A_1}{L_{cf}}\right)\right\}}\right]^{0.79}} + \frac{L_{cb}^2 C_b A_b u_*}{KA_1\left[1 - e^{\left\{-1.9\left(\frac{A_1}{L_{cb}}\right)\right\}}\right]^{0.79}} \quad (22)$$

where $A_f$ and $A_b$ are dimensionless enhancement factors by Nielsen to represent the current influence on the bed concentration $C_b$.

$$A_f = 0.5\left(\frac{U_{max}}{U_b}\right)^6$$

$$A_b = 0.5\left(\frac{U_{min}}{U_b}\right)^6$$

**2. Diffusive sediment transport-**

The diffusive sediment transport computed from

$$S_{diff} = \frac{1}{T}\int_{t=t'}^{t+T}\int_0^h C(y,t')\, u_w(y,t')dydt' \quad (23)$$

Where $S_{diff}$ = diffusive sediment transport per unit width due to wave

C(y, t') = instantaneous concentration at height y

$U_w$(y, t') = instantaneous wave velocity at height y

Using the fact the entrainment of sediment from ripple bed occur close to the moment that velocity changes direction Nielsen derived expression for the instantaneous load and further following were derived to calculate the diffusion sediment transport.

$$S_f = -A_f C_b U_b L_{cf}\left[\frac{\frac{2\pi L_{cf}}{T_w}}{1+\left(\frac{2\pi L_{cf}}{T_w}\right)^2}\right] \quad (24)$$

$$S_b = -A_b C_b U_b L_{cb}\left[\frac{\frac{2\pi L_{cb}}{T_w}}{1+\left(\frac{2\pi L_{cb}}{T_w}\right)^2}\right] \quad (25)$$

$$S_{diff} = S_f + S_b \quad (26)$$

Where T is wave period knowing the convective and diffusion sediment transport the total sediment transport can be calculated as

$$S_t = S_{con} + S_{diff} \quad (27)$$

**Limitation**

- The Nielsen formula gives good result for sediment transport rate.
- According to Nielsen method the transport rate increase as sediment size increase which is not true.
- $H_{sig}$ gives larger sediment transport than $H_{prob}$ which is not correct.
- $K_s$ maximum gives larger sediment transport rate than $K_s$ minimum

**e. The Bijker Equation:**

Bijker derived the total transport in to two parts, the bed load and suspended load transport. The bed load transport is calculated using the Kalinke-Frinjlink formula in which combined action of wave and current is accounted by a modification of the bed shear stress.

$$S_b = \frac{BDU\sqrt{g}}{C} e^{\left[\frac{-0.27\Delta\gamma_S d}{\gamma\tau_c\left\{1+0.5\left(\frac{\xi U_b}{U}\right)^2\right\}}\right]} \quad (28)$$

Sb = Bed load transport per unit width with void.
B = A coefficient which reflects the influence of breaking of waves
B = 5 for breaker zone
B = 1 beyond breaker zone
C = Chezy's coefficient=18 log (12h/$K_s$)
Ks = Bed roughness =3 $d_{90}$ +1.1 r(1-$e^{(-25r/1)}$)Rijn(1982) $k_s$ =25($r^2$/1) (swart1976)
μ = Ripple factors =$(c/c_{90})^{3/2}$
C90 = Chezy's coefficient based on $d_{90}$ =18 log(12h/$d_{90}$)
ξ = Dimensionless parameter = $c(f_w/2g)^{0.5}$
$\tau_c$= Bed shear stress due to current=pg $u^2/c^2$
$f_w$ = e(-5.977+5.213(A /k)^(-0.194))for 1.47<$A_b/K_s$<3000 and $f_w$ = 0.32 for $A_b/K_s$
$A_b$ = H/(2sinh(2πh/L)) = Amplitude of orbital displacement at bed
H&L are waves height and wave length respectively
$U_b$ be the amplitude of orbital velocity at the bed $U_b$ = $A_b$(2π/T)
Bijker assumed that the bed load transport take place in bed layer having a thickness of bed roughness $K_s$ and the concentration in this layer is assumed to be constant over the entire thickness of this layer concentration in bed layer

$$C_b = K_s = \frac{S_b}{6.4\left(\frac{\tau_0}{\rho}\right)} \quad (29)$$

Bijkar calculated suspended load transport as

$$S_t = \int_0^h C(y)U(y)dy \quad (30)$$

To compute the suspended load transport the velocity is approximated by Karman-Prandtl logarithmic velocity law. The concentration distribution is approximated by the equation

$$C(y) = C_b\left[\left(\frac{h-y}{y}\right)\left(\frac{k_s}{h-k_s}\right)\right]^{Z_*} \quad (31)$$

Z*(dimensionless parameter)=w√ρ{K($\tau_c$(1+0.5$(U_b/U)^2$)} thus

$$S_s = 1.083\left[I_1 ln\left(\frac{33h}{K_s}\right) + I_2\right]S_b \quad (32)$$

Where $I_1$ and $I_2$ are Einstein's integrals.
Knowing the bed load transport and suspended load transport the total load can be computed as

$$S_t = S_b + S_s \quad (33)$$

It is considered that concentrated to be constant over entire thickness. $K_s$ of the bed load layer, the load can therefore be computed as

$$L_b = C_b K_s \quad (34)$$

The suspended load is calculated as

$$L_s = \int_{k_s}^h C(y)dy = 4.63\, L_b I_1 \quad (35)$$

If suspended load and bed load are known the total load can be given as $L_t = L_b + L_s$.

**Limitations**

- Larger values of sediment transport with the use of $H_{sig}$, $H_{rms}$, $H_{prob}$ respectively.
- Larger sediment transport rates with the use of Ks maximum than the $K_s$.

### 3. Unsteady State of Flow

Naturally occurring flows are rarely in steady state conditions. Therefore a proposal to predict sedimentation in unsteady state condition was first mooted by the Austrian researcher Exner.

As with the de St. Venant equations, x represents a boundary attached downstream coordinate. Sediment mass balance is expressed in volume form (by dividing the mass balance by $\rho_s$) given below;

$$(1-\lambda_p)\frac{\partial\eta}{\partial t} = -\frac{\partial q_{bs}}{\partial x} - \frac{\partial q_{bm}}{\partial n} + \nu_s(\bar{c}_b - E_s) \quad (36)$$

In the above relation λp denotes the porosity of the bed deposit qbm and qbs. cb denotes the volume concentration of suspended sediment, averaged over turbulence, just above the bed. $E_s$ is sediment entrainment into suspension.

The work of Exner was extended by Balkrishna S. Chavan and Hradaya Prakash; and proposed following equation

$$\frac{\partial c}{\partial t} = -5 + 1.35\left\{\left(q\frac{dx}{dt}\right)^{0.96} + \left(q\frac{dx}{dt}\right)^{-0.5} - \left(q\frac{dx}{dt}\right)^{-0.25}\right\} \quad (37)$$

The proposed unsteady state equation predicts sedimentation precisely within 1.7% of the observed values of sediment.

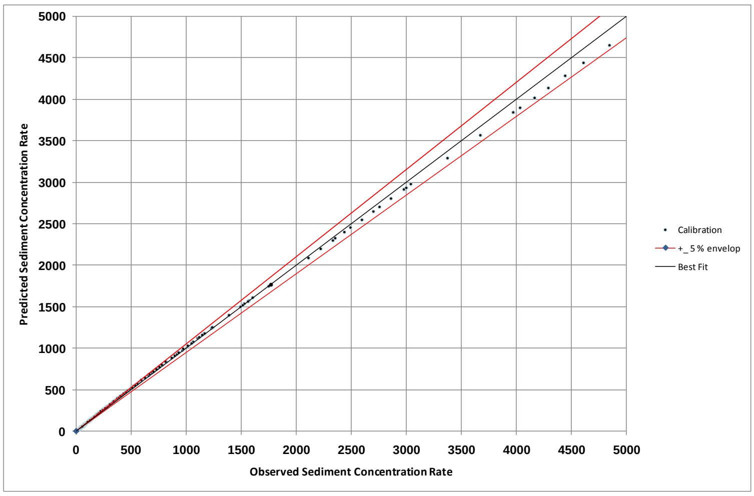


*Figure 5: Calibration of Sediment Concentration Rate*

It can be seen from the concentration sediment calibration curve that predicted exactly matches with the observed values upto 2000. However beyond 2500 observed values are smaller than predicted.

## Conclusion

- It is found that SPM formula gives larger value of sediment transport in deep water than the shallow water.
- The Bagnold – Bailard concept predicts current related sediment transport rate that are 10 to 50 times measured sediment transport in case of waves following currents. In case of waves opposing currents predicts these rates within 3 times of measured in the laboratories.
- The modified Englund and Hansen formula computes current related sediment transport rate too large in all cases and inconsistent for the median size of the bed material.
- The Nielsen model reflects the velocity profile rather well. The current related transport rate gives results within two times the measured transport for small transport rates (<0.001Kg/ms) and gives result 100 times or more for large sediment transport rate (>0.003Kg/ms).
- The Bijiker model predicts too small concentration magnitudes and too large fluid velocities in the near bed zone. These results in current related sediment transport rates that are 4 times for small transports (<0.001Kg/ms) and 4 times large transports (>0.003Kg/ms).
- Predicted values of sediment concentration by equation 38 are much better than earlier work which was based on steady state condition of flow.

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