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Research Article

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A Systematic Approach for the Optimal Solution of Multiple Sources to Multiple Destinations of Roads Networks

F. O. Ikpotokin[1], A. J. Tamber[2]*

Department of Computer Science, Ambrose Alli University, Ekpoma, Edo State, Nigeria
Department of Mathematics, Ambrose Alli University, Ekpoma, Edo State, Nigeria

Abstract In this work, we examine the road network of multiple sources to multiple destinations. A systematic approach was adopted to derive the shortest route from given multiple sources to multiple destinations. In doing this, the algorithm of the well-known Bellman functional relation for Dynamic programming problem was modified to enable the relation solve multiple sources to multiple destinations problems directly without splitting such network problems into either: single source to single destination, multiple sources to single destination or single source to multiple destinations as has been the case with Dynamic programming algorithm, A* search algorithm, Dijkstra's algorithm, Bellman-Ford-Moore algorithm, Floyd-Warshall algorithm, Johnson's algorithm and Veterbi algorithm.The proposed algorithm in this paper reduces the mathematical computations involved in the solution process and obtains the optimal shortest path like other algorithm. The proposed algorithm was used to solve the Nigeria road network problem of motorable roads from multiple sources in the coastal towns of Lagos, Warri, Port Harcourt and calabar to multiple destinations of border towns of Katsina, Sokoto and Maiduguri, and obtained the shortest possible routes for vehicular movement. The result was not compromised.

Keywords Road network, shortest route, multiple sources, multiple destinations and Modified dynamic algorithm

1. Introduction

Network of roads have become very important for road transporters, since it serves as a channel of moving people, goods and services from one place to another. In recent time, we have also observed that a lot of traffic congestion happens on our roads as a result of more influx of vehicles and poor state of some of the roads. These have caused a lot of delay in transportation of people, goods and services from their sources to respective destinations. It has been observed that alternative routes can be used for the same purpose of transporting people, goods and services from one location to the other, some of these roads can take a very long time which results in delay and even breakdown of vehicle, thus, increasing the cost of maintaining such vehicles on the path of management.

The most traditional path finding solutions are based on shortest path algorithms that tend to minimize the cost of travel from point to another [1]. It was stated that, multiple sources to multiple destination deals with many stops and attempts to achieve the shortest total distance. It plays an important role in saving gas, time and money and has application in a wide range of industries such as transportation, travel planning and delivery services. While the exact method exists, it is computationally very intensive, its computational complexity increases exponentially with the number of stops (destinations), which make such method practically infeasible in real world application. He added that, the algorithm proposed in his work applied a divide-and-conquer rule, i.e. it recursively breaks down large number of stops problem into small number of stops routing problem, thereby, greatly reduces the computational complexity and yet researches satisfactory results. Shortest route model is one of the network models which applications cover a wide range of areas such as telecommunications, agriculture, petroleum, education, military, road constructions, among others [2].

Earlier, [3], presented 42 applications of shortest path problem drawn from the fields of operations research, computer science, physical sciences, medicine, engineering and applied mathematics, some of which are: Matrix rounding problem, Locating object in space, Urban traffic flows, Routing of multiple commodities, Local access telephone network, Multi-item production planning, and so on.From the foregoing, this research work seeks to obtain the shortest possible route for vehicular movement from multiple sources to multiple destinations through multiple-roads network system so as to minimize the cost of fueling, maintenance and loss of customers and get to its destinations in good time.

The dynamic programming method for solving complex problems by breaking it down into a collection of simpler sub-problems, solving each of the sub-problems just once, and storing their solutions-ideally, using a memory-based data structure will be applied. The existing algorithm can be used to determine the shortest route between only two nodes Single source to single destination (SSSD). In addition, other algorithms like Dynamic programming algorithm, A^* search algorithm, Dijkstra's algorithm, Bellman-Ford-Moore algorithm, Floyd-Warshall algorithm, Johnson's algorithm and Veterbi algorithm are used to determine the shortest route from single source to single destination, single source to multiple destinations, multiple sources to single destination (by reversing the order of the network to get single source to multiple destinations) and multiple sources to multiple destinations (by splitting the entire network into individual single source to multiple destinations). However, the proposed algorithm will be able to handle the four network models above and does not need the reversal of the network order nor splitting the entire network into individual single source to multiple destinations. This implies a reduction in the number of iterations required to solve a given network problem as compared with other existing algorithm, and hence a reduction in processing time and storage space [4].

There was difficult to trace the history of the shortest path problem. One can imagine that, in very primitive societies (even animals), finding shortest paths (for instances, to food or water) is essential. He further added that, compared with other combinatorial optimization problem, like shortest spinning tree, assignment and transportation problem, the mathematical research in the shortest path problem started relatively late [5] and quoted [6] when he stated that, path problem were also studied at the beginning of 1950's in the context of alternate routing, that is, finding a second shortest route if the shortest route is blocked. In the work of [7], a number of existing shortest path search algorithms designed for the single pair shortest path problem include those stated earlier. They added that, among the above mentioned algorithm, Dijkstra's shortest path algorithm is the most commonly known to find the optimal shortest route from one origin to all vertices, while it can also be used to find the shortest route from one origin to one destination. Bidirection Dijkstra's algorithm is a variant of Dijkstra's algorithm with improved speed. They further noted that, A* search algorithm and its variants are the most popular path search algorithm designed for the shortest path problem implemented by major web-based map services to find the approximate shortest path between two nodes due to its low computational time and low memory consumption. Floyd's algorithm is an example of dynamic programming algorithm which is used for solving shortest path problems.

It was noted that, Dijkstra's algorithm or its variations are most generally utilized route finding algorithm for solving the shortest fasted and optimized path and others. He added that, Dijkstra's algorithm is sometime called the single-source most limited way on the grounds that, it understands the single-source shortest path problem on a weighted guided chart (G = V, E), where G is the graph, V is a situation which component are called vertices (intersections, nodes or junctions) and E is the set of ordered pair of vertices called coordinated edges (roads, segments or arcs).

They also added that, Floyd's algorithm is used to find all paths from the source node to every other destination node in a network using matrix technique. He finally stated that Bellman-Ford algorithm is also one of the algorithm to find shortest path, but works for negative weights by: detecting a negative cycle if any exists, and finding shortest simple path if no negative cycle exists [8].

Bellman's Dynamic Programming (BDP) Algorithm

It was stated that, after publishing several papers on dynamic programming (which is, in some sense, a generalization of shortest path methods), Bellman described equation (1) as the functional recursive equationfor the shortest path problem [5]:

$$v_n(S_n) = \text{Min}\ \{t_n(S_n) + v_{n-1}(S_{n-1}))\},\ S_n = 1, 2, 3, \ldots, n-1 \qquad (1)$$

Where:

$v_n(S_n)$ = optimal value over the current state and subsequent stages, given that we are in state S_n with n stages to go.

$t_n(S_n)$ = the distance in state S_n with n stages to go.

$v_{n-1}(S_{n-1})$ = optimal value before the current state and previous stages, given that we are in state S_n with n stages to go.

S_n = number of stages

n = number of states

$V_0(S_0) = t_0(S_0)$, $S_0 = 1, 2, 3, \ldots, N$

Immediate decision is S_{n-1}, and optimal decisions are made thereafter.

The Model

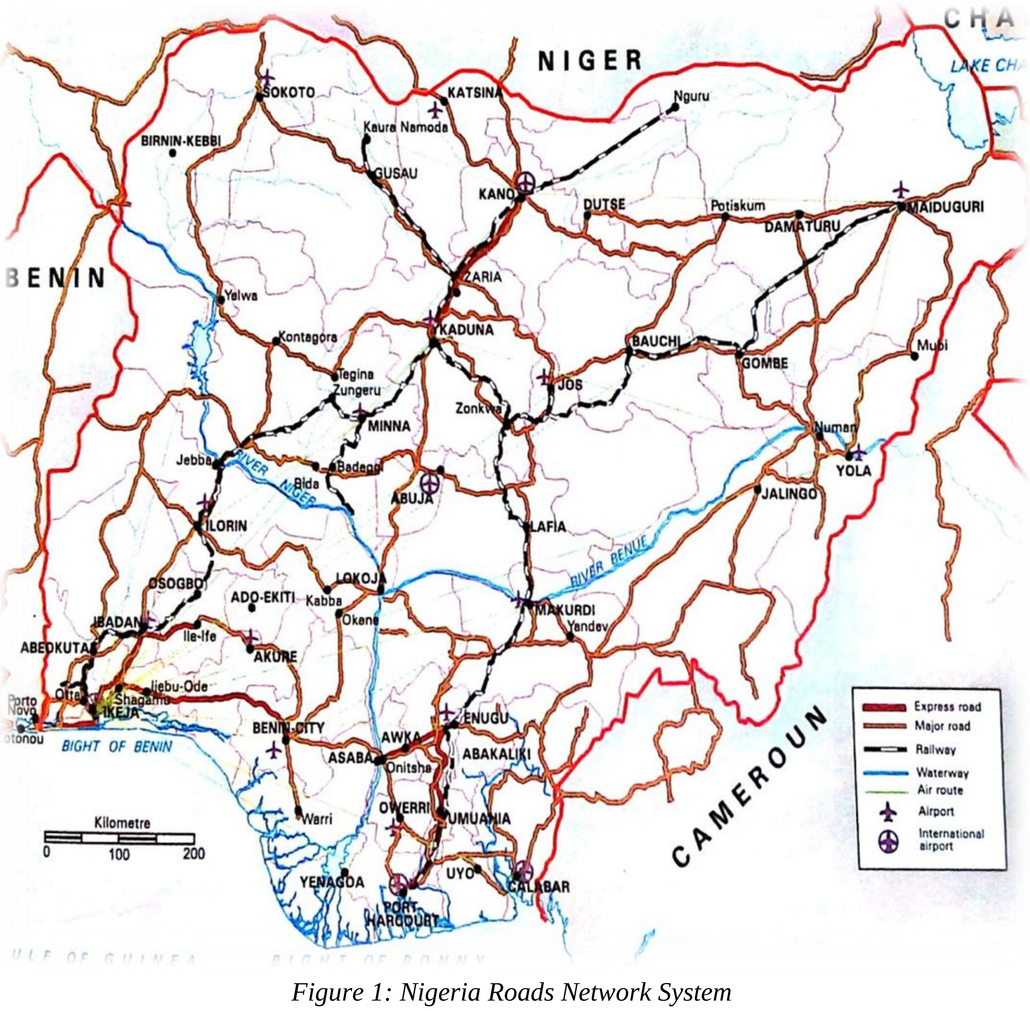

Figure 1: Nigeria Roads Network System

The Nigeria road network model of multiple sources to multiple destinations described in this paper is shown in Fig 3 was model from the Nigeria roads network map (see Fig 1) and the Kilometre Chart of Nigeria States' Capitals (see Fig 2). The road network of motorable road from multiple sources (coastal towns: Lagos, Port-Harcourt, Asaba/warri and Calabar) to multiple destinations (border towns: Katsina, Sokoto and Maiduguri) passed through intermediate routes is shown in Fig 1. The purpose algorithm is to find the shortest route from each of these sources to each of these destinations, by doing so, we adopt the modified version of the Dynamic programming algorithm proposed in this paper, which can solve the problem directly, with far less number of iterations, without splitting the network into single source to single destination or single source to multiple destinations. The ordinal numbers 1 to 75 in Fig 1 and Tables 1-12 does not represent the distance, but the town number for ease of reference.

The arrows in Fig 3 signify the direction of flow of possible route from the sources to the destinations. In order for all states (state capitals) to be reached, other states appear more than once. This is because the graph is a directed, therefore, the movement is only directed forward and for all states to be reached, we must model it this way. In addition, the distance between these cities are collected from kilometer chart (see Fig 2) within states capitals and depicted in tableaus 1 – 12. However, the distance between each pair of identical state is 0, e.g. OSO to OSO = 0, IBD to IBD = 0,…, YOL to YOL = 0 .

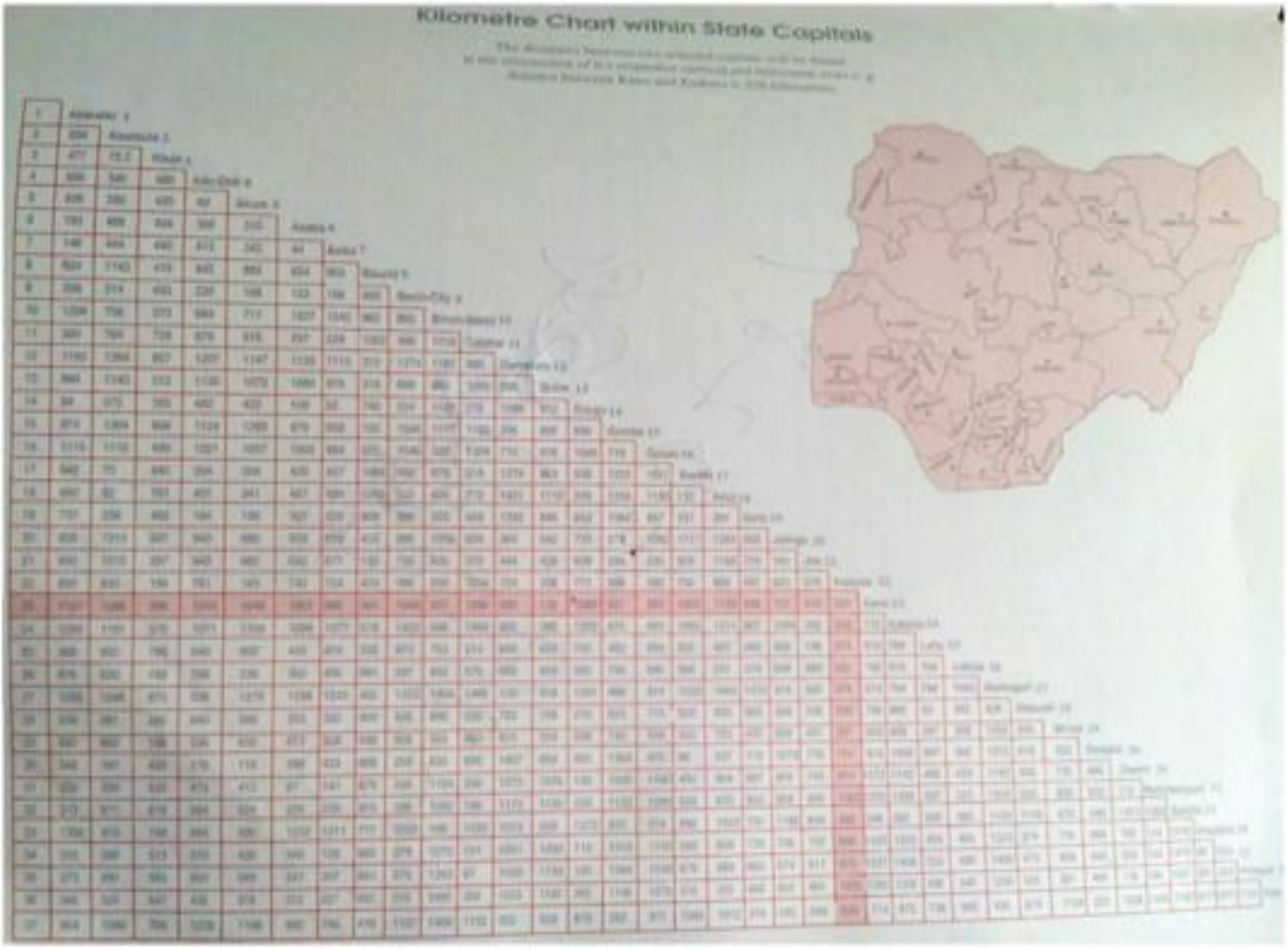

Figure 2: Kilometer Chart of Distances between Cities in Nigeria

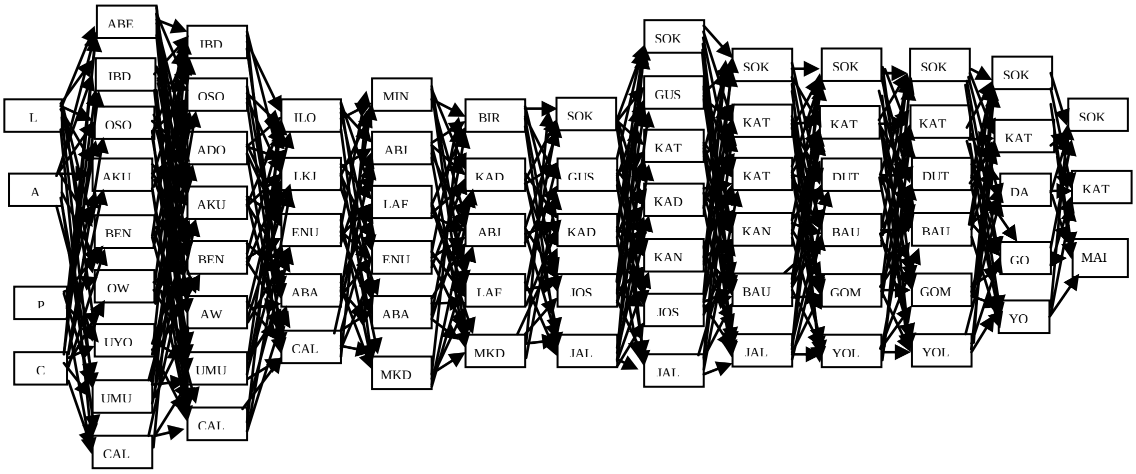

Figure 3: Nigeria road network of motorable road from multiple sources (coastal towns: Lagos, Port-Harcourt, Asaba/warri and Calabar) to multiple destinations (border towns: Katsina, Sokoto and Maiduguri).

The Proposed Algorithm

The generalized dynamic programming method by Bellman cannot solve multiple sources to multiple destinations problem directly. It can only determine the first destination in the multiple sources of multiple destinations problem. We also discover that, the generalized dynamic programming method by Bellman can solve the multiple sources to multiple destinations problem by decomposing it into single source to single destination then solve these networks individually before superimposing them to one network. However, the proposed algorithm which is a modification in the procedure of applying the Bellman generalized algorithm can solve multiple sources to multiple destination problems and obtain the shortest route from different sources to the different destinations. However, we discovered that the algorithm can also solve the other three categories of:

i. Single source to single destination network problem

ii. Single source to multiple destination network problem (without reversing the network problem order) and

iii. Multiple sources to single destination network problem (without reversing the network problem order).

The Procedure of the Proposed Algorithm

Step 1: Identify the problem decision variables and specify the objective function to be optimized under certain conditions, (if any)

Step 2: Decompose (or divide) the given network problem into a number of smaller sub-problems (or stages). Identify the state variables at each stage and write down the transformation function as a function of the state variables and decision variables at the next stage.

Step 3: If the last stage has more than one destination (state), decompose (or divide) the last stage, into individual destination (state), otherwise, proceed to step 4.

Step 4: Write down a general recursive relationship for computing the optimal policy. Decide whether to follow the forward or the backward method for solving the problem.

Step 5: Construct appropriate tables to show the required values of the return function at each stage as shown in the table 1 below.

Step 6: Determine the overall optimal policy or decisions and its value at each stage. There may be more than one such optimal policy.

Table 1: Required Values of the Return Function at Each Stage

Decision, d_n → States, S_n ↓	$f_n(s_{n,\,dn})$	Optimal return $f_n^*(s_n)$	Optimal decision d_n^*

Table1:
Stage1

	5	6	7	8	9	10	11	12	13
1	82	132	237	341	322	564	689	600	772
2	468	430	396	310	133	97	247	166	297
3	671	656	623	524	296	112	120	114	196
4	764	518	695	619	490	200	67	151	0

Table2:
Stage 2

	14	15	16	17	18	19	20	21
5	75	167	340	280	314	444	595	764
6	0	90	264	204	300	457	580	518
7	90	0	178	118	259	425	564	695
8	204	118	60	0	711	342	430	619
9	300	259	228	168	0	166	279	490
10	450	495	473	413	240	141	165	200
11	676	645	606	545	375	207	88	67
12	580	564	510	430	279	126	0	151
13	518	695	679	619	490	259	151	0

Table3:
Stage 3

	22	23	24	25	26
14	157	584	558	642	518
15	115	305	462	546	695
16	164	296	482	566	679
17	199	236	422	506	619
18	399	287	257	338	490
19	528	456	62	146	259
20	770	405	118	202	151
21	849	579	276	360	0

Table4:
Stage 4

	27	28	29	30	31	32
22	432	482	660	653	660	863
23	309	193	788	392	476	342
24	598	393	705	0	84	270
25	682	477	388	84	0	256
26	860	729	514	276	360	532

Table5:
Stage 5

	33	34	35	36	37
27	456	297	156	682	440
28	573	186	0	180	280
29	753	474	180	0	82
30	1120	772	393	705	270
31	1204	856	477	388	256
32	898	500	280	82	0

Table6:
Stage 6

	38	39	40	41	42
33	198	322	550	830	1266
34	480	260	0	278	822
35	748	490	186	297	691
36	840	650	474	196	450
37	1116	774	500	336	368

Table7:
Stage 7

	43	44	45	46	47	48	49
38	0	274	380	480	546	644	1188
39	274	0	463	260	260	330	1082
40	480	260	428	0	220	278	822
41	644	330	592	278	418	0	542
42	1188	1082	1084	822	731	542	0

Table8:
Stage 8

	50	51	52	53	54
43	0	380	546	777	1188
44	274	463	260	670	1082
45	380	0	172	518	1084
46	480	428	220	410	822
47	546	172	0	301	731
48	644	592	418	130	54.2
49	1188	1084	731	410	0

Table9:
Stage 9

	55	56	57	58	59	60

Table10:
Stage 10

	61	62	63	64	65	66

50	0	480	520	777	931	1187
51	480	0	338	410	566	875
52	546	172	136	301	457	714
53	777	516	312	0	155	416
54	1188	1084	542	410	279	142

55	0	380	520	777	931	1187
56	380	0	296	518	675	820
57	520	296	0	318	895	558
58	777	518	318	0	155	416
59	931	675	895	155	0	262
60	1187	820	558	416	262	0

Table11:
Stage 11

	67	68	69	70	71
61	0	380	1030	931	1187
62	380	0	603	671	875
63	520	296	288	895	558
64	777	581	312	155	416
65	931	675	256	0	262
66	1187	875	322	262	0

Table12:
Stage 12

	72	**73**	**74**
67	0	380	1123
68	380	0	794
69	1030	603	130
70	931	675	400
71	1187	875	436

Results

Subjecting the above tableaus which represent the distance from one town to the other in the network to the proposed algorithm, the following routes were obtained showing the shortest path through the network. The results are also depicted in Fig 4 below:

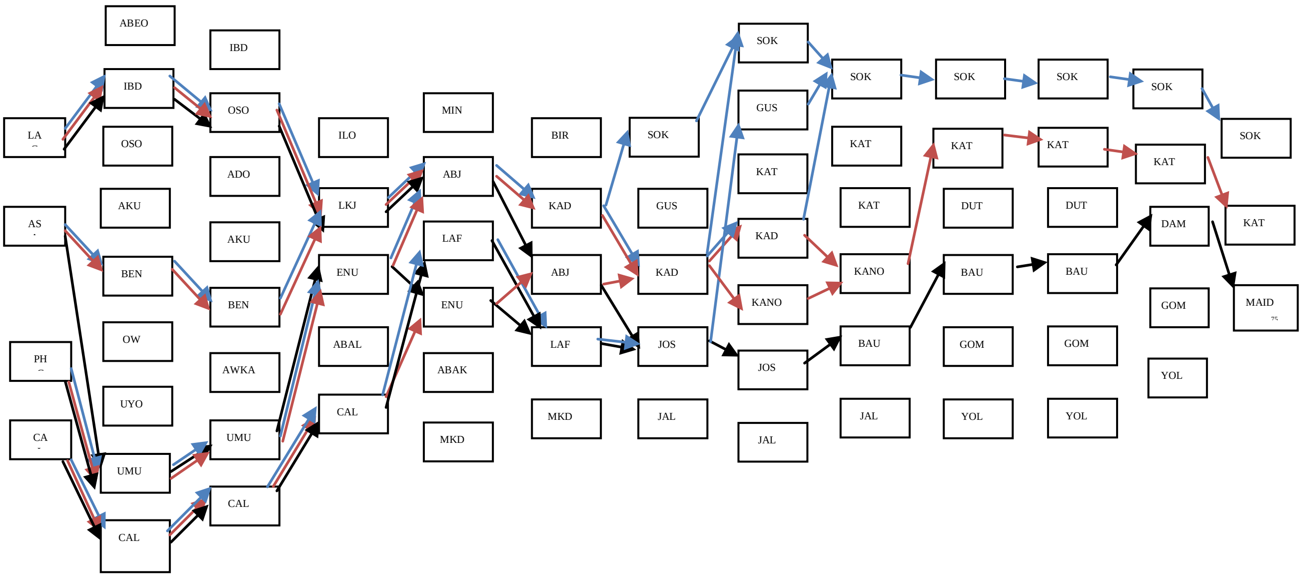

Figure 4: Optimal path of Nigeria road network of motorable road from multiple sources (coastal towns: Lagos, Port-Harcourt, Asaba/warri and Calabar) to multiple destinations (border towns: Katsina, Sokoto and Maiduguri)

1) a. Lagos to Sokoto: Lagos-Ibadan-Oshogbo-Lokoja-Abuja-Kaduna-Sokoto = 1375km
   b. Lagos to Katsina: Lagos- Ibadan-Oshogbo-Lokoja-Abuja-Kaduna-Kano-Katsina =1298km
   c. Lagos to Maiduguri: Lagos- Ibadan-Oshogbo-Lokoja-Abuja-Jos-Bauchi-Damaturu-Maiduguri =1589km
2) a. Asaba to Sokoto: Asaba-Benin-Lokoja-Abuja-Kaduna-Sokoto =1279km
   b. Asaba to Katsina: Asaba-Benin-Lokoja-Abuja-Kaduna-Kano-Katsina =1191
   c. Asaba to Maiduguri: Asaba-Umuahia-Enugu-Lafia-Jos- Bauchi-Damaturu-Maiduguri =1404km
3) a. Port Harcourt to Sokoto: PHC-Umuahia-Enugu- Abuja-Kaduna-Sokoto =1291km
   b. Port Harcourt to Katsina: PHC-Umuahia-Enugu- Abuja-Kaduna-Kano-Katsina =1203km
   c. Port Harcourt to Maiduguri: PHC-Umuahia-Enugu- Lafia-Jos- Bauchi-Damaturu-Maiduguri =1352km
4) a. Calabar to Sokoto: Calabar-Lafia-Jos-Gusau-Sokoto = 1314km
   b. Calabar to Katsina: Calabar-Enugu- Abuja-Kaduna-Kano-Katsina =1240km

c. Calabar to Maiduguri: Calabar-Lafia-Jos- Bauchi-Damaturu-Maiduguri =1282km.

Discussion

The proposed algorithm used 12 iterations to obtain the optimal solution of the Nigeria road network from the four coastal towns of Lagos, Warri, Port Harcourt and Calabar to multiple destinations of Katsina, Sokoto and Maiduguri through intermediate routes and obtain the shortest possible routes for vehicular movement without splitting as is the case with earlier methods. However, splitting the road network into single sources to single destinations we have 144 iterations, single sources to multiple destinations yields 48 iterations and multiple sources to single destinations yields 36 iterations. In addition, the proposed method also reduces the mathematical computation to 427 floating points arithmetic in the solution process to obtain the shortest routes from the four sources to the three destinations as shown in section 6, whereas, the earlier methods cannot solve this problem without splitting the network into either single sources to single destinations, single sources to multiple destinations or multiple sources to single destinations. Further, the proposed algorithm can obtain the shortest route from any number sources to any number destinations through any intermediary nodes in the network. From the foregoing this method can solve any non-negative edged directed graph to obtain the shortest route.

Conclusion

The proposed algorithm has successfully solve the model of Nigeria road network of multiple sources to multiple destinations presented in this paper in 12 iterations and 427 floating points arithmetic. There is no need for splitting the network into any smaller form like single sources to single destinations, single sources to multiple destinations and multiple sources to single destinations. The proposed algorithm is straight forward and unambiguous. In general, when applied to other road networks of multiple sources to multiple destinations, less number of iterations and floating point arithmetic are used to obtain the shortest route from the various sources to various destinations without splitting. Further, the proposed algorithm can also handle the other types of networks like single sources to single destinations, single sources to multiple destinations and multiple sources to single destinations directly, without splitting.

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