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Research Article

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Weighted Least Squares Estimation of Parameters in Heteroscedastic Regression Model Under Liner Constraint

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Abstract This paper, with the method of weighted least squares estimator ,gives an expression of estimated parameters in heteroscedastic regression model under liner constraint, the residual sums of squares of above model and unconstrained heteroscedastic regression model are compared by this conclusion, the model can be more researched and applied which has some practical and theoretical significance.

Keywords liner constraint; heteroscedastic; regression model; weighted least squares estimator

1. Introduction

For the estimation of classical linear regression model parameters, The most commonly used is the least squares estimate, and there are relatively mature theories [1], There are many ways for the estimator of parameters in heteroscedastic regression model under liner constraint, For example, the generalized least squares estimate and weighted least squares estimate, etc [2], but the paper on parameter estimation in Heteroscedastic Regression Model Under Liner Constraint is relatively small [3-4]. In this paper [4], the generalized least-square estimation of parameters in Heteroscedastic regression model Under Liner Constraint l is discussed, This paper discusses the estimation of regression model parameters Under linear constraint By the weighted least squares estimation, At the same time, the paper gives the relationship between the sum of squares for residuals of the model and the sum of the squares for residuals of Regression Model Under no Constraint.

2. Model overview

Consider the model

$$\begin{cases} Y = X\beta + \varepsilon \\ H\beta = c \\ E(\varepsilon) = 0, Var(\varepsilon) = V = diag(\sigma_1^2, \sigma_2^2, \cdots, \sigma_3^2) \end{cases} \quad (1)$$

among them, Y is $n\times 1$ observe vector, X is $n\times p$ Column full rank design matrix, ε is $n\times 1$ Random error vector,and $\varepsilon \sim N(0,V)$, $\sigma_i^2 \geq 0 \ (i=1,2,\cdots n)$. H is $q\times p$ Column full rank constraint matrix, c is $q\times 1$ vector, $\beta \in \{\beta | H\beta = c\}$ is $p\times 1$ parameter vector that we're evaluating.

3. Estimation of model parameters

For unconstrained heteroscedasticity linear regression model:

$$\begin{cases} Y = X\beta + \varepsilon \\ E(\varepsilon) = 0, Var(\varepsilon) = V = diag(\sigma_1^2, \sigma_2^2, \cdots, \sigma_3^2) \end{cases} \quad (2)$$

it's usually fitted by the least squares, In this method, the Deviation between the predicted value of y_i and the expected value of y_i is going to be multiplied by a weight w_i , The weighted a is inversely proportional to the variance of b, let $W=\{w_1,w_2,\cdots,w_n\}$,then the weighted least squares function is:

$$S(\beta_0,\beta_1,\cdots,\beta_{p-1})=\sum_{i=1}^{n}\omega_i(y_i-\beta_0-\beta_1x_{i,1}-\cdots-\beta_{p-1}x_{i,p-1})^2 \quad (3)$$

The parameter β in the model (1) is calculated using the Lagrangian multiplier, let

$$L(\beta,\lambda)=S(\beta_0,\beta_1,\cdots,\beta_{p-1})+\lambda'(H\beta-c) \quad (4)$$

Here, λ is the undetermined $q\times1$ vector, we take the partial derivative of a and b with respect to (4), and make it to zero, we get:

$$\frac{\partial L(\beta,\lambda)}{\partial\beta}=-2X'WY+2X'WY\beta+H'\lambda=0 \quad (5)$$

$$\frac{\partial L(\beta,\lambda)}{\partial\lambda}=H\beta-c=0 \quad (6)$$

We get by (5):

$$\hat{\beta}_w=(X'WX)^{-1}X'WY-\frac{1}{2}(X'WX)^{-1}H'\lambda \quad (7)$$

After the Consolidation, We get an estimate of a by Plugging (7) into (6):

$$\hat{\lambda}_w=2[H(X'WX)H']^{-1}\{H[(X'WX)^{-1}X'WY]-c\} \quad (8)$$

We get by Plugging (8) into (7):

$$\hat{\beta}_w=(X'WX)^{-1}X'WY-(X'WX)^{-1}\cdot H'[H(X'WX)H']^{-1} \cdot\{H[(X'WX)^{-1}X'WY]-c\} \quad (9)$$

4. The comparison of the sum of squares for residuals of the model

The weighted least squares estimate of the parameters of the linear regression model (2)with no constraints is

$$\hat{\beta}=(X'WX)^{-1}X'WY \quad (10)$$

We get by (9)and (10):

$$\hat{\beta}_w=\hat{\beta}-(X'WX)^{-1}H'[H(X'WX)H']^{-1}\cdot\{H[(X'WX)^{-1}X'WY]-c\} \quad (11)$$

We get the sum of squares for residuals of the model(1) by (11):

$$RSS\,\hat{\beta}_w=(Y-X\hat{\beta}_w)'(Y-X\hat{\beta}_w)=(Y-X\hat{\beta})'(Y-X\hat{\beta})$$

$$+\{(X'WX)^{-1}H'[H(X'WX)H']^{-1}(H\hat{\beta}-c)\}$$

$$\cdot X'X\{(X'WX)^{-1}H'[H(X'WX)H']^{-1}(H\hat{\beta}-c)\}$$

$$=(Y-X\hat{\beta})'(Y-X\hat{\beta})+(\hat{\beta}-\hat{\beta}'c)'X'X(\hat{\beta}-\hat{\beta}'c)$$

$$=RSS(\hat{\beta})+(\hat{\beta}-\hat{\beta}'c)XX'(\hat{\beta}-\hat{\beta}'c)>RSS(\hat{\beta})$$

Thus it can be seen that, Under the linear constraint, the weighted least-squares estimator for the regression model of the isovariance regression model is larger than the non-constraint.

5. Conclusion

In this paper, we discuss the regression model of the isovariance under linear constraint by the weighted least-squares method, and we derive the formula for the estimation of this model parameter, The formula can be used directly under different variance and the appropriate selection of weights, and Compare the size of the sum of squares of residuals in linear constraints and unconstrained situations. This paper has some theoretical and practical implications for further study and application of the heteroscedastic regression model Under the constraint.

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