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Research Article

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Different Approaches on the Matrix Division and Generalization of Cramer's Rule

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Abstract In this study, the different approaches of the matrix division and the generalization of Cramer's rule and some examples are given.

KeywordsMatrix, Division, Matrix Division, Cramer's Rule, Generalization of Cramer's Rule

I. Introduction

Recently, matrix division has been described as $\frac{A}{B} := \left[\frac{\left({}^{A}_{B}i_j\right)_{ji}}{|B|}\right]_{n\times n}$ for A and $B \epsilon M_n^n(\mathbb{R})$matrices, with $|B| \neq 0$, where $\left[{}^{A}_{B}i_j\right]$is the co-divisor matrix on B of A.

II. Different Approaches on the Matrix Division

Now, different computation of $\frac{A}{B}$ for A and $B \epsilon M_n^n(\mathbb{R})$ with A and B regular matrices, to find $\frac{A}{B}$, if it exist, proceed as follows:

Step 1. Form the augmented matrix $\left[B|A\right]$.

Step 2. Apply the Gauss-Jordan method to attempt to reduce $\left[B|A\right]$ to $\left[I_n \left| \frac{\left({}^{A}_{B}i_j\right)_{ji}}{|B|}\right.\right]$. This is written uniquely as $\left[I_n \left| \frac{A}{B}\right.\right]_{n\times n}$. Otherwise $\frac{A}{B}$ dos not exist.

Similarly, $\left[A|B\right] \sim \left[I_n \left| \frac{B}{A}\right.\right]$.

Lemma 1. Let A and B be regular matrices, with $n \times n$. Then, matrix division $\frac{A}{B}$ is regular too.

Proof. For A and B be regular matrices is $|A| \neq 0$ and $|B| \neq 0$. Then,

matrix of $\frac{A}{B}$ is regular from define.

Example 1. Graphs of matrices $A=\begin{bmatrix}1 & 1 & -1\\ 1 & 0 & -1\\ -1 & -1 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 1\\ 2 & 0 & 1\\ 1 & 1 & -1\end{bmatrix}$ and their divisions $\frac{A}{B},\ \frac{B}{A}$.

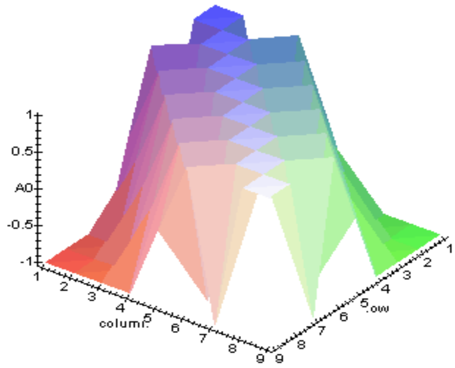


Figure 1: Graph of Matrix A

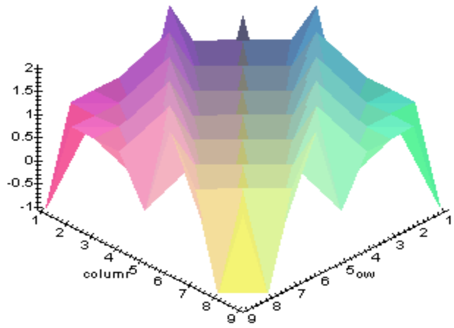


Figure 2: Graph of Matrix B

$$\left[B\middle|A\right]\sim\left[\begin{array}{ccc|ccc}1 & -1 & 1 & 1 & 1 & -1\\ 2 & 0 & 1 & 1 & 0 & -1\\ 1 & 1 & -1 & -1 & -1 & -1\end{array}\right]\Leftrightarrow\frac{A}{B}=\begin{bmatrix}0 & 0 & -1\\ 0 & -1 & -1\\ 1 & 0 & 1\end{bmatrix}$$

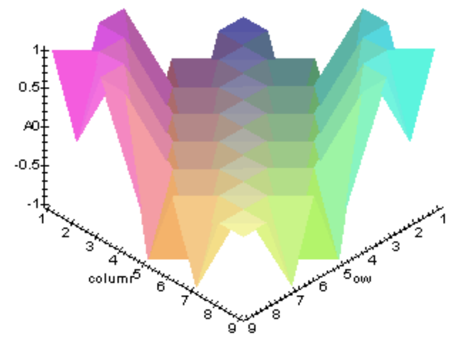


Figure :. Graph of Matrix $\frac{A}{B}$.

$$\left[A\middle|B\right]\sim\left[\begin{array}{ccc|ccc}1 & 1 & -1 & 1 & -1 & 1\\ 1 & 0 & -1 & 2 & 0 & 1\\ -1 & -1 & -1 & 1 & 1 & -1\end{array}\right]\Leftrightarrow\frac{B}{A}=\begin{bmatrix}1 & 0 & 1\\ -1 & -1 & 0\\ -1 & 0 & 0\end{bmatrix}$$

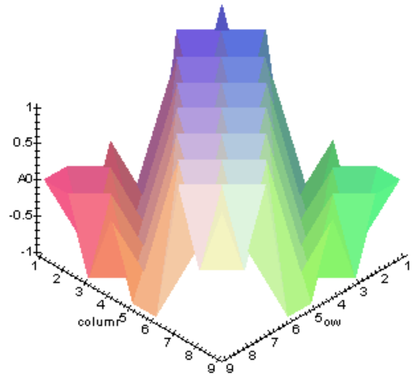


Figure 4: Graph of Matrix $\frac{B}{A}$.

Now, we have established with following theorem equivalent relation. We summarize the main ones in a new Theorem 1. for easy reference.

Theorem 1. Let A and B be $n\times n$ regular matrices. Then, the following are equivalent.

i. The system $AX=B$ has a unique solution.

ii. The matrix $\frac{B}{A}$ is invertible.

iii. The unknown matrix X is equal to $\frac{B}{A}$.

Proof.i.)$\Rightarrow$ iii.) It is obvious.

iii.) $\Rightarrow$ii.) If the system $AX=B$ has a unique solution then the solutions

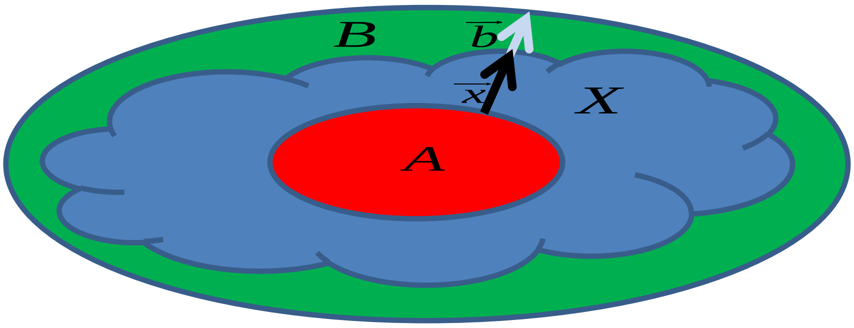
$$\left[A\middle|B\right]\sim\left[I_n\middle|\frac{B}{A}\right]\Leftrightarrow X=\frac{B}{A}.$$

ii.) $\Rightarrow$i.) If there is $\frac{B}{A}$ then matrix $\frac{B}{A}$ is invertible and $\left[\frac{B}{A}\middle|I_n\right]\sim\left[I_n\middle|\frac{A}{B}\right]\Leftrightarrow\left(\frac{B}{A}\right)^{-1}=\frac{A}{B}$,

$$\left[\frac{A}{B}\middle|I_n\right]\sim\left[I_n\middle|\frac{B}{A}\right]\Leftrightarrow\left(\frac{B}{A}\right)\left(\frac{A}{B}\right)=I_n\Leftrightarrow\left(\frac{A}{B}\right)^{-1}=\frac{B}{A}.$$

If $A=I_n$ then we certainly write A^{-1} as $\frac{I_n}{A}$. In [5] it is clamed that this can not be written.

III. Generalization of Cramer's Rule



Consider a systems of $n\times n$ linear equations for n^2 unknowns, represented in matrix multiplication form as follows:

$$AX = B, |B| \neq 0$$

where the n by n matrix A has a nonzero determinant, and the $X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \cdots & \vdots \\ x_{n1} & \cdots & x_{nn} \end{bmatrix}$ is a matrix of column vectors.

Then the following theorem states that in this case the system has a unique solution, whose individual values for the unknowns are given by:

$$x_{ji} = \frac{\det\left(\left[\left({}^{A}_{B}i_j\right)_{ji}\right]\right)}{\det(B)}, i, j = 1, \ldots, n.$$

Theorem 2. Let a system be $AX = B$, A, B regular matrices. Then, $x_{ji} = \frac{\det\left(\left[\left({}^{A}_{B}i_j\right)_{ji}\right]\right)}{\det(B)}$, $i, j = 1, \ldots, n$, where $\left[{}^{A}_{B}i_j\right]$ is the co-divisor matrix on B of A.

Proof. Let $A = \begin{bmatrix} a_{12} & \cdots & a_{1n} \\ \vdots & \cdots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$, $X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \vdots & \vdots \\ x_{n1} & \cdots & x_{nn} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \vdots & \vdots \\ b_{n1} & \cdots & b_{nn} \end{bmatrix}$, $|A|, |B| \neq 0$ be squares matrices. It is clear that $x_{ji} = \frac{\det\left(\left[\left({}^{A}_{B}i_j\right)_{ji}\right]\right)}{\det(B)}, i, j = 1, \ldots, n$ from division of matrices.

IV. Conclusions

The matrix division in [1] defined before by determinant coincides with the definition of matrices division given by writing the Gauss-Jordan method.

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