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Research Article

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Second Order Lagrangian

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Abstract Recently a Lagrangian dependent on the second order derivatives was shown to be successful in describing and solving long standing gravitational problems. This gives motivation to construct second order electromagratic Lagrangian, which may hopefully can open a new horizon to solve disasters electromagnetic problems like self charge energy problem. In this work second order electromagnetic Lagrangian was derived with vacuum current density terms. This Lagrangian is used to derive a new Maxwell's equations, which predicts that both electric charges and vacuum can generate photons. The new Hamiltonian consists of an additional terms representing source emitting or observing photons beside vacuum energy terms. The equation of motion reduces to the ordinary equation of motion and to ordinary Hamiltonian with additional source term in the absence of vacuum energy term.

Keywords Second order Lagrangian, electromagnetic field, Vacuum current density, emitting and absorbing term

1. Introduction

Electromagnetic waves (E.M.W) plays an important role in our day life now. This is since it is widely used in telecommunications and multimedia. They are described by Maxwell's equations. Maxwell's equations (M. E) unify electric and magnetic phenomena [1-2]. The M. E are derived from the laws of electricity and magnetism [3]. But later on, Lagrange formalism is utilized to derive them. Despite the remarkable success of this formalism, it suffers from some drawbacks. For instance, the Lagrange formalism, fails in describing some gravitational phenomena, namely that concerns the gravitational energy and the early universe [4]. Maxwell's equations also does not able to find finite charge self energy. Recently, attempts were made to use a generalized principle of least action to cure these defects. In this generalized version the Lagrangian consists of an additional terms that depends on the second derivatives of the generalized coordinates [4-6]. Motivated by the success of the generalized Lagrangian in most solving gravitational problems, the generalized Lagrangian is utilized. In this work one derives (M. E) [7-10]. This is done in section (3), sections (4) and (5) are devoted for discussion and conclusion.

2. Ordinary Lagrangian of Electromagnetic Field

The electromagnetic field Lagrangian is given by

$$L = \frac{1}{8\pi}\left(\frac{1}{c}\frac{\partial A}{\partial T} + \nabla\phi\right)^2 - \frac{1}{8\pi}(\nabla \times A)^2$$

$$= \frac{1}{8\pi} = (\partial_o A_i - \partial_i A_o)^2 - \frac{1}{8\pi}(\partial_i A_o - \partial_i A_o)^2$$

$$Px = \frac{\partial L}{\partial \partial_x q}$$

$$Px = \frac{\partial L}{\partial \partial_x A} = \frac{1}{4\pi c}\left(\frac{1}{c}\frac{\partial A_x}{\partial t} + \frac{\partial Q}{\partial x}\right) \quad (2.1)$$

$$P = \frac{1}{4\pi c}\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t} + \nabla\phi\right) = \frac{1}{4\pi c}\left(\partial_0 A_i - \partial_i A_o\right)$$

Where A_i's stands for magnetic potential, while ϕ represents the electric potential. The corresponding Hamiltonian is givens by

$$H = P\cdot\frac{\partial A}{\partial t} - L = \frac{1}{4\pi}\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t} + \nabla\phi\right)\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t}\right) - \mathrm{L}$$

$$= \frac{1}{4\pi}\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t} + \nabla\phi\right)\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t}\right) - \frac{1}{8\pi}\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t} + \nabla\phi\right)\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t}\right)$$

$$-\frac{1}{8\pi}\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t} + \nabla\phi\right).\nabla\phi + \frac{1}{8\pi}(\nabla\times A)^2$$

$$= \frac{1}{8\pi}\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t} + \nabla\phi\right).\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t}\right) - \frac{1}{8\pi}\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t} + \nabla\phi\right).\nabla\phi + \frac{1}{8\pi}(\nabla\times A)^2$$

$$= \frac{1}{8\pi}\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t} + \nabla\phi\right).\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t} + \nabla\phi\right) - \frac{1}{4\pi}\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t} + \nabla\phi\right).\nabla\phi + \frac{1}{8\pi}(\nabla\times A)^2$$

$$= 2\pi\, C^2P^2 - CP.\nabla\phi + \frac{1}{8\pi}(\nabla\times A)^2$$

The equation of motion and the Hamiltonians are derived from Lagrangian dependent on the field variables and their first derivatives.

3. New second order Lagrangian:

The equation of motion for second order Lagrangian takes the from

$$\frac{\partial L}{\partial A_u} - \partial_\mu\left[\frac{\partial L}{\partial \partial_\mu A_u}\right] + \partial_{\sigma\mu}\left[\frac{\partial L}{\partial \partial_{\sigma\mu} A}\right] = 0 \quad (3.1)$$

The corresponding Hamiltonian is also gives by

$$T_0^0 = H = \frac{\partial L}{\partial \partial_o A_i}\partial_o A_i - \partial_\sigma\left[\frac{\partial L}{\partial \partial_{0\sigma} A_i}\right] + \frac{\partial L}{\partial \partial_{o\sigma} A_i}\partial_{o\sigma} A_i - L \quad (3.2)$$

Where i is a dummy indices, beside σ.

The appropriate second order lagrangian that can give Maxwell's equations and the correct Hamiltonian given by:

$$L = c_1 {\eta^{\rho\sigma}}^2\left[\partial_\rho A_\sigma - \partial_\rho A_\sigma\right]^2 - c_2 A_\lambda\left[J_\psi^\lambda + J_v^\lambda\right]$$

$$-\mathrm{J}_\psi - J_A$$

$$J_\psi^\lambda = q\overline{\Psi}\gamma^\lambda\Psi = \text{charge current density}$$

$$J_A = c_3\left(\partial_{\lambda\rho}A_\rho - \partial_{\rho\rho}A_\lambda\right) = surce\ generating\ or\ absorbing\ field \quad (3.3)$$

J_u^μ vacuum current density

thus the system of matter have charge current density corresponding to rest mass energy which at the same time acts as a source emitting or absorbing field mediators. to find the equation of motion of electromagnetic field (E. M. F) one differentiate w. r. t A to get:

$$\frac{\partial L}{\partial A_v} = -c_2\left(J_\psi^\lambda + J_v^\lambda\right) \quad (3.5)$$

Also

$$\frac{\partial L}{\partial \partial_\mu A_v} = c_1\frac{\partial}{\partial \partial_\mu A_v}\left[{\eta^{\mu v}}^2\left(\partial_\mu A_v - \partial_v A_\mu\right)^2 + {\eta^{\mu v}}^2\left(\partial_v A_\mu - \partial_\mu A_v\right)^2\right]$$

$$= c_1\left[2{\eta^{\mu v}}^2\left(\partial_\mu A_v - \partial_v A_\mu\right)(1) + 2{\eta^{\mu v}}^2\left(\partial_v A_\mu - \partial_\mu A_v\right)(-1)\right]$$

$$= 4c_1{\eta^{\mu v}}^2\left(\partial_\mu A_v - \partial_v A_\mu\right) \quad (3.6)$$

Equations (5) and (6) can help in finding the equation of motion. To find the Hamiltonian, one needs to differentiate L w. r. t. to time derivative of magnetic potential $A_i$to get.

$$\frac{\partial L}{\partial \partial_o A_i}\partial_o A_i =$$

$$= \frac{\partial L}{\partial \partial_o A_i}\left[c_1 \eta^{oi^2}(\partial_o A_i - \partial_i A_o)^2 + c_1 \eta^{io^2}(\partial_i A_o - \partial_o A_i)^2\right]\partial_o A_i$$

$$c_1 \eta^{oi^2}[(\partial_o A_i - \partial_i A_o) + 2(\partial_i A_o - \partial_o A_i)(-1)]\partial_o A_i$$

$$= 2c_1(\partial_o A_i - \partial_i A_o)\partial_o A_i \qquad (3.7)$$

By choosing:

$$\partial_o = \frac{\partial}{\partial x_0} \qquad x_0 = ict \qquad A_0 = i\phi$$

$$x_1 = x, \qquad x_2 = y \qquad x_3 = z$$

$$A_1 = A_x \qquad A_2 = Ay \qquad A_3 = Az \qquad (3.8)$$

$$i = 1,2,3$$

One gets

$$\frac{\partial L}{\partial \partial_o A_i}.\partial_o A_i = 2c_1\left[-\frac{j}{c}\frac{\partial A_x}{\partial t} - j\frac{\partial \phi}{\partial x}\right]\left[-\frac{j}{c}\frac{\partial A_x}{\partial t}\right] + \cdots$$

$$= -2c_1\left[\frac{1}{c}\frac{\partial A_x}{\partial t} + \frac{\partial \phi}{\partial x}\right]\left[\frac{1}{c}\frac{\partial A_x}{\partial t}\right]$$

$$= -2c_1\left[\frac{1}{c}\frac{\partial A}{\partial t} + \nabla\phi\right]\left[-\frac{1}{c}\frac{\partial A}{\partial t}\right]$$

The terms differentiated with respect to the second order derivatives are given according to equation (3.3) and (3.4) to be.

$$\frac{\partial L}{\partial \partial_\mu A_v} = \frac{\partial}{\partial \partial_\mu A_v}\left[c^{\lambda gg}\left(\partial_{\lambda g} A_g - \partial_{\lambda g} A_\lambda\right)\right]$$

Let : $\mu = \lambda = o, \sigma = v = g = i, \qquad \mu = \sigma = g = 0 \quad v = \lambda = i$

$$= c^{\mu\sigma\sigma}\frac{\partial}{\partial \partial_{\mu\sigma} A_v}\left[(\partial_{\mu\sigma} A_v) - \partial_{\sigma\sigma} A_\mu\right]$$

$$= c^{\mu\sigma\sigma}\frac{\partial}{\partial \partial_{\mu\sigma} A_v}\left[\partial_{v\mu} A_\mu - \partial_{\mu\sigma} A_v\right]$$

$$= c^{\mu\sigma\sigma} - c^{v\sigma\sigma} = c_3 - c_3 = 0 \qquad (3.8)$$

Thus according to equations (3-1), (3-4) , (3-5) and (3-6) and (3-8) the equation of motion is given by

$$-\partial_\mu\left[4c_1 \eta^{\mu v^2}(\partial_\mu A_v - \partial_v A_\mu)\right] - c_2(J_\psi^v + J_v^v) = 0$$

By setting (3.9)

$$c_1 = -{}^1\!/_{8\pi} \qquad c_1 = -\frac{2}{c}$$

One gets the ordinary equation of motion

$$\partial_{\mu v} A_\mu - \partial_{\mu\mu} A_v = \frac{4\pi}{c}(J_\psi + J_v) \qquad (3.10)$$

The Hamiltonian can be found by a direct substitution of equations (3.8) , (3.7) and (3.8) in equation (3.2) to get

$$H = -2c_1\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t} + \nabla\phi\right)\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t}\right) - c_1 \eta^{oi^2}(\partial_o A_i - \partial_i A_o)^2$$

$$-c_1 \eta^{ij}\left(\partial_i A_j - \partial_i A_j\right)^2$$

$$+J_\psi + J_A + c_2 A_v(J_\psi^v + J_v^v)$$

According to equations (3.8) the Hamiltonian is given by

$$H = -2c_1\left(\frac{1}{c}\frac{\partial A}{\partial t} + \nabla\phi\right)\left(\frac{1}{c}\frac{\partial A}{\partial t}\right)$$

$$= -c_1\left(-\frac{i}{c}\frac{\partial A_x}{\partial t} - i\frac{\partial \phi}{\partial x}\right)\left(-\frac{i}{c}\frac{\partial A_x}{\partial t} - i\frac{\partial \phi}{\partial x}\right)$$

$$-c_1(\overline{\nabla} \times \underline{A}) + J_\psi + J_A + c_2 A_v(J_\psi^v + J_v^v)$$

$$-2c_1\left(\frac{1}{c}\frac{\partial A}{\partial t} + \nabla\phi\right)\left(\frac{1}{c}\frac{\partial A}{\partial t}\right)$$

$$+c_1\left(\frac{1}{c}\frac{\partial A}{\partial t}+\nabla\phi\right)^2-c_1\left(\vec{\nabla}\times\underline{A}\right)^2+J_\psi+J_A+c_2A_v\left(J_\psi^v+J_v^v\right) \quad (3.12)$$

$$=-2c_1\left[\left(\frac{1}{c}\frac{\partial A}{\partial t}+\nabla\phi\right)^2-\frac{1}{2}\left(\frac{1}{c}\frac{\partial A}{\partial t}+\nabla\phi\right)^2-\left(\frac{1}{c}\frac{\partial A}{\partial t}+\nabla\phi\right)(\nabla\phi)+\frac{1}{2}(\nabla\times A)^2\right]$$

$$+J_\psi+J_A+c_2A_v\left(J_\psi^v+J_v^v\right)$$

$$=-2c_1\left[\frac{1}{2}\left(\frac{1}{c}\frac{\partial A}{\partial t}+\nabla\phi\right)^2-\left(\frac{1}{c}\partial\frac{A}{\partial t}+\nabla\phi\right)+\frac{1}{2}(\nabla\times A)^2\right]$$

$$+J_\psi+J_A+c_2A_v\left(J_\psi^v+J_v^v\right)$$

$$=\frac{1}{8\pi}\left[\left(\frac{1}{c}\frac{\partial A}{\partial t}+\nabla\phi\right)^2-2\left(\frac{1}{c}\frac{\partial A}{\partial t}+\nabla\phi\right).\nabla\phi+(\nabla\times A)^2\right]$$

$$\frac{1}{8\pi}\left[6\pi^2c^2p^2-8\pi cp.\nabla\phi+\left(\nabla\times\underline{A}\right)^2\right]+J_\psi+J_A+c_2A_v\left(J_\psi^v+J_v^v\right)$$

$$H=2\pi c^2p^2-cp.\nabla\phi+\frac{1}{8\pi}\left(\nabla\times\underline{A}\right)^2+J_\psi+J_A+c_2A_v\left(J_\psi^v+J_v^v\right) \quad (3.13)$$

There

$$c_1=-\frac{1}{8\pi}$$

This Hamiltonian is the usual electromagnetic field Hamiltonian.

Discussion

This second order Lagrangian was shown to be successful in describing the gravitational phenomena by the so called generalized general relativity. This motivates us to try to construct new second order Lagrangian for the electromagnetic field, as shown by equation (3.3). In this Lagrangian matter energy manifests is self through two terms. The first term J_ψ represents energy stored in a charge which equals charge energy per parties γ^μ multiplied by the number of particles $|\psi|^2$. The second terms represents the contribution of charged system in generating or observing photons. This term, J_A, which represents the a field source manifests its role in absorbing or generating field through the equation of motion (3.10). Which shows dependence of J_A on second order derivative of A_μ.

It is very interesting to note that this terms gives no contribution to the equation of motion according to equations (3.8) and (3.1). At the same time the Hamiltonian of equation (3.13) shows the appearance of the terms J_A which describes absorption or emission of photons energy by charged systems, beside the terms J_ψ which represent the energy stored in electric charges.

It is also important to note that equation (3.10) shows that both electric charges and vacuum energy can generate electromagnetic field. The generation of e. m. field was proved by Cassimar effect. The Hamiltonian in equations (3.13) too also have a term recognizing the vacuum energy. This is also experimentally verified by Cassimar effect.

Conclusion

The Lagrangian depending on second order field derivatives shows that vacuum energy can generate electromagnetic field as well as electric charges. It shows also that the Hamiltonian consists of terms recognizing charge energy, source energy, and vacuum energy.

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