



Effect of Two Temperature and Rotation on Plane Waves of Generalized Thermoelasticity under Thermal Loading due to Laser Pulse in the Context of Three Theories

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Abstract In the present paper, we introduced the coupled theory, Lord–Schulman theory, and Green–Lindsay theory to study the rotation on a two-dimensional problem of thermoelasticity subject to thermal loading by a laser pulse. The material is a homogeneous isotropic elastic half-space and is heated by a non-Gaussian laser beam with a pulse. The method applied here is to use normal mode analysis to solve a thermal shock problem. Deformation of a body depends on the nature of the force applied as well as the type of boundary conditions. Numerical results for the temperature, displacement, and thermal stress components are given and illustrated graphically in the presence of rotation, reinforcement, and for different values of time.

Keywords Rotation, Generalized thermoelasticity, Laser pulse, Normal mode analysis

1. Introduction

In the classical theory of thermoelasticity, Fourier's heat conduction theory assumes that the thermal disturbances propagate at infinite speed, which is unrealistic from the physical point of view. Two different generalizations of the classical theory of thermoelasticity have been developed, which predict only the finite velocity of propagation of heat and displacement fields. The first one is given by Lord and Shulman [1]. The second developed a temperature rate dependent thermoelasticity by including temperature rate among the constitutive variables is given by Green and Lindsay [2] have introduced situations where very large thermal gradients or annular a high heating speed may exist on the boundaries [3].

Effects of rotation and relaxation times on plane waves in generalized thermo-elasticity are studied by Roychoudhuri [4]. The classical Fourier model, which leads to an infinite propagation speed of the thermal energy, is no longer valid [5]. Ahmad and Khan [6] studied the effect of rotation on thermoelastic plane waves in an isotropic medium.

Some problems in thermoelastic rotating media are due to, and Schoenberg and Censor [7], Puri [8], Singh and Kumar [9], Othman [10-13], Othman and Singh [14], Abd-Alla and Abo-Dahab [15]. Also normal mode analysis is used to solve a lot of problems in thermal flexibility, such as Lotfy and Abo-Dahab [16] and Othman and Song [17].

The so-called ultra-short lasers are those with pulse duration ranging from nano-seconds to Femto-seconds in general. In the case of ultra-short-pulsed laser heating, the high-intensity energy flux and ultra-short duration laser beam, have introduced situations where very large thermal gradients or an ultra-high heating speed may exist on the boundaries by Al-Qahtani and Datta [18] Othman and Song [19] have also studied the effect of



rotation on plane waves of generalized electro-magneto-thermo-visco-elasticity with two relaxation times. These problems are based on the more realistic elastic model since earth; the moon and other planets have angular velocity. Wang and Xu [20] have studied the stress wave induced by Pico-and Femto-second laser pulses in a semi-infinite metal by expressing the laser pulse energy as Fourier series.

Some researchers have investigated different problems of the laser pulse. Ultra short, tightly-focused laser pulses are employed in a wide range of fields, including particle acceleration [21, 22], high resolution microscopy was discussed by Dudovich et al. [23], Schwoerer et al. [24] studied X-ray generation and particle trapping was studied by Jiang, Tetsuya and Hiromi [25].

The present paper is to investigate the influence of the rotation and thermal loading due to laser pulse on the plane waves in a linearly thermoelastic isotropic medium. The problem has been solved numerically using normal mode analysis. Numerical results for the temperature, displacement components, and the stresses are represented graphically and the results are analyzed. The graphical results indicate that the effect of rotation, reinforcement, and laser pulse for different values of time on the plane waves in the thermoelastic medium are very pronounced.

2. Formulation of the problem and basic equations

We consider the problem of a rotating thermoelastic half-space ($x \geq 0$). Acting parallel to the boundary plane (taken as the direction of the z -axis). The surface of a half-space is heated uniformly by a laser pulse that is a function of x , y , and t . Thus, all quantities considered are independent of z and the third component of displacement vector vanishes. When all body forces are neglected the constitutive equation for a linear thermoelastic transversely isotropic medium.

When all body forces are neglected, the governing equations are:

(1) Strain-displacement relations

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \omega_{ij} = \frac{1}{2}(u_{i,j} - u_{j,i}), \quad (1)$$

where, $u_i = (u, v, 0)$ are the components of the displacement vector and e_{ij} are the components of strain tensor.

(2) Stress-displacement relations

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e \delta_{ij} - \gamma(1 + \nu_0) \frac{\partial}{\partial t} T \delta_{ij}, \quad (2)$$

(3) Heat conduction equation

$$K \Theta_{,ii} = \rho C_E (1 + \tau_0) \frac{\partial}{\partial t} T + (1 + n_0 \tau_0) (\gamma T_0 \dot{e} - \rho Q) \quad (3)$$

$$T = \Theta - a^* \Theta_{,ii}, \quad (4)$$

The plate surface is illuminated by laser pulse given by the heat input by Al-Qahtani and Datta [18]:

$$Q = I_0 f(t)g(y)h(x) \quad (5)$$

where, I_0 is the energy absorbed, the temporal profile $f(t)$ is represented as

$$f(t) = \frac{t}{t_0^2} \exp\left(-\frac{t}{t_0}\right) \quad (6)$$

where t_0 is the pulse rising time.

$$g(y) = \frac{1}{2\pi r^2} \exp\left(-\frac{y^2}{r^2}\right) \quad (7)$$

where r is the beam radius. The function of the depth, x , is

$$h(x) = z^* \exp(-z^* x) \quad (8)$$

From Equations (6)-(8) in Equation (5), we have:



$$Q = \frac{I_0 z^*}{2\pi r^2 t_0^2} t \exp\left(\frac{-y^2}{r^2} - \frac{t}{t_0}\right) \exp(-z^* x) \quad (9)$$

Where σ_{ij} are the stress components, λ, μ are the Lamé constants, $\gamma = (3\lambda + 2\mu)\alpha_t$,

α_t is the thermal expansion coefficient, δ_{ij} is the Kronecker delta, T is the temperature above the reference temperature T_0 , k is the thermal conductivity, n_0 is a parameter, τ_0, ν_0 are the relaxation times, ρ is the density, C_E is the specific heat at constant strain and θ is the conductive temperature.

Since the medium is rotating uniformly with an angular velocity $\boldsymbol{\Omega} = \Omega \mathbf{n}$ where, \mathbf{n} is a unit vector representing the direction of the axis of the rotation, the equation of motion in the rotating frame of reference has two additional terms (Schoenberg and Censor [11]): centripetal acceleration $\boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \mathbf{u})$ due to time varying motion only and Coriolis' acceleration $2\boldsymbol{\Omega} \wedge \dot{\mathbf{u}}$, then the equation of motion in a rotating frame of reference is

$$\rho[\ddot{u}_i + \{\boldsymbol{\Omega} \wedge \boldsymbol{\Omega} \wedge \mathbf{u}\}_i + 2(\boldsymbol{\Omega} \wedge \dot{\mathbf{u}})_i] = \sigma_{ij,j}, \quad i, j = 1, 2, 3. \quad (10)$$

Equations (1)-(3) are the field equations of the generalized linear thermoelasticity for a rotating media, applicable to the coupled theory, for generalizations, as follows:

1. The equations of the coupled (CT) theory, when

$$n_0 = 0, \tau_0 = \nu_0 = 0. \quad (11)$$

2. Lord-Shulman (L-S) theory, when

$$n_0 = 1, \nu_0 = 0, \tau_0 > 0. \quad (12)$$

3. Green-Lindsay (G-L) theory, when

$$n_0 = 0, \nu_0 \geq \tau_0 > 0. \quad (13)$$

The constitutive relations, using Eq. (1), can be written as

$$\sigma_{xx} = (\lambda + 2\mu)u_{,x} + \lambda v_{,y} - \gamma(1 + \nu_0) \frac{\partial}{\partial t} T, \quad (14)$$

$$\sigma_{yy} = \lambda u_{,x} + (\lambda + 2\mu)v_{,y} - \gamma(1 + \nu_0) \frac{\partial}{\partial t} T, \quad (15)$$

$$\sigma_{zz} = \lambda e - \gamma(1 + \nu_0) \frac{\partial}{\partial t} T, \quad (16)$$

$$\sigma_{xy} = \mu(u_{,y} + v_{,x}), \quad \sigma_{xz} = \sigma_{yz} = 0. \quad (17)$$

The equation of motion in the absence of body force:

$$\rho(\ddot{u} - \Omega^2 u - 2\Omega \dot{v}) = \mu \nabla^2 u + (\lambda + \mu)e_{,x} - \gamma(1 + \nu_0) \frac{\partial}{\partial t} T_{,x} \quad (18)$$

$$\rho(\ddot{v} - \Omega^2 v + 2\Omega \dot{u}) = \mu \nabla^2 v + (\lambda + \mu)e_{,y} - \gamma(1 + \nu_0) \frac{\partial}{\partial t} T_{,y} \quad (19)$$

For simplifications we shall use the following non-dimensional variables:

$$\{t', \tau'_0, \nu'_0\} = \omega^* \{t, \tau_0, \nu_0\}, \quad u'_i = \frac{\rho c_0 \omega^*}{\gamma T_0} u_i, \quad \{T', \Theta'\} = \frac{\{T, \Theta\}}{T_0}, \quad x'_i = \frac{\omega^*}{c_0} x_i, \quad \sigma'_{ij} = \frac{\sigma_{ij}}{\gamma T_0},$$

$$\Omega' = \frac{\Omega}{\omega^*}, \quad Q' = \frac{\gamma^2}{\rho c_0^2} Q. \quad (20)$$



Where $\omega^* = \rho C_E c_0^2 / K$ and $c_0^2 = (\lambda + 2\mu) / \rho$.

In terms of the non-dimensional quantities defined in (20), the above governing equations take the form (dropping the primes over the non-dimensional variables for convenience)

$$\ddot{u} - \Omega^2 u - 2\Omega \dot{v} = (1 - \beta) \nabla^2 u + \beta \frac{\partial e}{\partial x} - (1 + \nu_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial x}, \quad (21)$$

$$\ddot{v} - \Omega^2 v + 2\Omega \dot{u} = (1 - \beta) \nabla^2 v + \beta \frac{\partial e}{\partial y} - (1 + \nu_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial y}, \quad (22)$$

$$\nabla^2 \Theta = (1 + \tau_0 \frac{\partial}{\partial t}) \dot{T} + (1 + n_0 \tau_0 \frac{\partial}{\partial t}) (\varepsilon \dot{e} - \varepsilon_1 Q) \quad (23)$$

$$T = (1 - a \nabla^2) \Theta. \quad (24)$$

$$\text{Where, } \varepsilon = \frac{\gamma^2 T_0}{\rho^2 C_E c_0^2}, \quad \varepsilon_1 = \frac{\rho c_0^2}{\gamma^2 T_0 C_E \omega^*}, \quad \beta = \frac{(\lambda + \mu)}{\rho c_0^2}, \quad a = \frac{a^* \omega^{*2}}{c_0^2}.$$

Also, the constitutive relations (14)-(17) reduces to

$$\sigma_{xx} = u_{,x} + (2\beta - 1)v_{,y} - (1 + \nu_0 \frac{\partial}{\partial t}) T, \quad (25)$$

$$\sigma_{yy} = (2\beta - 1)u_{,x} + v_{,y} - (1 + \nu_0 \frac{\partial}{\partial t}) T, \quad (26)$$

$$\sigma_{zz} = (2\beta - 1)e - (1 + \nu_0 \frac{\partial}{\partial t}) T, \quad (27)$$

$$\sigma_{xy} = (1 - \beta)(u_{,y} + v_{,x}), \quad \sigma_{xz} = \sigma_{yz} = 0. \quad (28)$$

We shall consider only the two-dimensional problem. Assuming that all variables are functions of space coordinates x, y and time t and independent of coordinate z . So the displacement components are

$$u_x = u(x, y, t), \quad u_y = v(x, y, t), \quad u_z = 0.$$

We introduce the displacement potentials $\Phi(x, y, t)$ and $\Psi(x, y, t)$ which related to displacement components by the relations:

$$u = \Phi_{,x} + \Psi_{,y}, \quad v = \Phi_{,y} - \Psi_{,x}. \quad (29)$$

Using Eqs. (21)-(23), the two-dimensional equations of motion and the heat-conduction equation become, respectively

$$(\frac{\partial^2}{\partial t^2} - \Omega^2 - \nabla^2) \Phi + 2\Omega \dot{\Psi} + (1 + \nu_0 \frac{\partial}{\partial t})(1 - a \nabla^2) \Theta = 0 \quad (30)$$

$$[\frac{\partial^2}{\partial t^2} - \Omega^2 - (1 - \beta) \nabla^2] \Psi - 2\Omega \dot{\Phi} = 0 \quad (31)$$

$$\nabla^2 \Theta - (1 + \tau_0 \frac{\partial}{\partial t})(1 - a \nabla^2) \dot{\Theta} - (1 + n_0 \tau_0 \frac{\partial}{\partial t}) \varepsilon \nabla^2 \dot{\Phi} = -Q_0 f^*(y, t) \exp(-z^* x) \quad (32)$$

$$Q_0 = \varepsilon_1 \frac{I_0 z^*}{2\pi r^2 t_0^2}, \quad f^*(y, t) = [t + n_0 \tau_0 (1 - \frac{t}{t_0})] \exp(\frac{-y^2}{r^2} - \frac{t}{t_0}) \quad (33)$$

3. Normal mode analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as

$$[\Phi, \Psi, \Theta](x, y, t) = [\bar{\Phi}, \bar{\Psi}, \bar{\Theta}](x) \exp(\alpha t + i b y) \quad (34)$$



where, ω is a complex constant; b is the wave number in the y -direction, $\bar{\Phi}(x)$, $\bar{\Psi}(x)$ and $\bar{\Theta}(x)$ are the amplitudes of the field quantities. By using (34) then (30)–(32) take the form

$$(D^2 + s_1)\bar{\Phi} - s_2\bar{\Psi} - (s_3D^2 + s_4)\bar{\Theta} = 0 \quad (35)$$

$$(D^2 + s_5)\bar{\Psi} + s_6\bar{\Phi} = 0 \quad (36)$$

$$(s_7D^2 + s_8)\bar{\Theta} - (s_9D^2 - s_{10})\bar{\Phi} = -Q_0 f(y, t) \exp(-z^* x) \quad (37)$$

where

$$s_1 = \Omega^2 - \omega^2 - b^2, \quad s_2 = 2\Omega\omega, \quad s_3 = -a(1 + \nu_0\omega), \quad s_4 = (1 + \nu_0\omega)(1 + ab),$$

$$s_5 = \frac{\Omega^2 - \omega^2}{(1 - \beta)} - b^2, \quad s_6 = \frac{2\Omega\omega}{(1 - \beta)}, \quad s_7 = \omega(1 + a(1 + \tau_0\omega)), \quad s_8 = \omega(-b^2 - (1 + \tau_0\omega)(1 - ab^2))$$

$$s_9 = \omega\varepsilon(1 + n_0\tau_0\omega), \quad s_{10} = \omega b\varepsilon(1 + n_0\tau_0\omega)$$

$$f(y, t) = [t + n_0\tau_0(1 - \frac{t}{t_0})] \exp(\frac{-y^2}{r^2} - \omega t - iby - \frac{t}{t_0}). \quad (38)$$

Eliminating $\bar{\Psi}$ and $\bar{\Theta}$ using (35)–(37) we obtain

$$\{D^6 + AD^4 + BD^2 + C\}\bar{\phi} = -Q_0 N_1 f^*(y, t) \exp(-z^* x) \quad (39)$$

In a similar manner we arrive at

$$\{D^6 + AD^4 + BD^2 + C\}\bar{\psi} = -Q_0 N_1 f^*(y, t) \exp(-z^* x) \quad (40)$$

$$\{D^6 + AD^4 + BD^2 + C\}\bar{\theta} = -Q_0 N_2 f^*(y, t) \exp(-z^* x) \quad (41)$$

$$\text{Where, } A = \frac{s_5(s_7 - s_9s_3) + (s_8 + s_1s_7 - s_4s_9 + s_{10}s_3)}{(s_7 - s_9s_3)},$$

$$B = \frac{s_5(s_8 + s_1s_7 - s_4s_9 + s_{10}s_3) + s_1s_8 + s_4s_{10} + s_6s_2s_7}{(s_7 - s_9s_3)}, \quad C = \frac{s_1s_8s_5 + s_4s_{10}s_5 + s_6s_2s_8}{(s_7 - s_9s_3)},$$

$$N_1 = \frac{(s_3z^* + s_4)(z^* + s_5)}{(s_7 - s_9s_3)}, \quad N_2 = \frac{(z^* + (s_1 + s_5)z^* + s_1s_5 + s_6s_2)}{(s_7 - s_9s_3)}. \quad (42)$$

Equation (35) can be factorized as

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)\bar{\phi}(x) = -Q_0 N_1 f^*(y, t) \exp(-z^* x) \quad (43)$$

4. Solution of the problem

where, k_n^2 ($n = 1, 2, 3$) are the roots of the characteristic equation of (43).

The general solution of (39)–(41) is given by

$$\bar{\phi}(x, y, t) = \sum_{n=1}^3 L_n \exp(-k_n x + \omega t + iby) + Q_0 N_1 g_1 f^*(y, t) \exp(-z^* x) \quad (44)$$

$$\bar{\psi}(x, y, t) = \sum_{n=1}^3 H_{1n} L_n \exp(-k_n x + \omega t + iby) + Q_0 N_1 g_1 f^*(y, t) \exp(-z^* x) \quad (45)$$

$$\bar{\theta}(x, y, t) = \sum_{n=1}^3 H_{2n} L_n \exp(-k_n x + \omega t + iby) + Q_0 N_2 g_2 f^*(y, t) \exp(-z^* x) \quad (46)$$

where



$$H_{1n} = \frac{-s_6}{k_n^2 + s_5} L_n, H_{2n} = \frac{k_n^2 + s_1 - s_2 H_{1n}}{s_3 k_n^2 + s_4} L_n, g_1 = \frac{-1}{z^{*6} + Az^{*4} + Bz^{*2} + C}, n=1,2,3 \quad (47)$$

To obtain the components of the displacement vector, from (44) and (45) in (29)

$$u(x, y, t) = \sum_{n=1}^3 H_{3n} L_n \exp(-k_n x + \omega t + iby) - (z^* + \frac{2y}{r^2}) N_1 Q_0 g f^*(y, t) \exp(-z^* x) \quad (48)$$

$$v(x, y, t) = \sum_{n=1}^3 H_{4n} L_n \exp(-k_n x + \omega t + iby) + (z^* - \frac{2y}{r^2}) N_1 Q_0 g f^*(y, t) \exp(-z^* x) \quad (49)$$

$$H_{3n} = -k_n + ibH_{1n}, H_{4n} = k_n + ibH_{1n}, \quad n=1,2,3 \quad (50)$$

The general solutions of stresses are

$$\sigma_{xx} = \sum_{n=1}^3 H_{5n} L_n \exp(-k_n x + \omega t + iby) + N_3 Q_0 g f^*(y, t) \exp(-z^* x) \quad (51)$$

$$\sigma_{yy} = \sum_{n=1}^3 H_{6n} L_n \exp(-k_n x + \omega t + iby) + N_4 Q_0 g f^*(y, t) \exp(-z^* x) \quad (52)$$

$$\sigma_{zz} = \sum_{n=1}^3 H_{7n} L_n \exp(-k_n x + \omega t + iby) + N_5 Q_0 g f^*(y, t) \exp(-z^* x) \quad (53)$$

$$\sigma_{xy} = \sum_{n=1}^3 H_{8n} L_n \exp(-k_n x + \omega t + iby) + N_6 N_1 Q_0 g f^*(y, t) \exp(-z^* x) \quad (54)$$

where

$$H_{5n} = ibH_{4n} (2\beta - 1) - k_n H_{3n} - H_{2n} [1 + \omega v_0 - a(k_n^2 - b^2) - av_0 \omega (k_n^2 - b^2)]$$

$$H_{6n} = ibH_{4n} - k_n H_{3n} (2\beta - 1) - H_{2n} [1 + \omega v_0 - a(k_n^2 - b^2) - av_0 \omega (k_n^2 - b^2)]$$

$$H_{7n} = (2\beta - 1)(k_n^2 - b^2) - H_{2n} [1 + \omega v_0 - a(k_n^2 - b^2) - av_0 \omega (k_n^2 - b^2)]$$

$$H_{8n} = (1 - \beta)(ibH_{3n} - k_n H_{4n}), \quad n=1,2,3$$

$$N_3 = N_1 [(z^{*2} + \frac{2yz^*}{r^2}) + (2\beta - 1)(-\frac{2}{r^2} + \frac{4y^2}{r^4} - \frac{2y}{r^2} z^*)] +$$

$$-N_2 \{1 + v_0 [1 - a(z^{*2} - \frac{2}{r^2} + \frac{4y^2}{r^4})] (1 - \frac{2n_0 \tau_0}{t_0} - \frac{t}{t_0} + \frac{n_0 \tau_0 t}{t_0^2}) - a(z^{*2} - \frac{2}{r^2} + \frac{4y^2}{r^4})\}$$

$$N_4 = N_1 [(2\beta - 1)(z^{*2} + \frac{2yz^*}{r^2}) + (-\frac{2}{r^2} + \frac{4y^2}{r^4} - \frac{2y}{r^2} z^*)] +$$

$$-N_2 \{1 + v_0 [1 - a(z^{*2} - \frac{2}{r^2} + \frac{4y^2}{r^4})] (1 - \frac{2n_0 \tau_0}{t_0} - \frac{t}{t_0} + \frac{n_0 \tau_0 t}{t_0^2}) - a(z^{*2} - \frac{2}{r^2} + \frac{4y^2}{r^4})\}$$

$$N_5 = N_1 (z^{*2} - \frac{2}{r^2} + \frac{4y^2}{r^4}) - N_2 \{1 + v_0 [1 - a(z^{*2} - \frac{2}{r^2} + \frac{4y^2}{r^4})] (1 - \frac{2n_0 \tau_0}{t_0} - \frac{t}{t_0} + \frac{n_0 \tau_0 t}{t_0^2})$$

$$- a(z^{*2} - \frac{2}{r^2} + \frac{4y^2}{r^4})\}$$

$$N_6 = (1 - \beta) [z^* (\frac{4y}{r^2} - z^*) - \frac{2}{r^2} + \frac{4y^2}{r^4}] \quad (55)$$



5. Boundary conditions

In this section we need to consider the boundary conditions to determine the parameters L_n ($n=1,2,3$). We shall suppress the positive exponentials that are unbounded at infinity. The constants L_1, L_2 and L_3 have to be chosen such that the boundary conditions on the surface at $x = 0$ and $I_0 = 0$.

1. The mechanical boundary conditions

$$\sigma_{xy} = 0, \frac{\partial \bar{\phi}}{\partial x} = 0, \sigma_{yy} = -p_1 \exp[\alpha x + i b y] \quad (56)$$

where p_1 is the magnitude of the mechanical force.

Substituting the expressions of the considered variables in these boundary conditions, we can obtain the following equations satisfied by the parameters:

$$\sum_{n=1}^3 H_{8n} L_n = 0 \quad (57)$$

$$\sum_{n=1}^3 -k_n L_n = 0 \quad (58)$$

$$\sum_{n=1}^3 H_{6n} L_n = -p_1 \quad (59)$$

Invoking boundary conditions (52) at surface $x = 0$ of the plate, we obtain a system of four equations, (53)–(55). After applying the inverse of matrix method, we have the values of four constants L_n ($n=1,2,3$).

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} H_{81} & H_{82} & H_{83} \\ -k_1 & -k_2 & -k_3 \\ H_{61} & H_{62} & H_{63} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ -p_1 \end{bmatrix} \quad (60)$$

Hence we obtain the expressions for the displacements, the temperature distribution, and the other physical quantities of the plate surface.

6. Numerical results and discussion

In the view to illustrate the computational work, the following material constants at $T_0 = 293^\circ \text{C}$ are considered a copper material for an elastic solid with generalized thermoelastic solid as follow:

$$\begin{aligned} \lambda &= 7.76 \times 10^{10} \text{ kgm}^{-1} \text{ s}^{-2}, & \mu &= 3.86 \times 10^{10} \text{ kgm}^{-1} \text{ s}^{-2}, & \mu_e &= 4\pi(10)^{-7} \text{ Hm}^{-1}, & \zeta &= 1, \\ \mu_0 &= 4.0 \times 10^{11} \text{ dyne/cm}^2, & k &= 386 \text{ m}^{-1} \text{ k}^{-1}, & \alpha &= -1.28 \times 10^9 \text{ N/m}^2, & I_0 &= 10^5 \text{ J/m}^2, \\ \alpha_t &= 1.78 \times 10^{-5} \text{ k}^{-1}, & C_E &= 383.1 \text{ J/kgK}, & \omega &= \omega_0 + i\zeta & a^* &= 0.1 \times 10^7, & \omega_0 &= 2, \\ \varepsilon_0 &= 0.1, & p_1 &= 1, & b &= 0.9, & r &= 25 \mu\text{m}, & z^* &= 10^{-3} / \text{m}, & t_0 &= 5 \text{ ns}, & 0 &< x < 8. \end{aligned}$$

First: the computations were carried out in the presence laser pulse $I_0 = 10^5$ and on the surface of a plane $y = 110$, The numerical results for, the normal displacement components (u, v), the force stress component $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}$ are shown respectively in all Figures Numerical analysis has been carried out by taking z range from 0 to 8 and always begins from negative values in all Figures.

From figs. (1-6), display a comparison between the three thermoelastic theories (*i.e.* CT, L-S and G-L). In figures (1, 4, 5), it is shown that the values of coefficient considering (G-L) theory are less than the



corresponding value considering (L-S) theory less than it takes into account (CT) theory. In Fig. 2 it is shown that the values of coefficient considering (G-L) theory are less than the corresponding value considering (CT) theory less than it takes into account (L-S) theory in the range $0 < x < 1$, and that the values of coefficient considering (G-L) theory are less than the corresponding value considering (L-S) theory less than it takes into account (CT) theory in the range $x > 1$. In Fig. 3 it is shown that the values of coefficient considering (G-L) theory are less than the corresponding value considering (CT) theory less than it takes into account (L-S) theory in the range $0 < x < 0.6$, and that the values of coefficient considering (G-L) theory are less than the corresponding value considering (L-S) theory less than it takes into account (CT) theory in the range $x > 0.6$. In Fig. 6 it is shown that the values of coefficient considering (G-L) theory are less than the corresponding value considering (CT) theory less than it takes into account (L-S) theory in the range $0 < x < 1.4$, and that the values of coefficient considering (G-L) theory are less than the corresponding value considering (L-S) theory less than it takes into account (CT) theory in the range $x > 1.4$.

Figs. 7-12 display the influence of rotation parameter $\Omega = 0.1, 0.2, 0.3, 0.4$.

In Figs. (7, 8, 9, 11, 12) we note that the smaller values of the x increases with a decreasing of the rotation but in Fig. 10 do not affect by the variation of the rotation.

Figs. 13-18 display the influence of rotation parameter $t = 0.1, 0.2, 0.3, 0.4$ and we noted that the smaller values of the x increases with a decreasing of the time.

6. Conclusion

By comparing the figures obtained under the three thermoelastic theories, important phenomena, we observed the following remarks:

1. The values of all the physical quantities converge to zero with the increase of distance x and all functions are continuous.
2. The method that is used in the present article is applicable to a wide range of problems in hydrodynamics and thermoelasticity.
3. The presence of the laser pulse in the current model has an important role on the field quantities.
4. The different values of rotation, laser pulse and time in the current model are very strongly pronounced and very affective on the wave propagation phenomena.

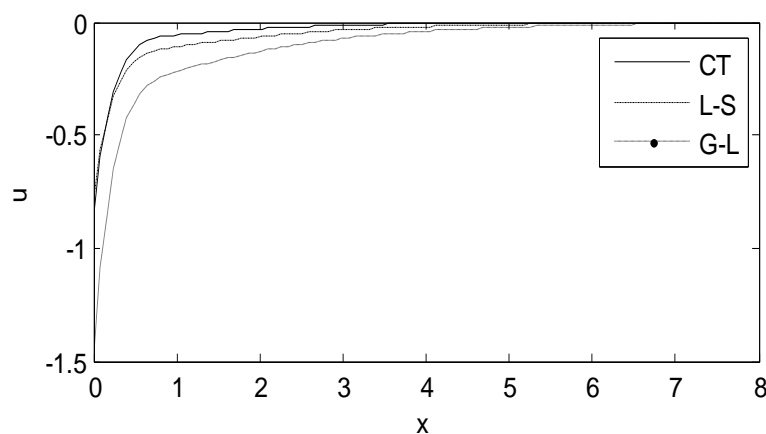


Figure 1: Displacement component u under three theories with respect to x



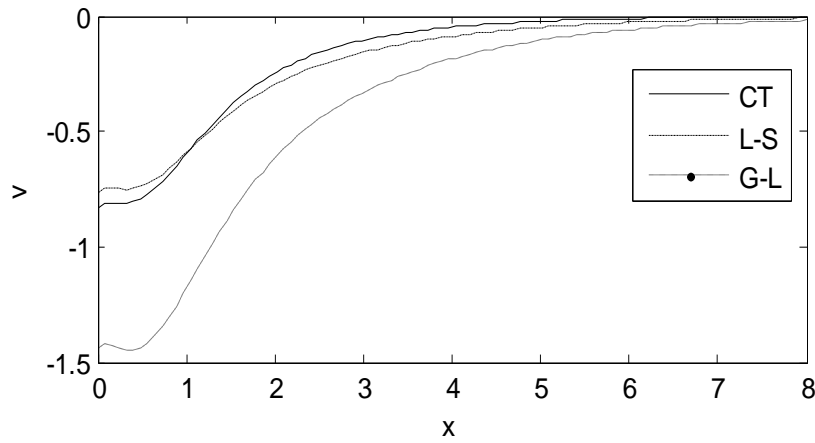


Figure 2: Displacement component v under three theories with respect to x

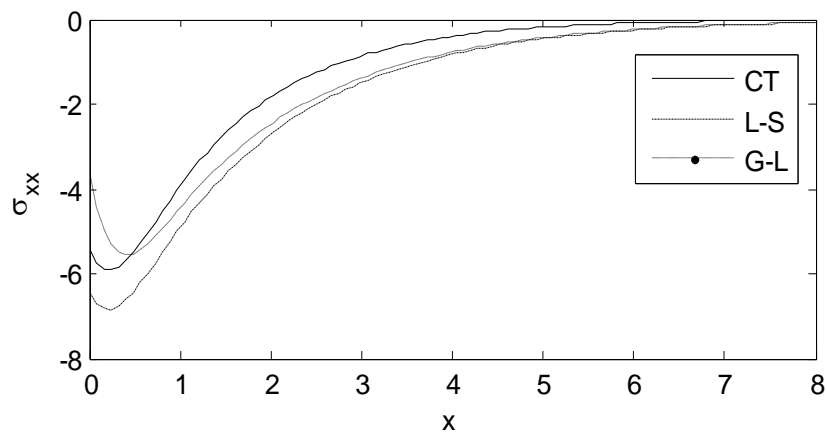


Figure 3: Distribution of σ_{xx} under three theories with respect to x

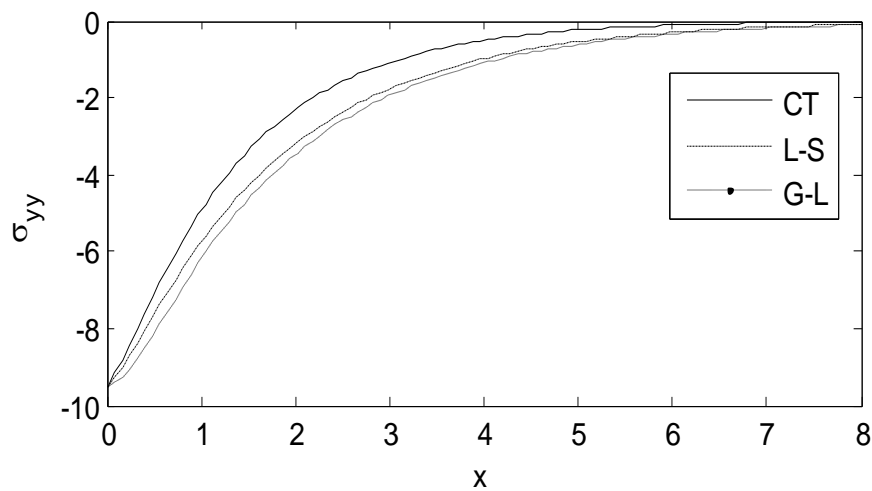


Figure 4: Distribution of σ_{yy} under three theories with respect to x

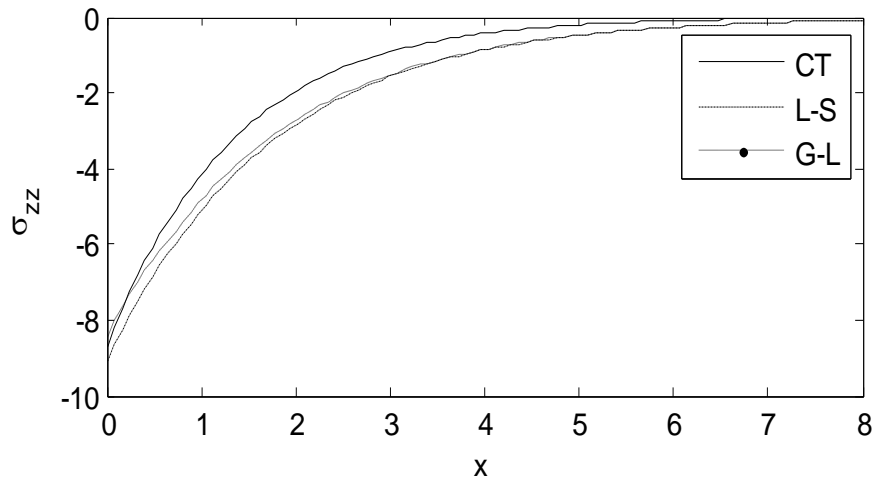


Figure 5: Distribution of σ_{zz} under three theories with respect to x

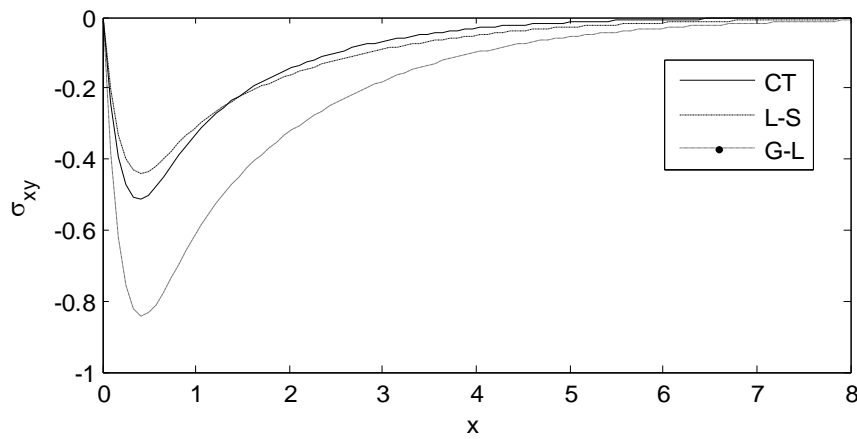


Figure 6: Distribution of the stress component σ_{xy} under three theories with respect to x

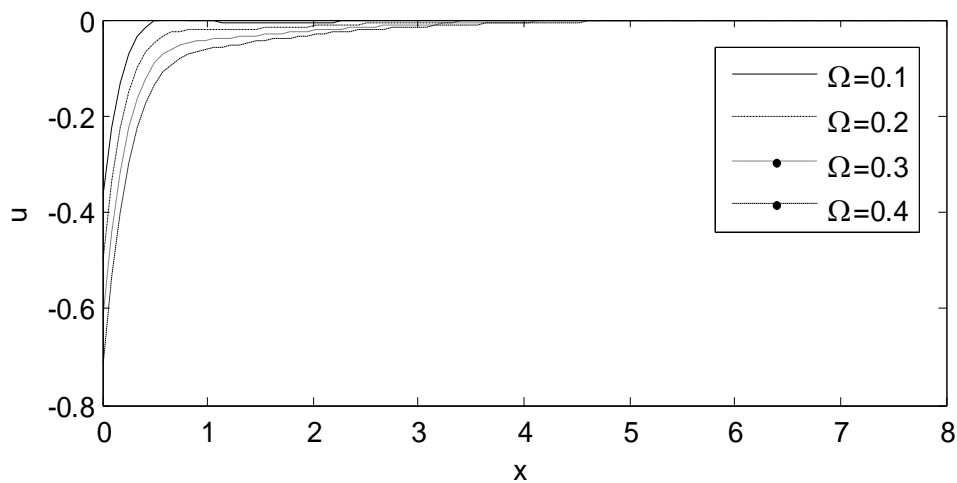


Figure 7: Displacement component u under effect of rotation with respect to x

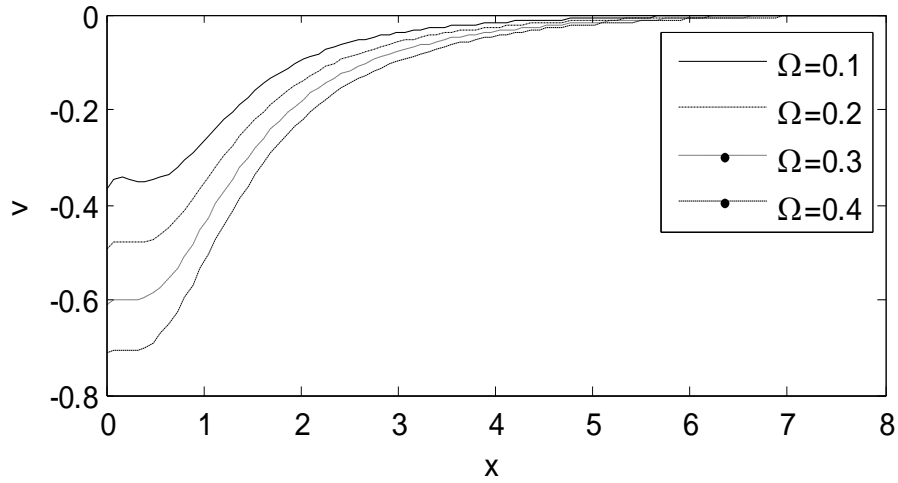


Figure 8: Displacement component u under effect of rotation with respect to x

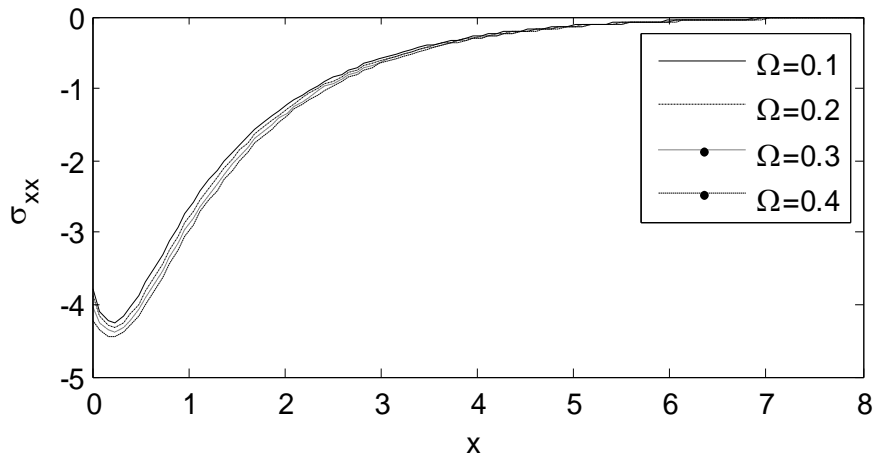


Figure 9: Distribution of σ_{xx} under effect of rotation with respect to x

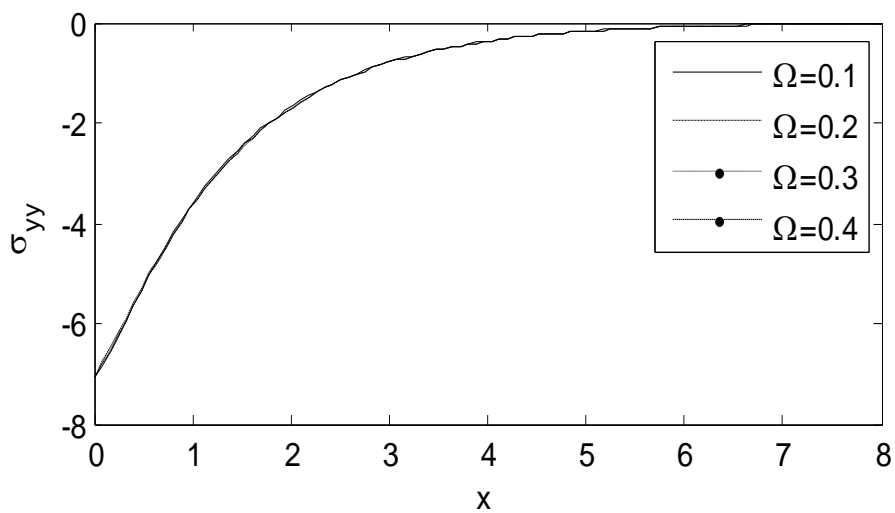


Figure 10: Distribution of σ_{yy} under effect of rotation with respect to x

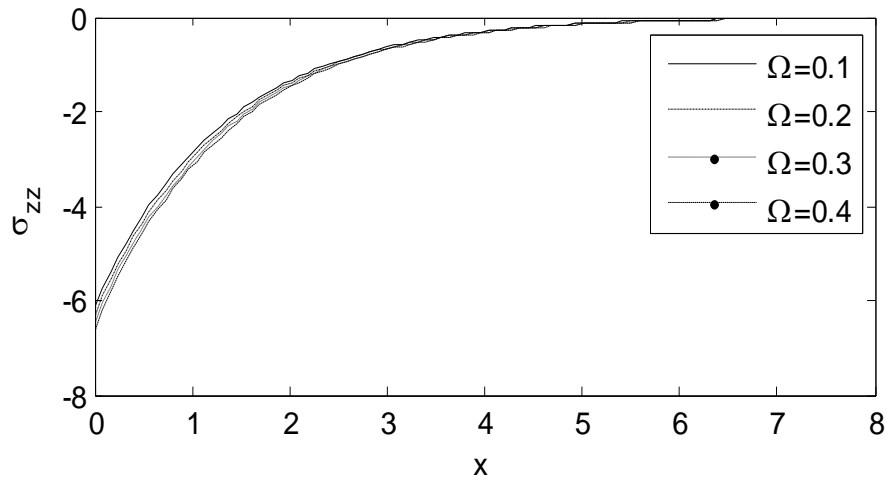


Figure 11: Distribution of σ_{zz} under effect of rotation with respect to x

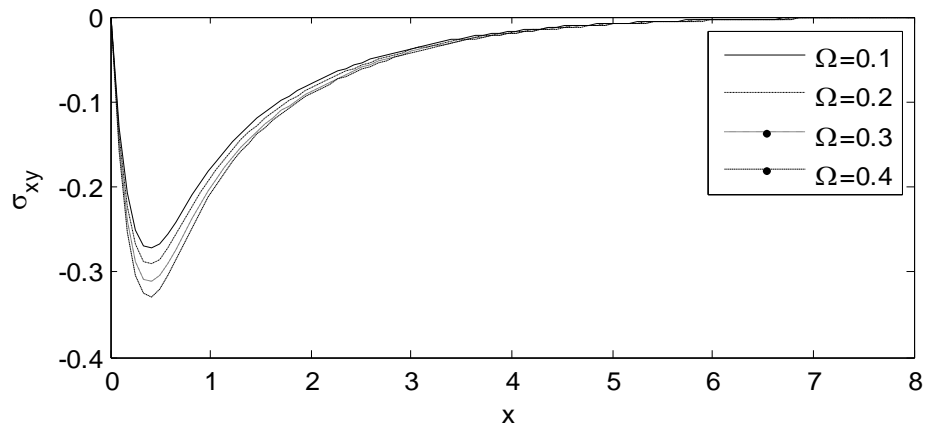


Figure 12: Distribution of σ_{xy} under effect of rotation with respect to x

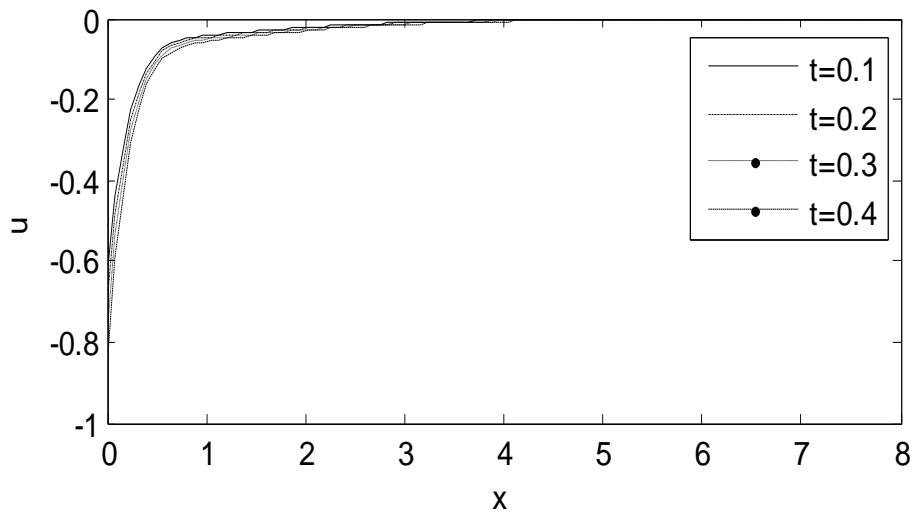


Figure 13: Displacement component u under the time with respect to x

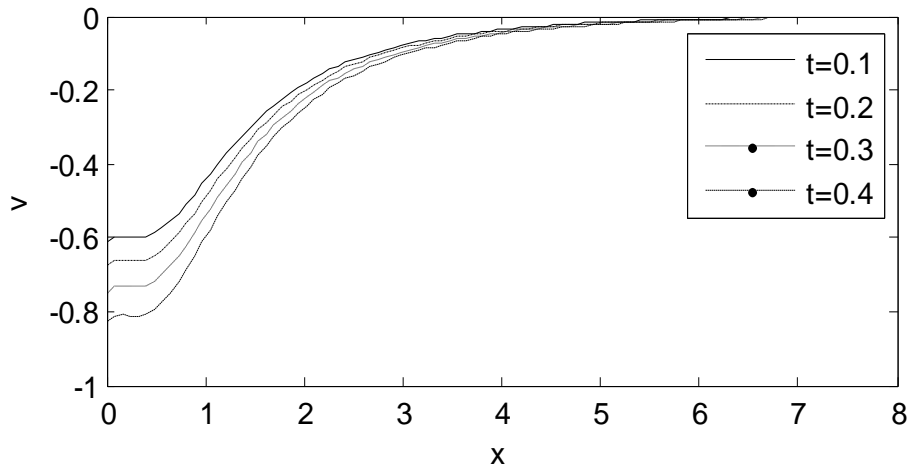


Figure 14: Displacement component under the time with respect to x

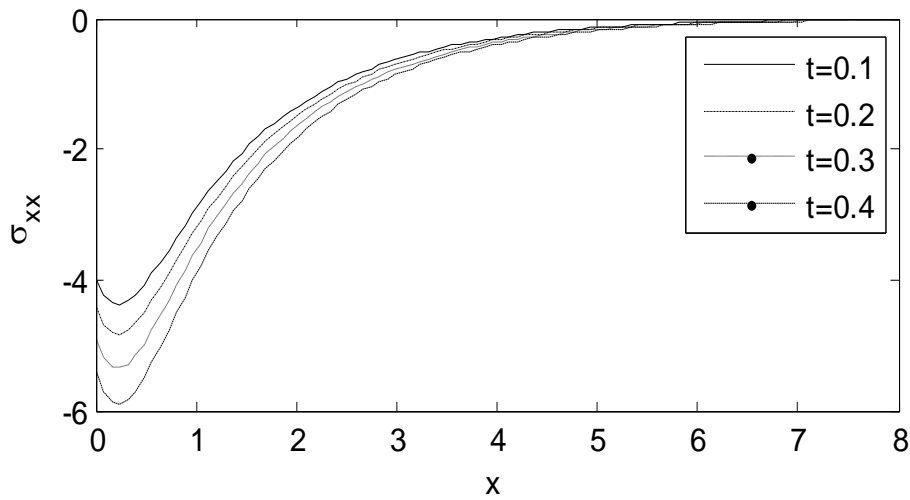


Figure 15: Distribution of σ_{xx} under the time with respect to x

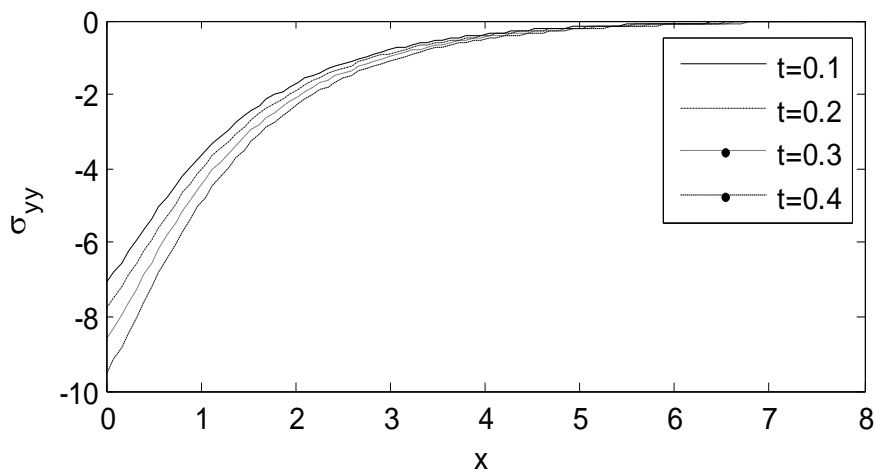


Figure 16: Distribution of σ_{yy} under the time with respect to x

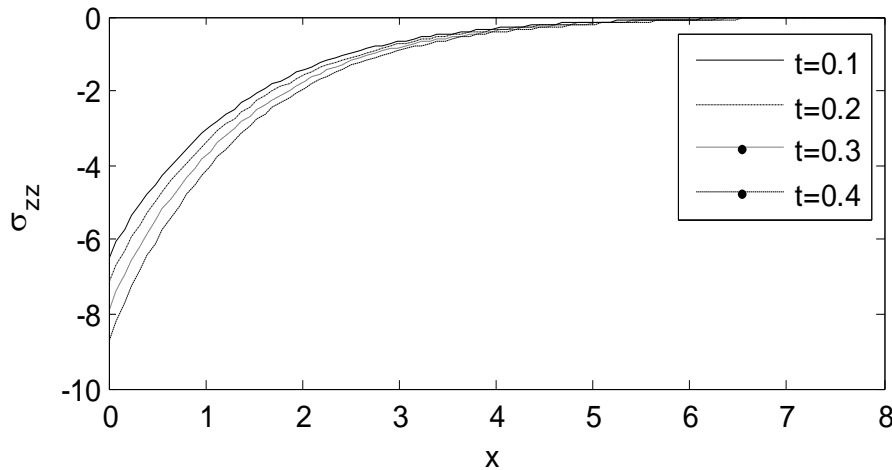


Figure 17: Distribution of σ_{zz} under the time with respect to x

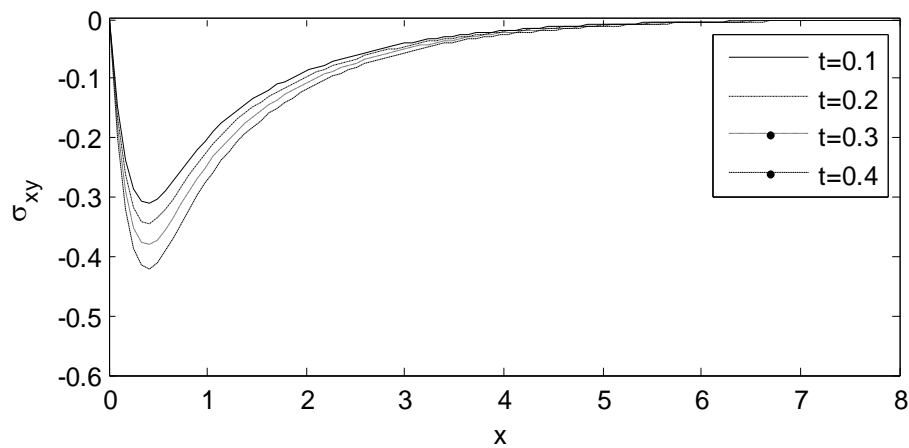


Figure 18: Distribution of the stress component σ_{xy} under the time with respect to x

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