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Properties of Finite Nuclei with Large Neutron Excess

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Abstract Designer nuclei with large neutron excess have been produced in the laboratories to understand the role played by excess neutrons in understanding the properties of large finite nuclei. Assuming the core of a nucleus is composed of neutron-proton pairs (np-pairs) and the unpaired neutrons constitute the surface region of the nucleus, we have used Bogoliubov technique to calculate the binding energy, binding fraction, specific heat, entropy, transition temperature and the nuclear radius of such nuclei.

Keywords Finite Nuclei, Large Neutron

Introduction

Superconductivity in condensed matter systems was discovered in 1911. Its microscopic explanation was given by a pairing theory [1] proposed in 1957. A series of applications [2-3] to nuclear structure were proposed in which pair-wise coupling of nucleons to a state of zero angular momentum takes place. In the theoretical and experimental studies of pairing phenomena in finite nuclei and infinitely extended nuclear systems, such as neutron star matter, the study of superfluidity and pairing has a long history [4-6]. In fact pairing lies at the heart of nuclear physics and the quantum many-body problem. Interest in nucleon-nucleon pairing has intensified in recent years due to experimental developments on two different fronts. In the field of astrophysics, a series of x-ray satellites has brought a flow of data on thermal emission from neutron stars, comprising both upper limits and actual flux measurements. The recent launching of the Chandra x-ray observatory provides further impetus to do more detailed theoretical investigations. In different laboratories, the expanding capabilities of radioactive-beam and heavy-ion facilities have aggressively led to exploration of nuclei far from stability, with special focus on neutron-rich nuclei [7-8]. Pairing plays a significant role in modeling the structure and behavior of these newly discovered nuclei.

As a rule, BCS is an approximate solution to the many-body problem, although it has been applied in nuclear structure calculations with some success [2-6]. Another method of solving the problem is to use the Hartree-Fock (HF) theory. Solutions of the HF equations describe various nuclear ground-state properties [9], but they do not include an explicit pairing interaction. A general way to include pairing interaction into the many body system will require solving the Hartree-Fock-Bogoliubov (HFB) equations [10]. There are some applications to both stable and weakly bound nuclei [11-12]. Another method of solving the problem is that instead of solving the full HFB equations; one may first calculate HF single particle wave functions and use these as a basis for solving the BCS equations [13]. For stable nuclei with large one or two-neutron separation energies, the HF+BCS approximation to HFB is valid, but the treatment is not able to adequately address weakly bound nuclei due to the development of a particle (generally neutrons) gas on or near the nuclear surface.

Now in nuclei with neuron number $N = Z$, the proton number, neutrons and protons occupy the same shell model orbitals. Consequently the large spatial overlaps between neutrons and proton single-particle wavefunctions are expected to enhance neutron-proton correlations (np-correlations) resulting in np-pairing. On the other hand most of our knowledge about nuclear pairing comes from nuclei with a sizable neutron excess (N>Z) where the isotopic spin $T=1$ neutron-proton (np) and proton-proton (pp) pairing dominates.

There is an indication that the spectra of the chain of tin isotopes point to a link between superfluidity in infinite star matter and the spectra of finite nuclei [14]. The link is provided by the $^{1}S_{O}$ partial wave of the nucleon-nucleon interaction. There could also exist proton and neutron BCS-like pairs. Such pair correlations are quite strong and reflect the well known coherence in the ground state of even-even nuclei. But the proton BCS-like pairing fields are not constant within an isotopic chain, or the proton pair matrix elements are not constant within the isotopic chain and such behavior is mainly caused by isoscalar neutron-proton pairing, showing that there are important neutron-proton correlations present in the ground state. On the other hand the proton and neutron occupation numbers show a much smoother behavior with increasing A. However, in the nuclear shell model the isotope $^{132}_{50}S_n$ could be chosen to have a closed-shell-core of $^{100}_{50}S_n$ and the neutron particles from $^{101}_{50}S_n$ to $^{132}_{50}S_n$ define model space. In this manuscript we shall use this concept to propose a nuclear model for a finite heavy nucleus with (N>Z).

Pair correlations for four nuclei were studied [15] for various isovector and isoscalar pairs. The behavior was most interesting in the J=0 proton and neutron pairs. There is a large excess of this pairing at low temperatures, indicating a ground-state coherence of even-even nuclei. Proton pair correlations are independent of the nucleus, whereas the neutron-pair correlations show a different behavior.

For some nuclei the vanishing of the $J=0$ neutron-proton correlations increase by about a factor of 3 after the $J=0$ proton and neutron pairs have vanished. As the neutron number increases, the magnitude of the correlation reaches its highest value at higher temperatures. But the isovector $J=1$ correlations have a different behavior as the temperature changes; the neutron-proton pair correlations are large at lower temperatures with increasing neutron excess and the correlations fade slowly with increasing temperature. Ultimately the correlations between the neutrons and protons are dominated by the neutron-proton (np-pairing) pairing, and much less by nn-pairing or pp-pairing.

From what has been mentioned above and the attempts, both theoretical and experimental, by many research workers, we come to the conclusion that the pairing is an essential feature of nuclear systems. We have to focus on the link between the nuclear-many-body problem and the underlying features of the nuclear force that may lead to pairing in nuclear systems. Both short-range and long-range correlations are central to the problem. However, it is the neutron-proton pairing that is most important in understanding the properties of finite nuclei.

Now a recent communication [16] contains the production of ‘designer’ atomic nuclei, which are new, rare isotopes with unusual numbers of neutrons or protons, or unusual decay modes for example super heavy isotopes of light elements, such as $^{11}_{3}Li$ have such a high ratio of neutrons to protons that the neutrons have a comparatively low binding energy. Quantum mechanically the wave function of the neutrons can extend far beyond the normal range of the nucleus. In the case of $^{11}_{3}Li$, the volume of the nucleus is roughly 10 times the volume of the normal $^{6}_{3}Li$ nucleus [16], and the length of the $^{11}_{3}Li$ nucleus is roughly 10fm $\left(10^{-15}m\right)$. A recent study of the charge radius of $^{11}_{3}Li$ provides key information for ab initio nuclear theory [18]. It is found that the size of $^{11}_{3}Li$ is that of a much heavier nucleus $^{220}_{88}Ra$ and it has a diffuse surface of neutrons.

Thus to study the properties of a large finite nucleus of mass number A with neutron excess or a light isotope with unusually large ratio of neutrons, N, to protons, Z, we have assumed that such a nucleus has a core composed of Z neutron-proton pairs, and this core is surrounded by the unpaired (N-Z) neutrons that stay in the surface region. In another article [19] on neutron-proton interactions and the new atomic masses, it is assumed that the core is not significantly altered, and δV_{PN} which is the interaction of the last proton(s) with the last neutron(s), by construction, largely cancels out the interaction of the last nucleon with the core. In our opinion, the above method of looking at the possible interaction in a nucleus is an oversimplification of the exact problem. Under no circumstances it can be assumed that the interaction between the nucleons in the surface region and or the core can be treated in isolation without disturbing the core. Thus for determining the properties of finite nuclei, especially binding energy, B, we have assumed that the core of the nucleus is composed of np-pairs such that the neutron and proton interact with each other harmonically, and the unpaired neutrons in the surface region interact with the neutron-proton pairs in the core. We have used the Bogoliubov technique to study the problem, and have calculated the binding energy B, the specific heat C, the entropy S, the temperature T and the transition temperature, T_C, of the nucleus. We have also calculated size of the nuclear radius R and the thickness of the surface region containing the excess neutrons.

2. Theoretical Derivations

We shall use Bogoliubov technique and the many-body theory to study the properties of a nucleus with large neutron excess. In this method we use a trial wave-function that exhibits the interaction of an unpaired neutron in the surface region with the np-pair in the core region of a large A nucleus. The trial wavefunction will be written as,

$$\psi = a_l^+ \left(U_k + V_k a_k^+ a_k^+ \right) |0\rangle \tag{1}$$

where $a_k^+ a_k^+$ will refer to the neutron-proton pair in the core of the nucleus, and a_l^+ will refer to the perturbing neutron that exists in the surface region of the nucleus. Thus if $H^/$is the interaction between the np-pair in the core of the nucleus and the unpaired neutron in the surface region of the nucleus, the expectation value of this interaction can be written as,

$$\langle \psi | H^/ | \psi \rangle = \langle 0 | a_l \left(U_k + V_k a_k a_k \right) H^/ \left(U_k + V_k a_k^+ a_k^+ \right) a_l^+ | 0 \rangle \tag{2}$$

where U_k and V_k are constants of the Bogoliubov transformation , and since we are dealing with fermions, we can write,

$$U_k^2 + V_k^2 = 1 \tag{3}$$

Depending upon the values of U_k and V_k, the trial wave function ψ can account for the following possibilities.

(1) If $U_k = 0$ and $V_k = 1$ then this will mean that ψ contains the term $a_l^+ a_k^+ a_k^+$, and this will mean that the np-pair and interacting neutron must exist to-gether for all the times. However this is not always true and we need not accept this possibility.

(2) If $U_k = 1$ and $V_k = 0$ then this will mean that ψ contains the term a_l^+ only, and hence it means that np-pair plays no role in ψ. This situation is not acceptable since the model is based on the existence of np-pairs in the wave function.

(3) If $U_k = \frac{1}{\sqrt{2}}$ and $V_k = \frac{1}{\sqrt{2}}$, then this will mean that the interacting neutron in the surface region exists as a separate entity since there will exist a term $\frac{1}{\sqrt{2}} a_l^+ |0\rangle$, and the np-pair also exists as a separate entity, and the existence of the term $\frac{1}{\sqrt{2}} a_l^+ a_k^+ a_k^+$ asserts that the unpaired neutron in the surface region interacts with np-pair in the core region.

Thus in our calculations we have used the values of

$$U_k = V_k = \frac{1}{\sqrt{2}} \tag{4}$$

Now the interaction term $H^/$ is written as,

$$H^/ = \beta x^3 + \gamma x^4 \tag{5}$$

where β and γ are the constants of interaction to be defined later. Since the neutron and proton in the np-pair are assumed to interact with each other harmonically, the displacement operator x is written in terms of the creation operator, a^{+}, and the annihilation operator, a, in the following form,

$$x = \frac{1}{\alpha\sqrt{2}}\left(a^{+} + a\right) \tag{6}$$

where

$$\alpha = \sqrt{\frac{\mu\omega}{\hbar}} \tag{7}$$

$\mu =$ reduced mass of the np-pair; ω refers to the natural frequency of oscillation of the oscillator which is a np-pair in our case.

Now the expectation value of $\mathrm{H}^{/}$ is written as,

$$\langle\psi|\mathrm{H}^{/}|\psi\rangle = \langle\psi|\beta x^{3}|\psi\rangle + \langle\psi|\gamma x^{4}|\psi\rangle \tag{8}$$

Substituting for x from Eq.(6) in (8) we get,

$$\begin{aligned}\langle\psi|\mathrm{H}|\psi\rangle = {} & \frac{\beta}{\alpha^{3}\sqrt{8}}\langle n,0|a_{l}\left(U_{k}+V_{k}a_{k}a_{k}\right)\left(a+a^{+}\right)^{3}\left(U_{k}+V_{k}a_{k}^{+}a_{k}^{+}\right)a_{l}^{+}|n,0\rangle + \\ & \frac{\gamma}{4\alpha^{4}}\langle n,0|a_{l}\left(U_{k}+V_{k}a_{k}a_{k}\right)\left(a+a^{+}\right)^{4}\left(U_{k}+V_{k}a_{k}^{+}a_{k}^{+}\right)a_{l}^{+}|n,0\rangle\end{aligned} \tag{9}$$

After very lengthy calculations we get

$$\langle\psi|\mathrm{H}^{/}|\psi\rangle = \frac{\gamma}{4\alpha^{4}}\left\{\begin{array}{l} U_{k}^{\ 2}\left(6n^{3}+24n^{2}+33n+15\right)+U_{k}V_{k}\left(4n^{4}+34n^{3}+104n^{2}+134n+60\right)+ \\ V_{k}U_{k}\left(4n^{4}+34n^{3}+104n^{2}+134n+60\right)+ \\ V_{k}^{\ 2}\left(6^{5}+78n^{4}+393n^{3}+948n^{2}1077n+450\right) \end{array}\right\} \tag{10}$$

$$= \mathrm{E}_{n}^{/}$$

Now the total binding energy E_{n} can be written as,

$$\mathrm{E}_{n} = \mathrm{E}^{0} + \mathrm{E}^{/} = \mathrm{Z}\left(n+\frac{1}{2}\right)\hbar\omega + \left(N-Z\right)\mathrm{E}_{n}^{/}, \quad n = 0,1,2,... \tag{11}$$

where E_{0} is the energy of the neutron-proton core (np-pairs) and $\mathrm{E}_{n}^{/}$ is the interaction energy of the (N-Z) unpaired neutrons in the surface region with the Z np-pairs in the core.

Substituting for $\mathrm{E}_{n}^{/}$ from Eq.(10) in Eq.(11) we get,

$$\mathrm{E}_{n} = \mathrm{Z}\left(n+\frac{1}{2}\right)\hbar\omega + (\mathrm{N}-\mathrm{Z})\frac{\gamma}{4\alpha^{4}}\left\{\begin{array}{l} U_{k}^{2}\left(6n^{3}+24n^{2}+33n+15\right)+ \\ 2U_{k}V_{k}\left(4n^{4}+34n^{3}+104n^{2}+134n+60\right)+ \\ V_{k}^{2}\left(6n^{5}+78n^{4}+393n^{3}+948n^{2}+1077n+450\right) \end{array}\right\} \tag{12}$$

Now since $U_{k} = V_{k} = \frac{1}{\sqrt{2}}$, Eq.(12) becomes,

$$\mathrm{E}_{n} = \mathrm{Z}\left(n+\frac{1}{2}\right)\hbar\omega + (\mathrm{N}-\mathrm{Z})\frac{\gamma}{4\alpha^{4}}\cdot\frac{1}{2}\left(6n^{5}+86n^{4}+467n^{3}+1180n^{2}+1378n+585\right) \tag{13}$$

3. Binding Fraction f

Binding fraction f is the binding energy per nucleon when the nucleus is in the ground state; i.e., it is the value of $\frac{\mathrm{E}_{n}}{\mathrm{A}}$ for $n = 0$. Using Eq.(13) we get,

$$f = \frac{E_0}{A} = \frac{Z}{A}\left(0 + \frac{1}{2}\right)\hbar\omega + \left(\frac{N-Z}{A}\right)\left(\frac{\gamma}{4\alpha^4}\right)\cdot\frac{1}{2}(585) \tag{14}$$

Here $\left(\frac{N-Z}{A}\right)$ is called the neutron excess parameter η,

$$\eta = \frac{N-Z}{A}$$

4. Specific Heat

To calculate the specific heat, C, it is necessary to include the probability amplitude Green's function. The corresponding energy activation factor is $e^{-\frac{\hbar\omega}{\kappa T}}$. Thus the total energy can be written as,

$$E_n = Z\left(n + \frac{1}{2}\right)\hbar\omega + (N-Z)\frac{\gamma}{4\alpha^4}\cdot\frac{1}{2}\times\left(6n^5 + 86n^4 + 467n^3 + 1180n^2 + 1378n + 585\right)e^{\frac{-\hbar\omega}{\kappa T}} \tag{16}$$

The specific heat C is written as,

$$C = \frac{\partial E_n}{\partial T} = (N-Z)\frac{\gamma}{8\alpha^4}\cdot\frac{\hbar\omega}{\kappa T^2}\left(6n^5 + 86n^4 + 467n^3 + 1180n^2 + 1378n + 585\right)e^{\frac{-\hbar\omega}{\kappa T}} \tag{17}$$

5. Transition Temperature T_C;

The transition temperature of a nucleus is given by,

$$\left(\frac{\partial C}{\partial T}\right)_{T=T_C} = 0 \tag{18}$$

Substituting for C from Eq.(17) in Eq.(18) gives,

$$T_C = \frac{\hbar\omega}{2\kappa} \tag{19}$$

6. Temperature of a Nucleus

The temperature T of a nucleus is given by [14],

$$T = \frac{C^{-1}}{2A}(8MeV) \tag{20}$$

where C is the specific heat and A is the mass number of the nucleus. A general value for T could be,

$$T = \frac{\lambda}{CA}(8MeV), \text{ where } \lambda \text{ is some parameter} \tag{21}$$

7. Size of the Nucleus

The size of the nucleus can be written as,

$$R = R_{np} + R_n \tag{22}$$

where R_{np} refers to the radius of the core containing np-pairs, and R_n is the width of that part of the nucleus that contains the unpaired neutrons and this region is invariably known as the surface region of the nucleus. This can be written as,

$$R_{np} = \sqrt{R_{np}^2} = \left[\langle\psi|2Zx^2|\psi\rangle\right]^{\frac{1}{2}} \text{ where } \psi \text{ is given by Eq.(1) and } x \text{ is given by Eq.(6)} \tag{23}$$

$$R_n = \sqrt{R_n^2} = \left[\langle\phi|(N-Z)y^2|\phi\rangle\right]^{\frac{1}{2}} \tag{24}$$

where $\phi = a_i^+|0\rangle$ (25)

If each neutron in the surface region is treated as an oscillator, then

$$y = \frac{1}{\alpha\sqrt{2}}\left(a + a^{+}\right) \tag{26}$$

The most important question to be answered is as to what role the excess neutrons in the surface region play in abnormally increasing the size of the nucleus. For instance [16] the size of ${}^{11}_{3}Li$ is ten times the size of the normal nucleus ${}^{6}_{3}Li$. If the neutron excess (N-Z) or the neutron excess parameter $\eta = \frac{N-Z}{A}$ can appear in the expression for R_n, we could then establish the role of neutron excess in increasing the size of the nucleus in an abnormal fashion.

Upon carrying out the substitutions in Eq.23 the expression for that part of the nucleus that contains the n-p pairs is obtained as,

$$R_{np} = \sqrt{\frac{57\hbar Z}{8\mu\omega}} \tag{27}$$

The thickness of the surface region that contains the unpaired neutrons becomes,

$$R_n = \sqrt{\frac{3\hbar(N-Z)}{8\mu\omega}} \tag{28}$$

The role of the neutron excess in determining the thickness of the surface region can now be obtained from Eq.28 .

8. Entropy S;

The expression for entropy S is,

$$dS = \frac{dQ}{T} \quad or \quad \int dS = \int \frac{dQ}{T} = \int \frac{mCdT}{T} \tag{29}$$

where m =mass of the nucleus

Carrying out the integration and substituting for C from Eq.(17) we get,

$$S = (N-Z)\frac{\gamma}{8\alpha^4}\cdot\frac{\hbar\omega}{\kappa}\left(6n^5 + 86n^4 + 467n^3 + 1076n^2 + 1378n + 585\right) \left(\frac{\kappa}{\hbar\omega T}e^{\frac{-\hbar\omega}{\kappa T}} + \frac{\kappa^2}{\hbar^2\omega^2}e^{\frac{-\hbar\omega}{\kappa T}}\right) \tag{30}$$

9. Numerical Calculations

Since γx^4 must have the dimensions of energy ML^2T^{-2}, the dimensions of γ should be $ML^{-2}T^{-2}$, since x which is the displacement operator has the dimension of length L. Therefore, a parameter a_0 which is assumed to be fundamental to the perturbation parameters γ has been introduced. This parameter a_0 is defined as the bond length between the nucleons in the nucleus. This bond length, which is also called the radius constant is taken as

$$a_0 = 1.3\times10^{-13} A^{\frac{1}{3}}\ \text{cm} \tag{31}$$

The perturbation parameter can therefore be defined as,

$$\gamma = \frac{\hbar\omega}{a_0^4} \tag{32}$$

The following values for different physical quantities have been used.

Planck's constant/ $2\pi = \hbar$ is given as 1.054×10^{-27} erg-s.

The neutron-proton reduced mass μ is given as 8.369×10^{-25} gm.

Boltzmann's constant κ is given as 1.3807×10^{-16} erg/K

The angular radian frequency $\omega = 6 \times 10^{22} S^{-1}$

10. Variation of binding fraction f with mass number A , Z, N and η

Giving different values to A, Z and N for different nuclei and using Eq.14 we obtain the values of binding fraction f presented in Table 1. In Fig. 1 the values of binding fraction are plotted against mass number A and the resulting points approximated to a linear characteristic. The graph shows that for medium heavy and heavy nuclei our values for f fit to within 0.5 MeV those obtained by the semi-empirical mass formula and the experimental data [24,26] and those obtained by us with the anharmonic perturbation method [22].

Fig.2 and 3 show the variation of the binding fraction f with the neutron number N and the proton number Z respectively.

Further we can define a neutron excess parameter η such that,

$$\eta = \frac{N - Z}{A}.$$

For different values of A, Z, and N we can get the values of the neutron excess parameter η. The values of f are given in table 1.

Table 1: Variation of the binding fraction f (MeV/nucleon) with A, Z, N and η for $n = 0$.

A	Z	N	η	f
15	7	8	0.067	17.257
39	19	20	0.026	10.493
55	25	30	0.091	10.922
56	26	30	0.071	10.663
84	36	48	0.143	10.202
101	44	57	0.129	9.830
108	47	61	0.13	9.725
137	56	81	0.182	9.231
153	63	90	0.176	9.100
161	66	95	0.18	9.019
163	66	97	0.19	8.955
169	69	100	0.183	8.945
177	72	105	0.186	8.876
207	82	125	0.208	8.586
226	88	138	0.221	8.413
235	92	143	0.217	8.405
237	93	144	0.215	8.410
239	94	145	0.213	8.415
262	105	157	0.198	8.449

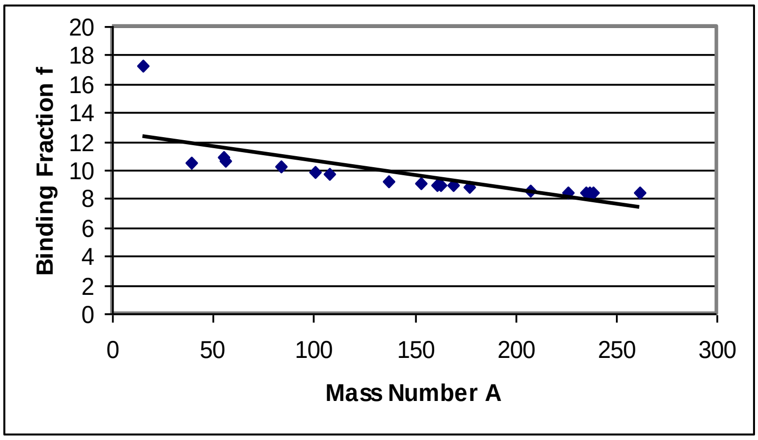

Figure 1: Binding fraction $f(MeV / Nucleon)$ against mass number A

11. Variation of C with E_n

When Eq.16 is inserted in Eq.17 we can express the specific heat C in terms of E_n and , proton number Z this relationship is,

$$C = \frac{\hbar\omega}{\kappa T^2}\left[E_n - Z\left(n + \frac{1}{2}\right)\hbar\omega\right] \tag{33}$$

This expression shows that C varies directly as the difference between E_n and the proton number energy $Z\left(n+\frac{1}{2}\right)\hbar\omega$. The variation of C with E_n is shown in Table 3 for ^{161}Dy and ^{163}Dy where it is seen that the specific heat for the heavier isotope is larger[14,22]. The variation of C against E for ^{11}Li and ^{220}Ra is shown in Fig. 3. This figure also shows that the peak specific heat for ^{11}Li is around six times larger than that for the much heavier nucleus ^{220}Ra. This is because of the much larger neutron excess parameter.

Using Eq.(17), we have calculated the variation of C with T for the three nuclei ^{11}Li, ^{163}Dy and ^{220}Ra and this is shown in Fig. 4. The value of the transition temperature $T_C\kappa = \frac{\hbar\omega}{2}$, turns out to be 19.602MeV. The specific heat curves are S-shaped and these results are similar to the those obtained earlier using other methods [14,28]. The S-shaped curve is interpreted as the liquid –gas phase transition in nuclear matter [25,30] and atomic nucleus[6,22]. The values of C are positive and this is mainly due to the Coulomb interaction [29]. The values of T_C vary [7,14,26] between 10-20MeV and[14,22] T_C=18MeV .

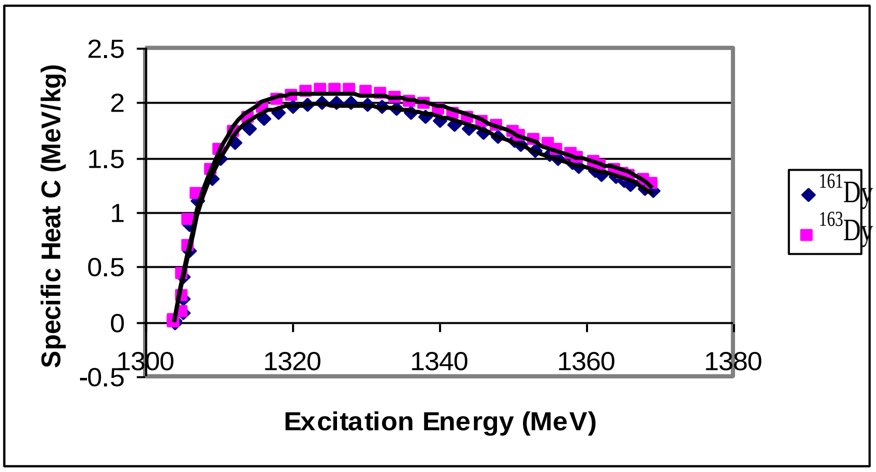

Figure 2: Variation of specific heat C against excitation energy E for $^{161}_{66}Dy$ and $^{163}_{66}Dy$ using eq.(33)

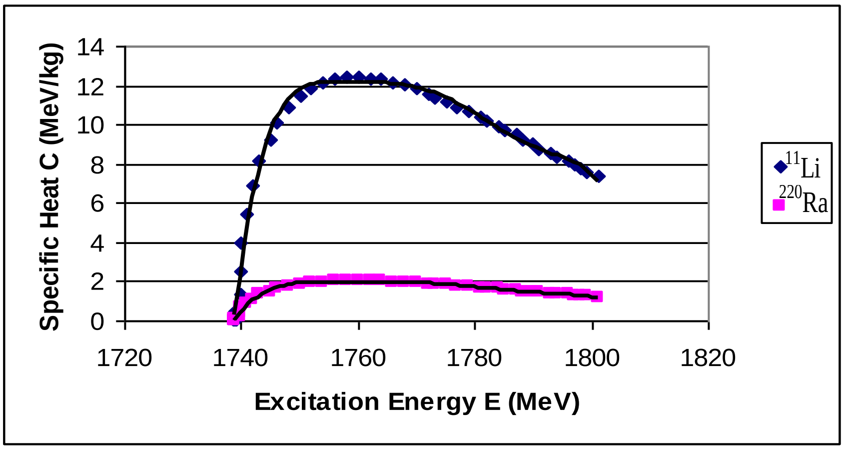

Figure 3: Variation of specific heat C against excitation energy E for $^{11}_{3}Li$ and $^{220}_{88}Ra$ using eq.(33).

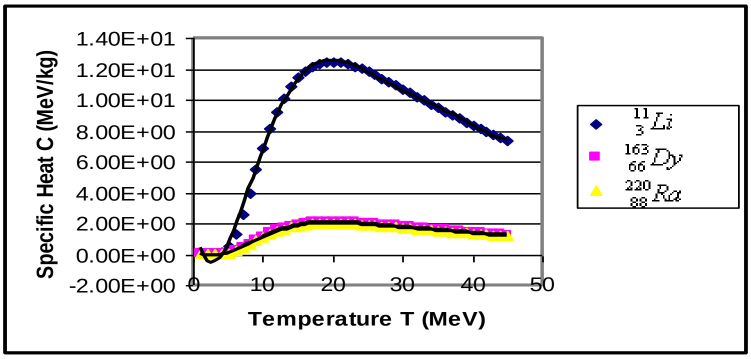

Figure 4: Variation of specific heat C against temperature T for $^{11}_{3}Li$, $^{163}_{66}Dy$ and $^{220}_{88}Ra$ using eq.(17).

12. Variation of nuclear radius R with mass number A

Using Eqs. (27) and (28) we have calculated the variation of core radius R_n and the thickness of the surface region R_{np} with mass number A. Fig. 5 show the variation of these parameters with mass number. Using our method the calculation of the size of $^{11}_{3}Li$ confirms that it is approximately the size [16] of $^{220}_{88}Ra$.

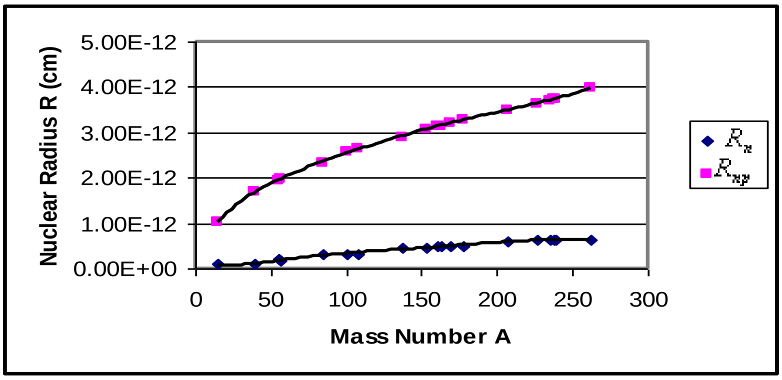

Figure 5: Variation of n-p pair core radius R_{np} (cm) and the surface thickness R_n (cm) against mass number A using eqs. (27) and (28).

13. Variation of S with T

Using Eq.(32), we have calculated the variation of entropy S with temperature T for the nuclei $^{11}_{3}Li$, $^{163}_{66}Dy$ and $^{220}_{88}Ra$. Fig. 6 shows this variation of S with T for these nuclei.

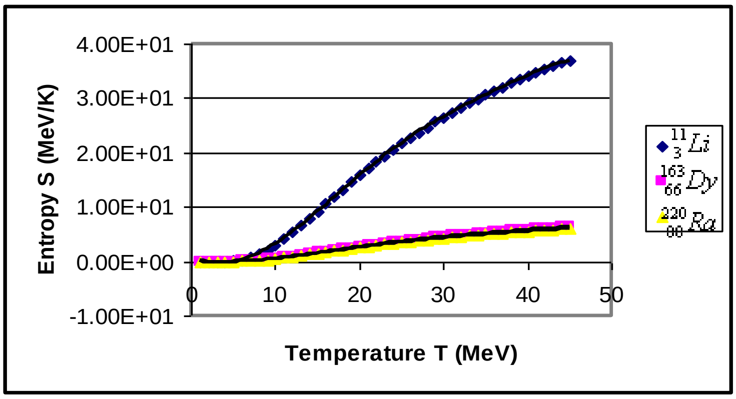

Figure 6. Variation of entropy S (MeV/T) against temperature T (MeV) for $^{11}_{3}Li$, $^{163}_{66}Dy$ and $^{220}_{88}Ra$,respectively.

14. Variation of S with E_n

Using Eq.(32) and Eq.(16), we have calculated the variation of entropy S with excitation energy E_n for the nuclei ${}^{11}_{3}Li$, ${}^{163}_{66}Dy$ and ${}^{220}_{88}Ra$. Figure 7 shows this variation. Using Eq.(16) in Eq.(30), we obtained the entropy as,

$$S = \left[E_n - Z\left(n + \frac{1}{2}\right)\hbar\omega\right]\left(\frac{1}{T} + \frac{\kappa}{\hbar\omega}\right) \qquad (34)$$

According to this equation the variation of S with E as shown in Fig.7 can be explained. This expression shows S varies directly as the total energy of the nucleus E which is the expression inside the square brackets. For large nuclei E_n is large and so is the entropy S. The repulsive energy of the protons must be subtracted from E_n to obtain the total energy of the nucleus E. The resulting variation of S is almost a linear[14,22] function of E.

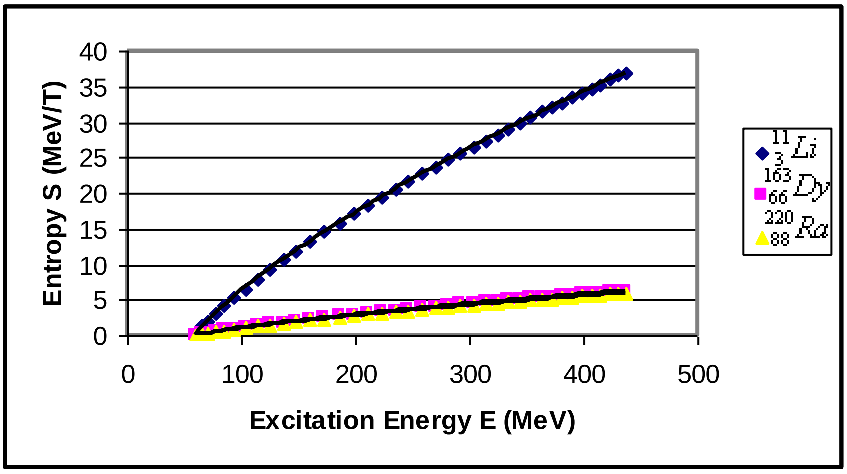

Figure 7: Variation of entropy S (MeV/T) against excitation energy E (MeV) for ${}^{11}_{3}Li$, ${}^{163}_{66}Dy$ and ${}^{220}_{88}Ra$ respectively using eq.(34).

15. Discussion and Conclusion

Assuming that the neutron-proton pairing, rather than the neutron-neutron or proton-proton pairing, is at the heart of nuclear theory; a large finite nucleus with large neutron excess is considered to be composed of a core containing neutron-proton pairs, and the unpaired excess neutrons constitute the so-called surface region of the nucleus, we surmised the trial wave function that exhibits the interaction of an unpaired neutron in the surface region with the n-p pair in the core region. The Bogoliubov technique and many body theory has been used to obtain the different properties of the nucleus.

Table1 shows that the binding fraction, f, is large compared to the values known in the literature[22,26], but the shape of the graph showing variation of B and f against A maintains the shape known in the books on nuclear theory [24]. But the large values of B and f emphatically indicate that nuclei or systems with large neutron excess could be strongly bound. Hence neutron stars as strongly bound systems could be an acceptable possibility.

The variation of specific heat C, with excitation energy, E is exhibited in Figs.2 and 3. Fig 2 shows that the variation in C has the same shape for different nuclei but the heavier isotope has slightly large C values and this agrees with the results obtained earlier [14,22].

In Fig.7, the variation of C against E is exhibited for two very different nuclei. Although ${}^{11}_{3}Li$ is a light nucleus, it has a very large neutron excess parameter, η=0.454; whereas ${}^{220}_{88}Ra$ is a large nucleus, its neutron excess parameter η=0.20. Here the variation of C with E is very large for $^{11}_{3}Li$ compared to the similar variation for $^{220}_{88}Ra$ .This shows that for nuclei with large values of η, variation of C with E will be large compared to the nuclei with small η. Similar trend is shown for the variation of C against T as shown in Fig.4. The variation of S against T, as shown in Fig. 6, and that of S against E as shown in Fig.7, also shows the same trend. It should be so since S and C are related to each other.

Fig.5 exhibits the variation of the nuclear core radius, R_n, and the variation of the surface thickness, R_{np}, with A. The graph shows that the surface thickness, R_{np}, increases faster with A, than the core radius, R_n. This points to the fact that the nuclei with large neutron excess will have a large diffused surface region [15]. Our results show that the size of $^{11}_{3}Li$ is quite large [16, 17].

Some of the calculations on specific heat C_v have been done for the nuclei that we considered, and we find that the graphs of C_v against T are more or less similar to what we have got. However, the theoretical approach we adopted is different from what has been done in these papers (30, 31, 32, 33).

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