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Two-temperature generalized thermoelastic rotating medium with voids and initial stress: comparison of different theories

Mohamed I. A. Othman, Ezaira R. M. Edeeb

Department of Mathematics, Faculty of Science, Zagazig University, P.O. Box 44519, Zagazig, Egypt

Abstract A new model of generalized rotation-thermoelastic in an isotropic elastic medium with voids and two-temperature is established. The entire elastic medium is rotated with a uniform angular velocity. The formulation is applied under three theories of generalized thermoelasticity: Lord-Schulman with one relaxation time, Green-Lindsay with two relaxation times, as well as the coupled theory. The normal mode analysis is used to obtain the exact expressions for the considered variables. Some particular cases are also discussed in the context of the problem. Numerical results for the considered variables are obtained and illustrated graphically. Comparisons are also made with the results predicted by different theories (CD), (L-S), (G-L) in the absence and presence of rotation, initial, as well as two-temperature parameters.

Keywords Rotation; Initial Stress; Two-temperature; Generalized thermoelasticity; Voids; Normal mode analysis

1. Introduction

Generalized theories of thermoelasticity have been developed to overcome the infinite propagation speed of thermal signals predicted by the classical coupled dynamical theory of thermoelasticity [1]. The subject of generalized thermoelasticity covers a wide range of extensions of the classical theory of thermoelasticity. We recall the two earliest and most well-known generalized theories proposed by Lord and Shulman [2], Green and Lindsay [3]. In the model of (L-S), Fourier's law of heat conduction is replaced by the Maxwell-Cattaneo law, which introduces one thermal relaxation time parameter in Fourier's law, whereas in the model of (G-L), two relaxation parameters are introduced in the constitutive relations for the stress tensor and the entropy. Othman [4] studied the (L-S) theory under the dependence of the modulus of elasticity on the reference temperature in two-dimensional generalized thermoelasticity. Othman [5] investigated the effect of rotation on plane waves in generalized thermoelastic medium with two relaxation times.

Theory of linear elastic materials with voids is an important generalization of the classical theory of elasticity. The theory is used for investigating various types of geological and biological materials for which classical theory of elasticity is not adequate. Othman and Atwa [6] developed the response of micropolar thermoelastic medium with voids due to various sources under (G-N) theory, Othman et al. [7] studied the effect of the gravitational field and temperature dependent properties on a two-temperature thermoelastic medium with voids under (G-N) theory, Cowin and Nunziato [8] developed a theory of linear elastic materials with voids. Puri and Cowin [9] studied the behavior of the plane waves in a linear elastic material with voids. The domain of influence theorem in the linear theory of elastic materials with voids was discussed by Dhaliwal and Wang [10]. Dhaliwal and Wang [11] developed a heat flux dependent theory of thermoelasticity with voids. Ciarletta and Scarpetta [12] discussed some results on thermoelasticity for dielectric materials with voids.

The initial stresses develop in the medium due to various reasons, and it is of paramount interest to study the effect of these stresses on the propagation of elastic waves. A lot of systematic studies have been made on the propagation of elastic waves. Abd-Alla and Alsheikh [13] showed the effect of the initial stresses on the reflection and transmission of plane quasi-vertical transverse waves in piezoelectric materials. Recently, Abbas and Kumar [14] studied the response of the initially stressed generalized thermoelastic solid with voids to thermal source.

The thermoelastic plane waves without energy dissipation in a rotating body have studied by Chanderashekhariah and Srinath [15]. Othman [16, 17] used the normal mode analysis to study the effect of rotation on plane waves in generalized thermo-elasticity with one and two relaxation times. Schoenberg and Censor [18] studied the effect of rotation on elastic waves.

The two temperatures theory of thermoelasticity was introduced by Chen and Gurtin [19], Abbas and Zenkour [20] have studied the two-temperature generalized thermoelastic interaction in an infinite fiber-reinforced anisotropic plate containing a circular cavity with two relaxation times. Othman et al. [21] studied the effect of rotation on micropolar generalized thermoelasticity with two-temperature using a dual-phase-lag model. Youssef [22] has developed the theory of two-temperature generalized thermoelasticity based on the (L-S) model. Youssef and El-Bary [23] solved a two-temperature generalized thermoelasticity problem with the variable thermal conductivity.

In the present paper, we will study two-temperature generalized rotation-thermoelastic medium with void and initial stress comparison of different theories. The normal mode method is used to obtain the exact expression for the considered variables. A comparison is carried out between the considered variables as calculated from the generalized thermoelastic pours medium based on (L-S), (G-L), and coupled theories in the absence and presence of rotation. A comparison is also made between the three theories with and without initial stress and two-temperature.

2. Formulation of the Problem

We consider a homogeneous thermoelastic half-space under hydrostatic initial stress and two-temperature rotating uniformly with angular velocity $\boldsymbol{\Omega} = \Omega \boldsymbol{n}$, where $\boldsymbol{n}$ is a unit vector representing the direction of the axis of rotation. All quantities considered are functions of the time t and the coordinates x and y. The displacement equation of motion in the rotating frame has two additional terms [23]: the centripetal acceleration $\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{u})$ due to the time-varying motion only and the Coriolis acceleration $2\boldsymbol{\Omega} \times \boldsymbol{u}_{,t}$ where $\boldsymbol{u} = (u, v, 0)$ is the dynamic displacement vector and $\boldsymbol{\Omega} = (0, 0, \Omega)$ is the angular velocity. These terms, do not appear in non-rotating media. The rectangular coordinate system (x, y, z) has originated on the surface $y = 0$, and y-axis pointing vertically into the considered medium.

3. Basic Equations

The basic governing equations of a linear thermoelastic rotation medium and initial stress with voids two-temperature under three theories are:

$$\sigma_{ji,j} = \rho[u_{i,tt} + \{\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{u})\}_i + (2\boldsymbol{\Omega} \times \boldsymbol{u}_{,t})_i], \tag{1}$$

$$\rho[u_{i,tt} + \{\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{u})\}_i + 2(\boldsymbol{\Omega} \times \boldsymbol{u}_{,t})_i] \tag{2}$$

$$\alpha \nabla^2 \phi - b\, e - \xi \phi - \omega_0 b\, \dot{\phi} + m(1 + \upsilon_0 \frac{\partial}{\partial t})T = \rho \chi \ddot{\phi}, \tag{3}$$

$$K \nabla^2 T = \rho C_E (1 + \tau_0 \frac{\partial}{\partial t})\dot{T} + \beta T_0 (1 + n_0 \tau_0 \frac{\partial}{\partial t})\dot{e} + m T_0 (1 + n_0 \tau_0 \frac{\partial}{\partial t})\phi. \tag{4}$$

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda e - \beta(1 + \upsilon_0 \frac{\partial}{\partial t})T]\delta_{ij} - p(\omega_{ij} + \delta ij), \tag{5}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{6}$$

$$\omega_{ij} = \tfrac{1}{2}(u_{j,i} - u_{i,j}), \quad \text{i, j=1,2,3.} \tag{7}$$

The thermodynamic temperature, T is related to the conductive temperature, θ as

$$T = \theta - a\theta_{,ii} \tag{8}$$

$\lambda,\ \mu$ are the Lame' constants, $\alpha, b, \xi, \omega_0, m, \chi$ are the material constants due to the presence of voids, $\beta = (3\lambda + 2\mu)\alpha_t$ such that α_t is the coefficient of thermal expansion, e is the dilation, e_{ij} components of strain tensor, δ_{ij} is the Kronecker delta, $i, j = x, y$, ρ is the density, C_E is the specific heat at constant strain, n_0 is a parameter, $\tau_0,\ \upsilon_0$ are the thermal relaxation times, K is the thermal conductivity, T_0 is the reference temperature is chosen so that $\left|(T - T_0)/T_0\right| \ll 1$, ϕ is the change in the volume fraction field, σ_{ij} are the components of stress tensor, p is the initial stress, ω_{ij} is the rotation tensor, a is the two temperature parameter.

$$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j, \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

The components of stress tensor are

$$\sigma_{xx} = \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + 2\mu\frac{\partial u}{\partial x} + b\phi - \beta\left(1 + \upsilon\frac{\partial}{\partial t}\right)T - p \tag{9}$$

$$\sigma_{yy} = \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + 2\mu\frac{\partial v}{\partial y} + b\phi - \beta\left(1 + \upsilon\frac{\partial}{\partial t}\right)T - p, \tag{10}$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - \frac{p}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial v}{\partial y}\right). \tag{11}$$

The basic governing equations of a rotating initially stressed linear thermoelastic material with voids under the influence of two-temperature will be

$$\left(\mu - \frac{p}{2}\right)\nabla^2 u + \left(\lambda + \mu + \frac{p}{2}\right)\frac{\partial e}{\partial x} + b\frac{\partial \phi}{\partial x} - \beta\left(1 + \upsilon_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} = \rho\left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega\frac{\partial v}{\partial t}\right], \tag{12}$$

$$\left(\mu - \frac{p}{2}\right)\nabla^2 v + \left(\lambda + \mu + \frac{p}{2}\right)\frac{\partial e}{\partial y} + b\frac{\partial \phi}{\partial y} - \beta\left(1 + \upsilon_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y} = \rho\left[\frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega\frac{\partial u}{\partial t}\right], \tag{13}$$

$$\alpha\nabla^2\phi - be - \xi\phi - \omega_0 b\dot{\phi} + m\left(1 + \upsilon_0\frac{\partial}{\partial t}\right)T = \rho\chi\ddot{\phi}, \tag{14}$$

$$K\nabla^2 T = \rho C_E\left(1 + \tau_0\frac{\partial}{\partial t}\right)\dot{T} + \beta T_0\left(1 + n_0\tau_0\frac{\partial}{\partial t}\right)\dot{e} + mT_0\left(1 + n_0\tau_0\frac{\partial}{\partial t}\right)\phi. \tag{15}$$

To facilitate the solution of the problem, introduce the following dimensionless variables

$$(x', y') = \frac{\omega_1^*}{c_0}(x, y), \quad (u', v') = \frac{\omega_1^*}{c_1}(u, v), \quad \sigma'_{ij} = \frac{\sigma_{ij}}{\mu_0}\sigma_{ij}, \quad \phi' = \frac{\omega_1^*\chi_0}{c_1^2}\phi, \quad T' = \frac{T}{T_0}, \ t' = \omega_1^* t,$$

$$\Omega' = \frac{\Omega}{\omega_1^*}, \ c_1^2 = \frac{\lambda_0 + 2\mu_0}{\rho}, \ \omega_1^* = \frac{\rho C_E c_1^2}{k}, \ \upsilon_0 = \omega_1^*\upsilon_0, \ \tau_0' = \omega_1^*\tau_0, \ \theta' = \frac{\theta}{T_0}, \ p' = \frac{p}{\mu} \tag{16}$$

In terms of non-dimensional quantities defined in Eq. (16) the governing Eqs. (12)-(15) reduce to (dropping the prime for convenience)

$$\nabla^2 u + A_1\frac{\partial e}{\partial x} + A_2\frac{\partial \phi}{\partial x} - A_3\left(1 + \upsilon_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} = A_4\frac{\partial^2 u}{\partial t^2}, \tag{17}$$

$$\nabla^2 v + A_1 \frac{\partial e}{\partial y} + A_2 \frac{\partial \phi}{\partial y} - A_3 (1 + \upsilon_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial y} = A_4 \frac{\partial^2 v}{\partial t^2}, \tag{18}$$

$$\nabla^2 \phi - A_5 \mathrm{e} - A_6 \phi - A_7 \frac{\partial \phi}{\partial t} + A_8 (1 + \upsilon_0 \frac{\partial}{\partial t}) T = A_9 \frac{\partial^2 \phi}{\partial t^2}, \tag{19}$$

$$\nabla^2 \theta - A_{10} (\frac{\partial}{\partial t} + \mathrm{n}_0 \tau_0 \frac{\partial^2}{\partial t^2}) \phi - \varepsilon (\frac{\partial}{\partial t} + \mathrm{n}_0 \tau_0 \frac{\partial^2}{\partial t^2}) e = (\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}) T. \tag{20}$$

Also, the constitutive Eqs. (9)-(11) reduces to

$$\sigma_{xx} = A_{11} [\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}] + 2 \frac{\partial u}{\partial x} + A_{12} \phi - A_{13} (1 + \upsilon_0 \frac{\partial}{\partial t}) T - p, \tag{21}$$

$$\sigma_{yy} = A_{11} [\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}] + 2 \frac{\partial v}{\partial y} + A_{12} \phi - A_{13} (1 + \upsilon_0 \frac{\partial}{\partial t}) T - p, \tag{22}$$

$$\sigma_{xy} = (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) - \frac{p}{2} (\frac{\partial v}{\partial x} - \frac{\partial v}{\partial y}). \tag{23}$$

Where $A_1 = \dfrac{2\lambda + \mu(2+p)}{\mu(2-p)}$, $A_2 = \dfrac{2bc_1^2}{\omega_1^{*2} \chi \mu (2-p)}$, $A_3 = \dfrac{2\beta T_0}{\mu(2-p)}$, $A_4 = \dfrac{2\rho c_1^2}{\mu(2-p)}$, $A_5 = \dfrac{b\chi}{\alpha}$,

$A_6 = \dfrac{\xi c_1^2}{\alpha \omega_1^{*2}}$, $A_7 = \dfrac{\omega_0 c_1^2}{\alpha \omega_1^{*}}$, $A_8 = \dfrac{m T_0 \chi}{\alpha}$, $A_9 = \dfrac{\rho c_1^2 \chi}{\alpha}$, $A_{10} = \dfrac{m c_1^2}{\rho C_E \omega_1^{*2} \chi}$, $A_{11} = \dfrac{\lambda}{\mu}$,

$A_{12} = \dfrac{b c_1^2}{\mu \omega_1^{*2} \chi}$, $A_{13} = \dfrac{2\beta T_0}{\mu(2-p)}$, $\varepsilon = \dfrac{\beta}{\rho C_E}$.

We define displacement potentials ψ_1 and the vector potential ψ_2 which related to displacement components u and v as,

$$u = \frac{\partial \psi_1}{\partial x} + \frac{\partial \psi_2}{\partial y}, \qquad v = \frac{\partial \psi_1}{\partial y} - \frac{\partial \psi_2}{\partial x}, \tag{24}$$

$$e = \nabla^2 \psi_1, \qquad (\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}) = \nabla^2 \psi_2. \tag{25}$$

By substituting from Eq. (25) in Eqs. (17)-(20), this yields

$$[(1 + A_1)\nabla^2 - A_4 \frac{\partial^2}{\partial t^2} + A_4 \Omega^2] \psi_1 - 2A_4 \Omega \frac{\partial}{\partial t} \psi_2 + A_2 \phi - A_3 (1 + \upsilon_0 \frac{\partial}{\partial t})(1 - a^* \nabla^2) \theta = 0, \tag{26}$$

$$2A_4 \Omega \frac{\partial}{\partial t} \psi_1 + [\nabla^2 - A_4 \frac{\partial^2}{\partial t^2} + A_4 \Omega^2] \psi_2 = 0, \tag{27}$$

$$-A_5 \nabla^2 \psi_1 + (\nabla^2 - A_6 - A_7 \frac{\partial}{\partial t} - A_9 \frac{\partial^2}{\partial t^2}) \phi + A_8 (1 + \upsilon_0 \frac{\partial}{\partial t})(1 - a^* \nabla^2) \theta = 0, \tag{28}$$

$$-\varepsilon (\frac{\partial}{\partial t} + \mathrm{n}_0 \tau_0 \frac{\partial^2}{\partial t^2}) \nabla^2 \psi_1 - A_{10} (\frac{\partial}{\partial t} + \mathrm{n}_0 \tau_0 \frac{\partial^2}{\partial t^2}) \phi + \nabla^2 \theta - (\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2})(1 - \mathrm{a}^* \nabla^2) \theta = 0. \tag{29}$$

$$T = (1 - a^* \nabla^2) \theta, \qquad a = \frac{a^* \omega_1^{*2}}{c_1^2}. \tag{30}$$

3. NORMAL MODE ANALYSIS

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form

$$[u, v, T, \theta, \psi_1, \psi_2, \phi, \sigma_{ij}](x, y, t) = [u^*, v^*, T^*, \theta^*, \psi_1^*, \psi_2^*, \phi^*, \sigma_{ij}^*](y)\exp[i(\omega t + cx)], \quad (31)$$

Where $[u^*, v^*, T^*, \theta^*, \psi_1^*, \psi_2^*, \phi^*, \sigma_{ij}^*]$ are the amplitudes of the function $[u, v, T, \theta, \psi_1, \psi_2, \phi, \sigma_{ij}]$, ω is the complex time constant, $\mathrm{i} = \sqrt{-1}$ and c is the wave number in *x*-direction.

Using Eq. (31) into Eqs. (26)-(29), then we have,

$$(\mathrm{D}^2 - S_2)\psi_1^* - S_3\psi_2^* + S_4\phi^* + [S_5\mathrm{D}^2 - S_6]\theta^* = 0, \quad (32)$$

$$S_7\psi_1^* + (\mathrm{D}^2 - S_8)\,\psi_2^* = 0, \quad (33)$$

$$-A_5(\mathrm{D}^2 - c^2)\psi_1^* + (\mathrm{D}^2 - S_9)\phi^* - (S_{10}\mathrm{D}^2 - S_{11})\theta^* = 0, \quad (34)$$

$$-S_{12}(\mathrm{D}^2 - c^2)\psi_1^* - S_{13}\phi^* + (S_{14}\mathrm{D}^2 - S_{15})\theta^* = 0. \quad (35)$$

Where, $S_1 = 1 + A_1,\ \ S_2 = c^2 - [\dfrac{A_4\omega^2 + A_4\Omega^2}{S_1}],\ \ S_3 = \dfrac{2\mathrm{i}A_4\omega\Omega}{S_1},\ \ S_4 = \dfrac{A_2}{S_1},\ \ S_5 = \dfrac{A_3a^*(1+\mathrm{i}\upsilon_0\omega)}{S_1},$

$$S_6 = \frac{a^*A_3(1+\mathrm{i}\upsilon_0\omega)(1+a^*c^2)}{S_1},$$

$$S_7 = 2\mathrm{i}A_4\Omega\omega,\ \ S_8 = c^2 - A_4\omega^2 - A_4\Omega^2,\ \ S_9 = c^2 + A_6 + \mathrm{i}\omega A_7 - A_9\omega^2,$$

$$S_{10} = A_8a^*(1+\mathrm{i}\upsilon_0\omega),\ S_{11} = A_8(1+\mathrm{i}\upsilon_0\omega)(1+a^*c^2),\ S_{12} = -\mathrm{i}\varepsilon\omega(1+\mathrm{i}n_0\tau_0\omega)\ ,$$

$$S_{13} = \mathrm{i}A_{11}\omega(1+i\,\upsilon_0\omega),\quad S_{14} = 1 + \mathrm{i}\omega a^*(1+\mathrm{i}\,\tau_0\omega),\quad S_{15} = c^2 - \mathrm{i}\omega(1+\tau_0\mathrm{i}\omega)a^*c^2,$$

$$\mathrm{D} = \frac{\mathrm{d}}{\mathrm{dy}}.$$

Eliminating ψ_2^*, ϕ^* and T^* between Eqs. (32)-(35), we get the following eighth ordinary differential equation satisfied with ψ_1^*:

$$[\,\mathrm{D}^8 - A\,\mathrm{D}^6 + B\,\mathrm{D}^4 - E\,\mathrm{D}^2 + F\,]\psi_1^*(\mathrm{y}) = 0. \quad (36)$$

In a similar manner we arrive at

$$[\,\mathrm{D}^8 - A\,\mathrm{D}^6 + B\,\mathrm{D}^4 - E\,\mathrm{D}^2 + F\,](\psi_1^*, \psi_2^*, \phi^*, \theta^*)(\mathrm{y}) = 0. \quad (37)$$

Where
$$A = \frac{1}{(S_{14} + S_5S_{12})}[S_{15} + S_{14}S_9 - S_{10}S_{13} + S_8S_{14} + S_2S_{14} - A_5S_4S_{14} - S_4S_{10}S_{12} + S_5S_9S_{12} + S_5S_{12}c^2$$
$$- A_5S_5S_{13} + S_5S_8S_{12} + S_6S_{12}]$$

$$B = \frac{1}{(S_{14} + S_5S_{12})}[S_9S_{15} - S_{11}S_{13} + S_8S_{15} + S_8S_9S_{14} - S_8S_{10}S_{13} + S_2S_{15} + S_2S_9S_{14} - S_2S_{10}S_{13} + S_2S_8S_{14}$$

$$+ S_3S_7S_{14} - A_5S_4S_{15} - A_5S_4S_{14}c^2 - S_4S_{11}S_{12} - S_4S_{10}S_{12}c^2 - A_5S_4S_8S_{14} - S_4S_8S_{10}S_{12} + S_5S_9S_{12}c^2$$

$$- S_6S_9S_{12}c^2 + S_5S_8S_9S_{12} + S_5S_8S_{12}c^2 - A_5S_5S_8S_{13} + S_6S_9S_{12} + S_6S_{12}c^2 - A_5S_6S_{13} + S_6S_8S_{12}]$$

$$E=\frac{1}{(S_{14}+S_5S_{12})}[S_8S_9S_{15}-S_8S_{11}S_{13}+S_2S_9S_{15}-S_2S_{11}S_{13}+S_2S_8S_{15}+S_2S_8S_9S_{14}+S_2S_8S_{10}S_{13}$$

$$+S_3S_7S_{15}+S_3S_7S_9S_{14}-S_3S_7S_{10}S_{13}-A_5S_4S_{15}c^2+S_4S_{11}S_{12}c^2-A_5S_4S_8S_{15}-A_5S_4S_8S_{14}c^2-S_4S_8S_{11}S_{12}$$
$$-S_4S_8S_{10}S_{12}c^2+S_5S_8S_9S_{12}c^2-A_5S_5S_8S_{13}c^2+S_6S_9S_{12}c^2-A_5S_6S_{13}c^2+S_6S_8S_9S_{12}+S_6S_8S_{12}c^2$$
$$-A_5S_6S_8S_{13}]$$
$$F=\frac{1}{(S_{14}+S_5S_{12})}[S_2S_8S_9S_{15}-S_2S_8S_{11}S_{13}+S_3S_7S_9S_{15}-S_3S_7S_{11}S_{13}-A_5S_4S_8S_{15}c^2-S_4S_8S_{11}S_{12}c^2$$

$$+S_6S_8S_9S_{12}c^2-A_5S_6S_8S_{13}c^2].$$

Equation (36) can be factored as

$$[(\mathrm{D}^2-k_1^2)(\mathrm{D}^2-k_2^2)(\mathrm{D}^2-k_3^2)(\mathrm{D}^2-k_4^2)]\psi_1^*(\mathrm{y})=0. \tag{38}$$

Where k_n^2 $(\mathrm{n}=1,2,3,4)$ are the roots of the characteristic equation of Eq. (36).

The solution of Eq. (36) bound as $y\to\infty$, is given by:

$$\psi_1^*=\sum_{n=1}^{4}R_n e^{-k_n y}, \tag{39}$$

$$\psi_2^*=\sum_{n=1}^{4}H_{1n}R_n e^{-k_n y}, \tag{40}$$

$$\phi^*=\sum_{n=1}^{4}H_{2n}R_n e^{-k_n y}, \tag{41}$$

$$\theta^*=\sum_{n=1}^{4}H_{3n}R_n e^{-k_n y}. \tag{42}$$

$$T^*=\sum_{n=1}^{4}H_{4n}R_n e^{-k_n y}. \tag{43}$$

Where, R_n $(\mathrm{n}=1,2,3,4)$ are some constants.

To obtain the components of the displacement vector, from (39) and (40) in (24)

$$u^*=\sum_{n=1}^{4}G_{1n}R_n e^{-k_n y}, \tag{44}$$

$$v^*=\sum_{n=1}^{4}G_{2n}R_n e^{-k_n y}, \tag{45}$$

From Eqs. (39)-(45) in (21)-(23) to obtain the components of the stresses

$$\sigma_{xx}^*=\sum_{n=1}^{4}H_{5n}R_n e^{-k_n y}, \tag{46}$$

$$\sigma_{yy}^*=\sum_{n=1}^{4}H_{6n}R_n e^{-k_n y}, \tag{47}$$

$$\sigma_{xy}^{*} = \sum_{n=1}^{4} H_{7n} R_n e^{-k_n y}. \tag{48}$$

Where

$$H_{1n} = \frac{-S_7}{(k_n^2 - S_8)}, \qquad H_{2n} = \frac{A_5(k_n^2 + c^2) + (S_{10}k_n^2 - S_{11})[S_3 H_{1n} - (k_n^2 - S_2)]}{(k_n^2 - S_9) + S_4(S_{10}k_n^2 - S_{11})},$$

$$H_{3n} = \frac{S_3 H_{1n} - (k_n^2 - S_2) - S_4 H_{2n}}{S_5 k_n^2 - S_6}, \qquad H_{4n} = [1 - a(k_n^2 - c^2)]H_{1n,}, \qquad G_{1n} = ic - k_n H_{1n},$$

$$G_{2n} = k_n + icH_{2n}, \; H_{5n} = A_{11}(icG_{1n} + k_n G_{2n}) + 2icG_{1n} + A_{12}H_{2n} - A_{13}H_{4n},$$

$$H_{6n} = A_{11}(icG_{1n} + k_n G_{2n}) - 2k_n G_{2n} + A_{12}H_{2n} - A_{13}H_{4n},$$

$$H_{7n} = (k_n G_{1n} + icG_{2n}) + \frac{p}{2}(k_n G_{1n} - icG_{2n}).$$

4. Boundary conditions

In this section, we need to consider the boundary conditions at $y = 0$, in order to determine the parameter R_n $(n = 1, 2, 3, 4)$.

(1) The thermal boundary condition that the surface of the half-space is subjected to

$$T = P_1 e^{i(\omega t + cx)}. \tag{49}$$

(2) The mechanical boundary condition

$$\sigma_{yy} = -P_2 e^{i(\omega t + cx)}, \qquad \sigma_{xy} = 0, \qquad \frac{\partial \phi}{\partial y} = 0. \tag{50}$$

Where, P_1 is the applied constant temperature to the boundary P_2 and is the magnitude of the applied force in of the half-space.

Using the expressions of the variables into the above boundary conditions (49), (50), we obtain

$$\sum_{n=1}^{4} H_{4n} R_n = -P_1, \tag{51}$$

$$\sum_{n=1}^{4} H_{6n} R_n = 0, \tag{52}$$

$$\sum_{n=1}^{4} -k_n H_{2n} R_n = 0, \tag{53}$$

$$\sum_{n=1}^{4} H_{7n} R_n = P_2. \tag{54}$$

Invoking boundary conditions (51)-(54) at the surface $y = 0$ of the plate, we obtain a system of four equations, (51)-(54). After applying the inverse of matrix method, then get the values of the four constants R_n $(n = 1, 2, 3, 4)$.

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{pmatrix} \begin{pmatrix} H_{41} & H_{42} & H_{43} & H_{44} \\ H_{61} & H_{61} & H_{61} & H_{61} \\ -k_1 H_{21} & -k_1 H_{21} & -k_1 H_{21} & -k_1 H_{21} \\ H_{71} & H_{71} & H_{71} & H_{71} \end{pmatrix}^{-1} \begin{pmatrix} -p_1 \\ 0 \\ 0 \\ p_2 \end{pmatrix} \tag{55}$$

Hence, we obtain the expressions for the displacements, the temperature distribution, and the other physical quantities of the plate surface.

5. Numerical results and discussion

Copper material was chosen for purposes of numerical evaluations and the constants of the problem were taken as follows:

$\lambda = 7.76\times 10^{10}\ N.m^{-2}$, $\mu = 3.86\times 10^{10}\ kg.m^{-1}.s^{-2}$, $\alpha_t = 1.78\times 10^{-5}\ k^{-1}$, $\rho = 8954\ kg.m^{-3}$,

$C_E = 383.1\ J.kg^{-1}.k^{-1}$, $T_0 = 293\ K$, $\beta = 2.68\times 10^{6}\ N/m^2$ deg, $\omega_1^* = 3.58\times 10^{11}/s$.

The voids parameters are

$\chi = 1.753\times 10^{-15}\ m^2$, $\xi = 1.475\times 10^{10}\ N/m^{0}$ $b = 1.13849\times 10^{10}\ N/m^2$, $\alpha = 3.688\times 10^{-5}\ N$,

$m = 2\times 10^{6}\ N/m^2$ deg, $\omega_0 = 0.0787\times 10^{-3}\ N/m^2 s$.

The comparisons were carried out for

$x = 0.1$, $t = 0.1$, $\omega = \zeta_0 + i\zeta_1$, $\zeta_0 = -0.1$, $\xi_1 = 1$, $p_1 = 2$, $p_2 = 0.01$, $\tau_0 = 0.05$, $\upsilon_0 = 0.5$, $c = 2$, $\Omega = 0.1$, $p = 0.1$, $a = 0.1$, $0 \le y \le 3$.

The above numerical technique, was used for the distribution of the real parts of the displacement component u, thermodynamic temperature distribution θ, the conductive temperature T, the stress components σ_{yy}, σ_{xy} and the change in the volume fraction field ϕ with distance in three theories, for the following cases:

(i) Figs. 1-6 show the comparisons between the considered variables in the absence and presence of initial stress (i.e. $P = 0, 0.1$) at $\Omega = 0.1$, $a = 0.1$

(ii) Figures 7-12 show comparisons among the considered variables for two different values of the non-dimensional, two-temperature parameter a $(a = 0, 0.1)$ where $a = 0$ indicates one-type of temperature and $a = 0.1$ indicates two-type of temperature in the presence of an initial stress (i.e. $P = 0.1$) and rotation (i.e. $\Omega = 0.1$).

With and without rotation effect are shown graphically in Figs. 13-18 in the case of two different values of rotation ($\Omega = 0, 0.1$) at $P = 0.1$, $a = 0.1$.

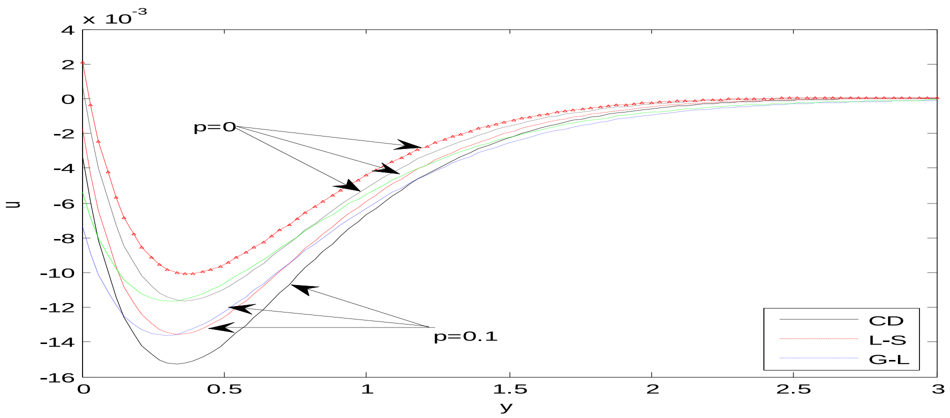

Figure 1: Horizontal displacement distribution u in the absence and presence of initial stress

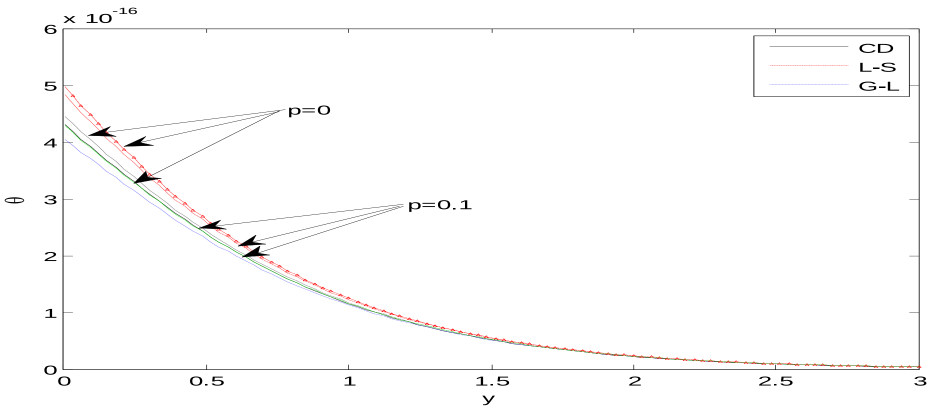

Figure 2: The distribution of the conductive temperature θ in the absence and presence of initial stress

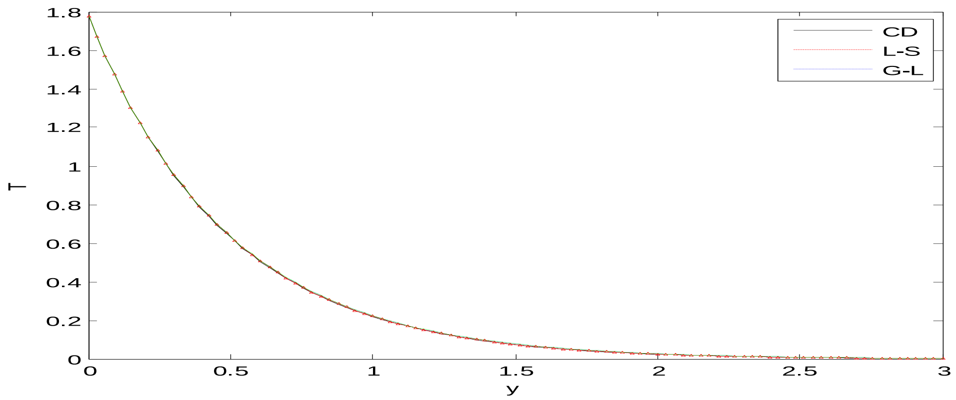

Figure 3: Conductive temperature distribution T in the absence and presence of initial stress

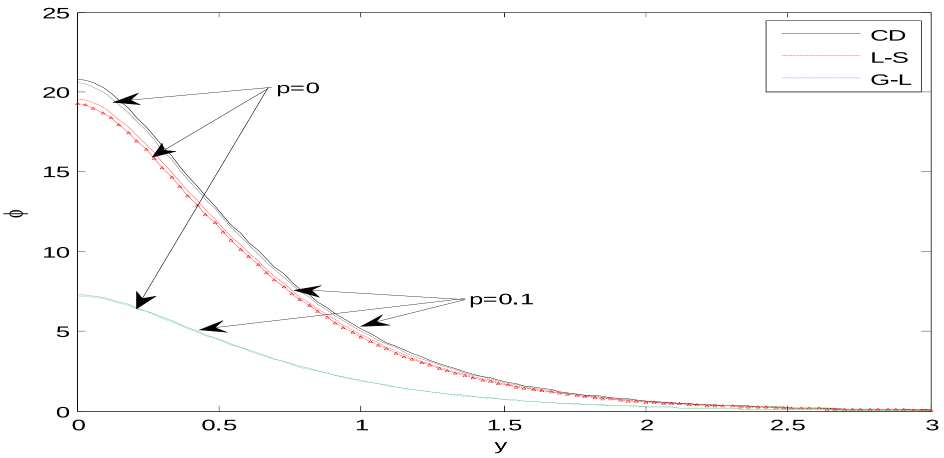

Figure 4: Change in volume fraction field distribution ϕ in the absence and presence of initial stress

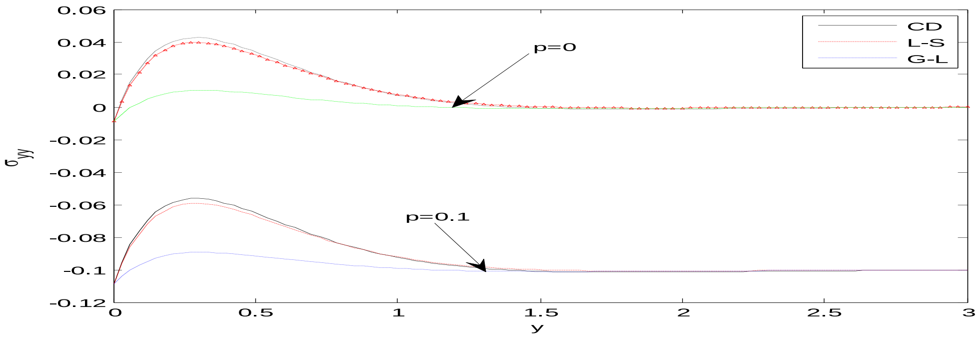

Figure 5: Distribution of strain component σ_{yy} in the absence and presence of initial stress

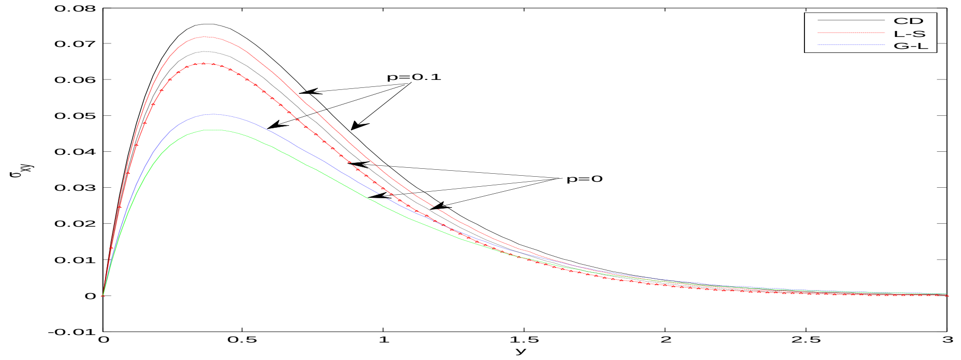

Figure 6: Distribution of stress component σ_{xy} in the absence and presence of initial stress

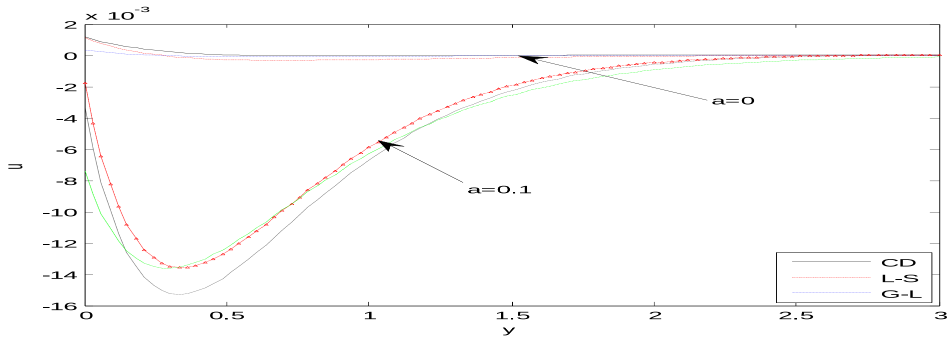

Figure 7: Horizontal displacement distribution u for two different values of a two-temperature parameter $a = 0, 0.1$

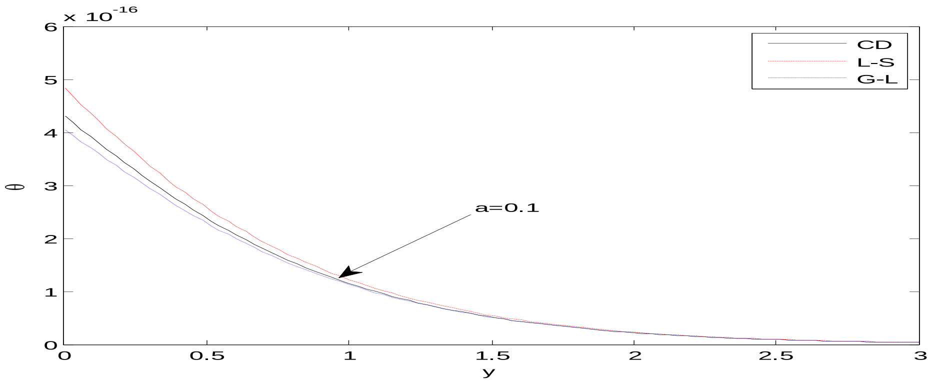

Figure 8 (a): Conductive temperature distribution θ for in the presence of two-temperature parameter $a = 0.1$

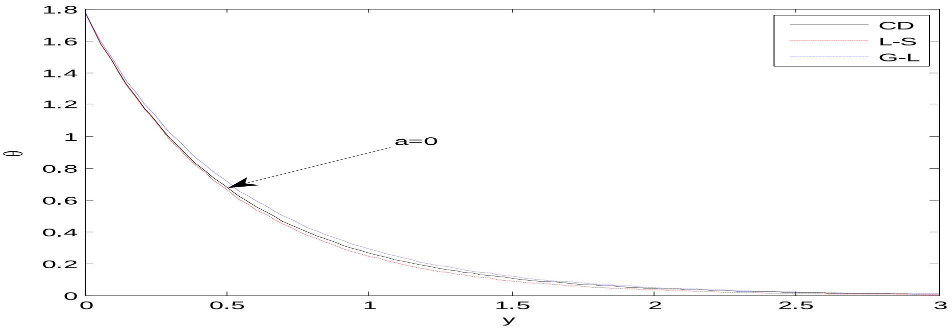

Figure 8(b):Conductive temperature distribution θ *for in the absence of two-temperature parameter* $a = 0$

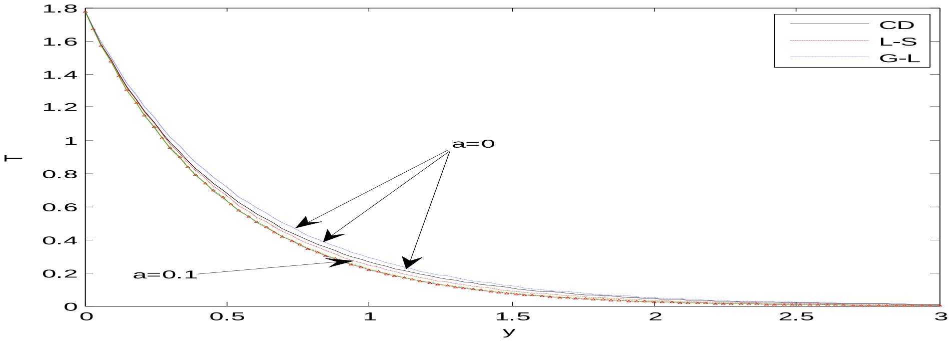

Figure 9: Thermodynamic temperature distribution T *for two different values of two-temperature parameter* $a = 0, 0.1$

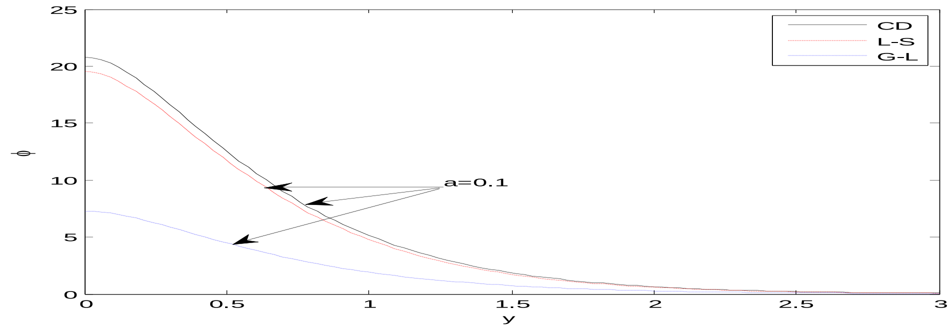

Figure 10(a:) The distribution of the volume fraction field ϕ *in the presence of two-temperature parameter* $a = 0.1$

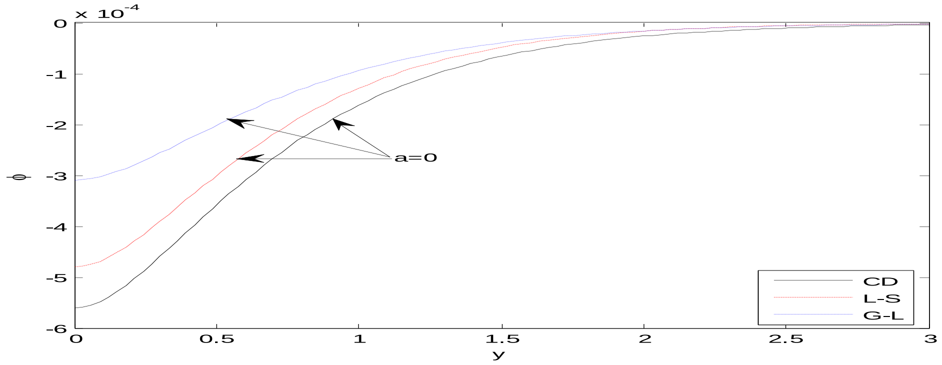

Figure 10(b): The distribution of the volume fraction field ϕ *in the absence of two-temperature parameter* $a = 0$

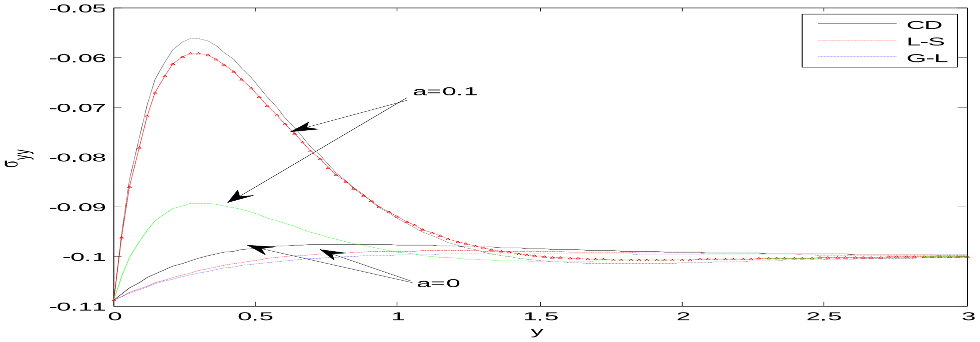

Figure 11: Distribution of stress component σ_{yy} *for two different values of two-temperature parameter* $a = 0, 0.1$

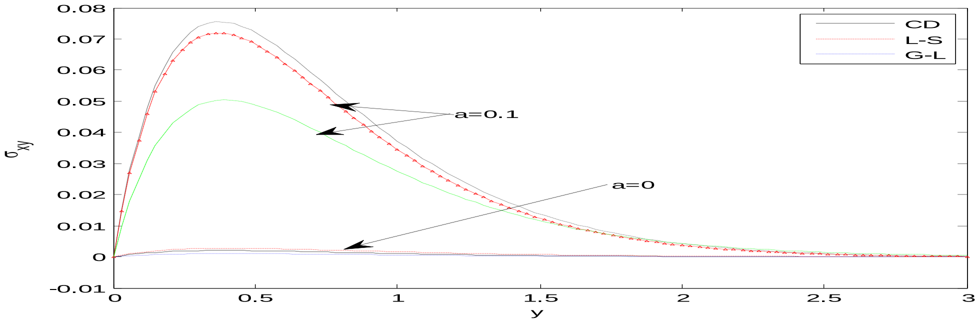

Figure 12: Distribution of stress component σ_{xy} *for two different values of two-temperature parameter* $a = 0, 0.1$

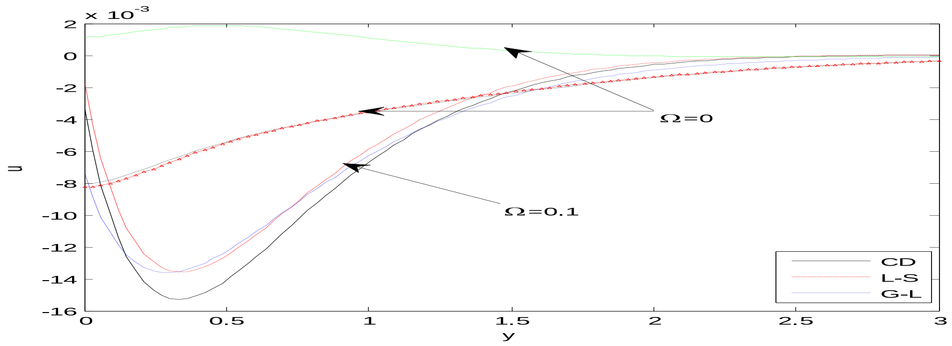

Figure 13: Horizontal displacement distribution u *in the absence and presence of rotation*

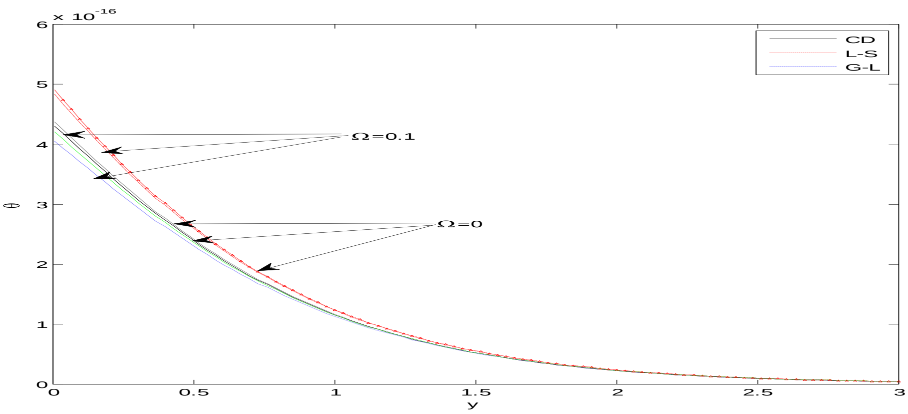

Figure 14: Conductive temperature distribution θ *in the absence and presence of rotation*

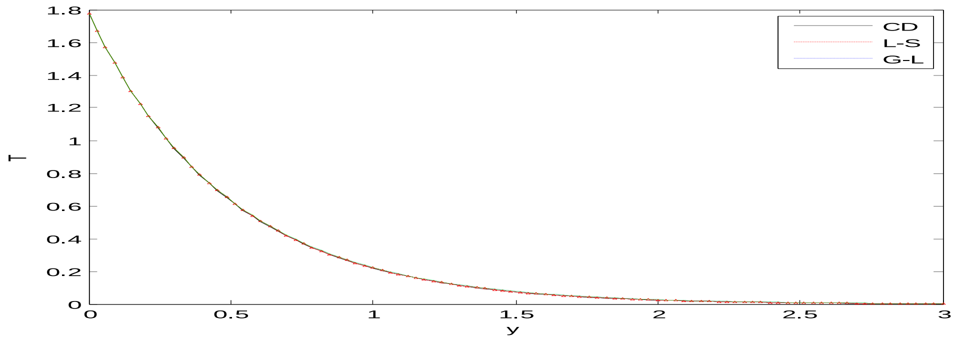

Figure 15: Thermodynamic temperature distribution T in the absence and presence of rotation

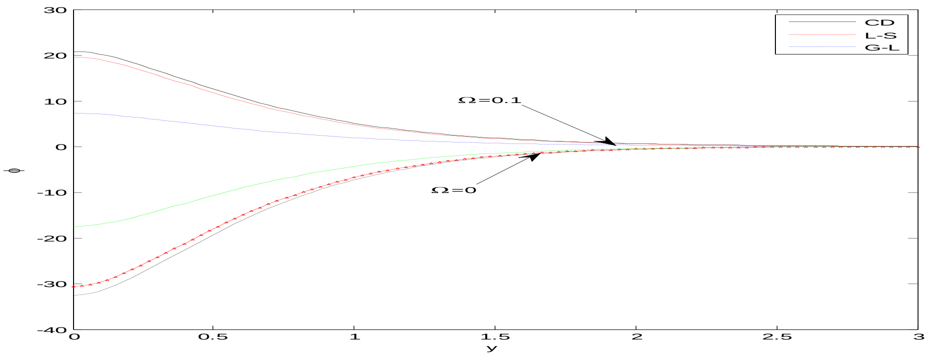

Figure 16: Distribution of the volume fraction field ϕ in the absence and presence of rotation

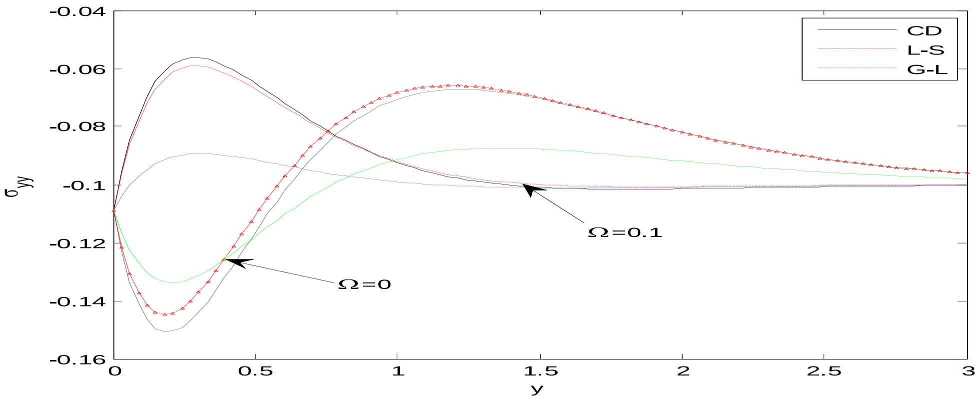

Figure 17: Distribution of stress component σ_{yy} in the absence and presence of rotation

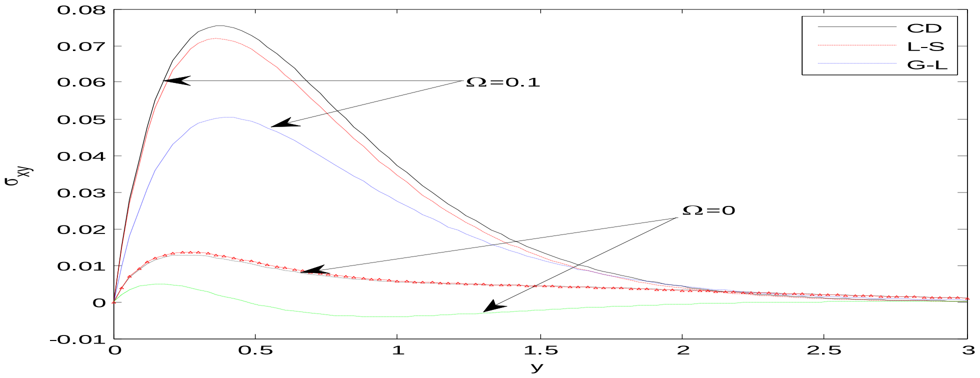

Figure 18: Distribution of stress component σ_{xy} in the absence and presence of rotation

The computations were carried out for a value of time $t = 0.1$. All the considered variables depend not only on the variables t, x and y, but also depend on the thermal relaxation times τ_0 and υ_0.

Figure 1 depicts that the displacement component u with distance y has been shown. In the context of the three theories, the values of u decreases in the range $0 \le y \le 0.3$, then increase in the range $0.3 \le y \le 3$, for $p = 0.1, 0$. Figures 2 and 3 demonstrate that the distribution of the conductive temperature θ and the thermodynamic temperature T begin from a positive value and satisfies the boundary conditions at $y = 0$. In the context of the three theories and in the absence and presence of initial stress (i.e. $P = 0, 0.1$), θ and T decrease in the range $0 \le y \le 3$. We also notice that the initial stress has no great effect on the distribution of the conductive temperature θ and the thermodynamic temperature T. Figure 4 expresses that the distribution of change in the volume fraction field ϕ begins from a positive value for ($P = 0, 0.1$),in the context of the three theories, however it decreases with the increase of the initial stress for $y > 0$. It was observed that the initial stress has a great effect on the distribution of ϕ. Figure 5 depicts that the distribution of the stress component σ_{yy} begins from negative value. In the context of the three theories, σ_{yy} increases in the beginning to a maximum value in the range $0 \le y \le 0.4$, then decreases in the range $0.4 \le y \le 3$ for $p = 0, 0.1$. It shows that the values of σ_{yy} at $p = 0$ are higher than that at $p = 0.1$. Figure 6 shows that the distribution of the stress component σ_{xy} reaches a zero and satisfies the boundary conditions at $y = 0$. In the context of the three theories, the values of σ_{xy} increases in the beginning to a maximum value in the range $0 \le y \le 0.3$, then decreases in the range $0.3 \le y \le 3$ for $(P = 0, 0.1)$. Figure 7 exhibits that the distribution of the vertical displacement u begins from negative value in the case of $a = 0$. In the context of the three theories and in the absence and presence of two-temperature, the values of the displacement u start by decreasing, then increasing. Figures 8a, b depict that the distribution of the temperature θ begins from a positive value for $a = 0.1, 0$ in the context of the three theories; it noticed that the distributions of θ increases with the increase of the two-temperature for $y > 0$, and finally goes to zero. Figure 9 demonstrates that the distribution of the temperature T begins from a positive value and satisfies the boundary conditions at $y = 0$. In the context of the three theories and in the absence and presence of two-temperature, T decreases in the range $0 \le y \le 3$. The values of the temperature T converge to zero as $y \ge 3$. It is noticed that the two-temperature has no great effect on the distribution of the temperature T. Figure 10a, b show that the distribution of change in the volume fraction field ϕ begins from a positive value for ($a = 0, 0.1$), in the context of the three theories, however it decreases in the range $0 \le y \le 3$ for $a = 0$. However, in the other cases, ϕ increases in the range $0 \le y \le 3$. It explains that all the curves converges to zero. Figure 11 shows that the distribution of stress component σ_{yy} begins from the negative value for $a = 0, 0.1$, in the context of three theories. We observe that the distribution σ_{yy} increases with the decrease of the time value. Figure 12 explains that the distribution of the stress component σ_{xy}, in the context of the three theories starts with zero for $a = 0, 0.1$, and increases in the range $0 \le y \le 0.3$, then, it decreases until attaining zero for $a = 0.1$, which agrees with the boundary conditions. It is also clear that the two-temperature parameter a acts to decrease the values of σ_{xy}. Figure 13 depicts that the distribution of the displacement u always begins from negative values. In the context of the three theories, in the absence and presence of the rotation, the values of u increase with the increase of the rotation. We can also observe from this figure that the values of u in the presence of rotation are lower than that in the absence of rotation for $y > 0$. Figure 14 demonstrates that the distribution of the conductive temperature θ, in the context of the three theories, decreases in the range $0 \le y \le 3$ for $\Omega = 0, 0.1$. Figure 15 determines that the distribution of the thermodynamic temperature T always begins from a positive value and satisfies the boundary condition at $y = 0$. In the context of the three theories, the values of the temperature T decrease in the range $0 \le y \le 3$, for $\Omega = 0, 0.1$. Figure 16 expresses the distribution of the change in the volume fraction field ϕ in the context of the three theories, for $\Omega = 0.1, 0$, decreasing with the increase of the rotation for $y > 0$. It explained that all the curves converge to zero. Figures 17 depicts that the distribution of the stress component σ_{yy} in the context of the three theories, for $\Omega = 0.1$ decreases in the range $0 \le y \le 3$, and increases in the range $0 \le y \le 3$, for $\Omega = 0$. It is observed that the distribution of σ_{yy} in the context of the three theories, is inversely proportional to the rotation for $y > 0$. Figure 18 shows that the distribution of the stress component σ_{xy} with distance in the context of the three theories, increases in the range $0 \le y \le 0.4$, then decreases in the range $0.4 \le y \le 3$, $\Omega = 0.1, 0$.

6. Conclusion

By comparing the figures obtained under the three theories, important phenomena are observed:

1. Analytical solutions based upon the normal mode analysis for thermoelastic problem in solids have been developed and utilized.
2. The value of all the physical quantities converges to zero with an increase in the distance y and all functions are continuous.
3. There are significant differences in the field quantities under (CD), (L-S) and (G-L) theories.
4. The presence and absence of rotation and initial stress effect in the current model is of significance.
5. The two-temperature effect for thermoelastic medium with voids is an interesting problem of thermo-mechanical.
6. The initial stress, the rotation and the two-temperature effect are significant in the current model since the amplitudes of these quantities are varying (increasing or decreasing) under the effect of the used fields.

References

[1]. Biot, M.A. (1956). Thermoelasticity and irreversible thermodynamics. *Journal of Applied Physics*, 27(3): 240-253.

[2]. Lord, H.W. & Shulman, Y. (1967). A generalized dynamical theory of thermoelasticity. *Journal of Mechanics and Physics of Solids*, 15: 299-339.

[3]. Green, A. E., & Lindsay, K. A. (1972). Thermoelasticity. *Journal of Elasticity*, 2: 1-7.

[4]. Othman, M. I. A. (2002). Lord-Shulman theory under the dependence of the modulus of elasticity on the reference temperature in two dimensional generalized thermoelasticity. *Journal of Thermal Stresses*, 25(11): 1027-1045.

[5]. Othman, M.I.A. (2004). Effect of rotation on plane waves in generalized thermoelasticity with two relaxation times. *International Journal of Solids and Structures*, 41(11-12): 2939-2956.

[6]. Othman, M. I. A., & Atwa, S. Y. (2012). Response of micropolar thermoelastic medium with voids due to various sources under Green-Naghdi theory. *Acta Mechanica Solida Sinica*, 25(2): 197-209.

[7]. Othman, M. I. A., Zidan, M. E. M., & Hilal, M. I. M. (2014). Effect of gravitational field and temperature dependent properties on two-temperature thermoelastic medium with voids under G-N theory. *Computers, Materials & Continua*, 40(3): 179-201.

[8]. Cowin, S. C., & Nunziato, J. W. (1983). Linear theory of elastic materials with void. *Journal of Elasticity*, 13: 125-147.

[9]. Puri, P., & Cowin, S. C. (1985). Plane waves in linear elastic materials with voids. *Journal of Elasticity*, 15: 167-183.

[10]. Dhaliwal R. S., & Wang, J. (1994). Domain of influence theorem in the theory of elastic materials with voids. *International Journal of Engineering Science*, 32: 1823-1828.

[11]. Dhaliwal, R. S., & Wang, J. (1995). A heat-flux dependent theory of thermoelasticity with voids. *Acta Mechanica*, 110, 33-39.

[12]. Ciarletta, M. & Scarpetta, E. (1995). Some results on thermoelasticity for dielectric materials with voids. *Journal of Applied Mathematics and Mechanics*, 75: 707-714.

[13]. Abd-Alla, A. N., & Alsheikh, F. A. (2009). The effect of the initial stresses on the reflection and transmission of plane quasi-vertical transverse waves in piezo-electric materials. *Journal of Computational and Theoretical Nanoscience*, 3(5): 233–240.

[14]. Abbas, I. A., & Kumar, R. (2014). Response of thermal source in initially stressed generalized thermoelastic half-space with voids. *Journal of Computational and Theoretical Nanoscience*, 11: 1472-1479.

[15]. Chanderashekhariah, D. S., & Srinath, K. S. (1997). Thermoelastic plane waves without energy dissipation in a rotating body. *Mechanical Research Communications*, 24: 551-560.

[16]. Othman, M. I. A. (2005). Effect of rotation and relaxation time on a thermal shock problem for a half-space in generalized thermo-viscoelasticity. *Acta Mechanica*, 174: 129-143.

[17]. Othman, M. I. A. (2010). The effect of rotation and thermal shock on a perfect conducting elastic half-space in generalized magneto-thermoelasticity with two relaxation times. *Mechanics and Mechanical Engineering*, 14: 31-55.

[18]. Schoenberg, M., & Censor, D. (1973). Elastic waves in rotating media. *Quarterly of Applied Math*ematics, 31: 115-125.

[19]. Chen, P. J., & Gurtin, M. E. (1968). On a theory of heat conduction involving two temperatures. *Zeitschrift für Angewandte Mathematik und Physik* (*ZAMP*), 19: 614-627.

[20]. Abbas, I. A., & Zenkour, A. (2014). Two-temperature generalized thermoelastic interaction in an infinite fiber-reinforced anisotropic plate containing a circular cavity with two relaxation times. *Journal of Computational and Theoretical Nanosciences*, 11(1): 1-7.

[21]. Othman, M. I. A., Hasona W. M., & Abd-Elaziz, E. M. (2014). Effect of rotation on micro-polar generalized thermoelasticity with two temperature using a dual-phase-lag model. *Canadian Journal of Physics*, 92: 149-158.

[22]. Youssef, H. M. (2006). Theory of two-temperature generalized thermoelasticity. *IMA Journal of Applied Mathematics*, 71(3): 383-390.

[23]. Youssef, H. M., & El-Bary, A. A. (2010). Two-temperature generalized thermoelasticity with variable thermal conductivity. *Journal of Thermal Stresses*, 33: 187-201.