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# A Time Series Model of Poverty Incidence in Nigeria

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**Abstract** This paper attempts to search for an optimal Autoregressive Integrated Moving Average model that best forecast absolute poverty incidence in Nigeria. The study uses absolute poverty data in Nigeria for 35 years from 1980-2014. The data was obtained as secondary data from Central Bank of Nigeria, Federal Office of Statistics, National Bureau of Statistics and International Monetary Fund World Economic Outlook. Time series plots, Ng & Perron modified unit root test and KPSS stationarity test were employed to check the graphical and statistical properties of the series. The results indicate that the series is integrated of order one, I(1) and the ACF and PACF plots of the stationary series suggest a mix ARMA (p,q) model for the series. ARIMA (p,d,q) model in line with Box-Jenkins procedure were then employed to model the poverty time series data. The result shows that ARIMA (4,1,4) was the best candidate to model poverty incidence in Nigeria. It was generally observed from the tests of residuals of the modeled equation that, the model was good, valid and adequate in describing absolute poverty situation in Nigeria. Accuracy measures such as Root Mean Square Error, Mean Absolute Error, Mean Absolute Percentage Error and Theil Inequality Coefficient were used to evaluate the forecast ability of the model and an out-of sample forecast mode was best for the model. The modeled ARIMA (4,1,4) was then used to forecast future poverty values in Nigeria. The forecast indicates a linear growth in poverty level in Nigeria.

**Keywords** Absolute poverty, ARIMA model, Accuracy measures, Forecast, Nigeria.

## 1. Introduction

Nigeria has a population of more than 170 million, the largest in Africa and a fast-growing economy. According to the Nigeria economic report released in July 2014, Nigeria has one of the world's highest economic growth rates averaging 7.4% [1]. Agriculture is the mainstay of the economy, contributing about 40 per cent of GDP. The agriculture sector employs approximately two-thirds of the country's total labour force and provides a livelihood for about 90 per cent of the rural population. Nigeria's huge agricultural resource base offers great potential for growth. For a country with massive wealth and a huge population to support commerce, a well-developed economy, plentiful agricultural resources and oil wealth, the level of poverty still remains high over the last decade. More than 70 per cent of Nigerians live on less than US$1.25 a day [2]. Poverty is especially severe in rural areas, where up to 80 per cent of the population lives below the poverty line, and social services and infrastructure are limited. The country's poor rural women and men depend on agriculture for food and income. About 90 per cent of Nigeria's food is produced by small-scale farmers who cultivate small plots of land and depend on rainfall rather than irrigation systems [3].

The poorest groups eke out a subsistence living but often go short of food, particularly during the pre-harvest period. The productivity of the rural population is also hindered by ill health, particularly HIV/AIDS, tuberculosis and malaria. Rural infrastructure in Nigeria has long been neglected. Investments in health, education and water supply have been focused largely on the cities. As a result, the rural population has extremely limited access to services such as schools and health centres, and about half of the population lacks access to safe drinking water. Neglect of rural infrastructure affects the profitability of agricultural production. The lack of rural roads impedes the marketing of agricultural commodities, prevents farmers from selling their produce at reasonable prices, and leads to spoilage. Limited accessibility cuts small-scale farmers off from sources of inputs, equipment and new technology, and this keeps agricultural yields at a very low level [4].

Poverty in Nigeria is therefore explained by the combined factors of inadequate food supply and limited entitlement to food as the most rudimentary manifestations of poverty is hunger and mal-nutrition, [5, 6 & 7]. In this study, we shall focus on the aspect of poverty that relate to the basic necessities of life (absolute poverty).

## 2. Conceptual and Theoretical Framework

### 2.1 The Concept of Poverty

According to the World Bank Economic Report (2014), poverty is pronounced deprivation in well-being, and comprises many dimensions. It includes low incomes and the inability to acquire the basic goods and services necessary for survival with dignity [1]. Poverty also encompasses low levels of health and education, poor access to clean water and sanitation, inadequate physical security, lack of voice, and insufficient capacity and opportunity to better one's life. Fundamentally, poverty is a denial of choices and opportunities, a violation of human dignity. It means lack of basic capacity to participate effectively in society. It means not having enough to feed and clothe a family, not having a school or clinic to go to; not having the land on which to grow one's food or a job to earn one's living, not having access to credit. It means insecurity, powerlessness and exclusion of individuals, households and communities. It means susceptibility to violence, and it often implies living in marginal or fragile environments, without access to clean water and sanitation, United Nations Report (2006).

According to Encyclopedia Americana (1989) poverty can be seen from two different perspectives: (i) a state of "moneylessness" which means both an insufficiency of cash and chronic inadequacy of resources of all types to satisfy basic human needs, such as, nutrition, rest, warmth and body care; and (ii) a state of "powerlessness" meaning those who lack the opportunities and choices open to them and whose lives seem to them to be governed by forces and persons outside their control [7]. Aku, *et al.* [8] saw poverty from five dimensions of deprivation: (i) personal and physical deprivation experienced from health, nutritional, literacy, educational disability and lack of self confidence; (ii) economic deprivation drawn from lack of access to property, income, assets, factors of production and finance; (iii) social deprivation as a result of denial from full participation in social, political and economic activities; (iv) cultural deprivation in terms of lack of access to values, beliefs, knowledge, information and attitudes which deprives the people the control of their own destinies; and (v) political deprivation in term of lack of political voice to partake in decision making that affects their lives. Haralambos & Holborn (2000) assert that poverty is a situation where a person is unable to acquire the minimum necessities that make for well-being. Poverty is marked by the inability to get good livelihood, have good house to live in, support oneself without depending on others, inability to acquire good healthcare, good educational training etc [9].

### 2.2 Types of Poverty

Haralambos & Holborn (2000) identifies three kinds of poverty. These are absolute, relative and subjective poverty. They describe absolute poverty as a state where the living condition is really critical and there is difficulty in survival. Absolute poverty refers to the complete absence of the basic necessities of life. The World Bank defines extreme poverty as living on less than US $1.25 at 2005 Purchasing Power Parity (ppp) per day, and moderate poverty as living on less than $2 a day [9].

According to Gordon (1998), absolute poverty is the absence of any two of the following eight basic needs: (i) Food: body mass index must be above 16; (ii) Safe drinking water: water must not come from solely rivers and ponds, and must be available nearby (less than 15 minutes walk each day); (iii) Sanitation facilities: toilets or latrines must be accessible in or near the home; (iv) Health: treatment must be received for serious illnesses and pregnancy; (v) Shelter: homes must have fewer than four people living in each room; floor must not be made of dirt, mud, or clay; (vi) Education: every one must attend school or otherwise learn to read and write; (vii) Information: every one must have access to newspapers, radios, televisions, computers, or telephones at home and (viii) Access to services: This item is normally used to indicate the complete access to education, health, legal, social and financial (credit) services [10].

Relative poverty views poverty as socially defined and dependent on social context, hence relative poverty is a measure of income inequality. Usually, relative poverty is measured as the percentage of population with income less than some fixed proportion of median income. The Economic aspects of poverty focus on material needs, typically including the necessities of daily living, such as food, clothing, shelter, or safe drinking water. Poverty in this sense may be understood as a condition in which a person or community is lacking in the basic needs for a minimum standard of well-being and life, particularly as a result of a persistent lack of income, [10]. According to Haralambos & Holborn (2000), relative poverty is in terms of adjustment from people of a particular society of what is taken as a reasonable and acceptable standard of living and way of life due to the conditions of the day [9].

According to Nweze and Ojowu (2002), Subjective Poverty is a concept of poverty which is expressed in a range of non-material and intangible qualities; it is based on respondents' perception of their standard of living [11]. The feeling of whether one is poor or not depends on the absolute minimum standard of living below which one is categorized as poor [12].

### 2.3 Measures of Poverty

Related to the definition and types of poverty is the measurement of poverty. According to Foster *et al.* (1984), the most frequently used measurements are: (i) the head count poverty index given by the percentage of the population that live in the households with a consumption per capita less than the poverty line; (ii) poverty gap index which reflects the depth of poverty by taking into account how far the average poor person's income is from the poverty line; and (iii) the distributional sensitive measure of squared poverty gap defined as the means of the squared proportionate poverty gap which reflects the severity of poverty. Recent studies by United Nations Development Programme (UNDP) advocates the use of the Human Development Index (HDI) [12]. According to UNDP (2009), HDI combine three components in the measurement of poverty: (i) life expectancy at birth (longevity); (ii) education attainment and; (iii) improved standard of living determined by per capita income. The first relates to survival-vulnerability to death at a relatively early age. The second relates to knowledge being excluded from the world of reading and communication. The third relates to a decent living standard in terms of overall economic provisioning [14].

### 2.4 Poverty Trends in Nigeria

Taking a critical look at the poverty situation in Nigeria as indicated in Table A1 and Figure A1, one would observe that the incidence of poverty in Nigeria is increasing with the increasing number of people living in absolute poverty.

**Table A1:** Estimated Population and Poverty Rate in Nigeria (1980-2012)

| Year | Estimated Total Population (in millions)[a] | Estimated No of Non-Poor (in millions)[b] | Estimated No of Poor People (in millions)[c] | Percentage (%) Poor[d] |
|---|---|---|---|---|
| 1980 | 64.6 | 45.5 | 18.1 | 28.1 |
| 1982 | 68.4 | 44.2 | 24.2 | 32.0 |
| 1985 | 75.4 | 40.5 | 34.9 | 46.3 |
| 1987 | 80.4 | 43.9 | 36.5 | 45.4 |
| 1990 | 86.6 | 48.6 | 38.0 | 44.0 |
| 1992 | 91.3 | 52.3 | 39.0 | 42.7 |
| 1995 | 98.9 | 39.6 | 59.3 | 60.0 |
| 1997 | 104.0 | 36.4 | 67.6 | 65.5 |
| 2000 | 111.3 | 34.3 | 77.0 | 74.0 |
| 2002 | 116.4 | 30.4 | 86.0 | 88.0 |
| 2005 | 137.5 | 47.2 | 90.3 | 65.7 |
| 2007 | 143.3 | 58.3 | 85.0 | 59.3 |
| 2010 | 152.2 | 53.4 | 98.8 | 64.9 |
| 2012 | 164.6 | 52.3 | 112.3 | 68.2 |

**Sources:** (a) National Population Commission [15]; Central Bank of Nigeria [16] Annual Report and Statement of Account (various issues), Federal Office of Statistics (1998) Annual Abstract Statistics (various issues) [17]; IMF 2012 World Economic Outlook (b) & (c) Computed by the Authors from (a) and (d). (d) Federal Office of Statistics (now National Bureau of Statistics) 2010 Poverty Profile for Nigeria [18]; IMF 2012 World Economic Outlook.

Since the mid 1980s the rate of poverty in Nigeria has been on the increase. For instance, in 1980 the rate was 28.1% and it has risen to about 45.4% and 65.5% in 1887 and 1995 respectively [19]. Some of the reasons behind this persistent increase include among others; the effects of the global economic crisis witnessed in the early 1980s, the negative effects of Structural Adjustment Programme (SAP) introduced in 1986, political instability, bad governance, corruption, and the collapse of public infrastructures [20 & 21]. The poverty situation in Nigeria was even worse in the early 2000s particularly in 2002 when the incidence recorded the highest rate of 88.0% in the history of Nigeria. This percentage rate represents in absolute term 86 million people out of an estimated population of about 116.4 million people.

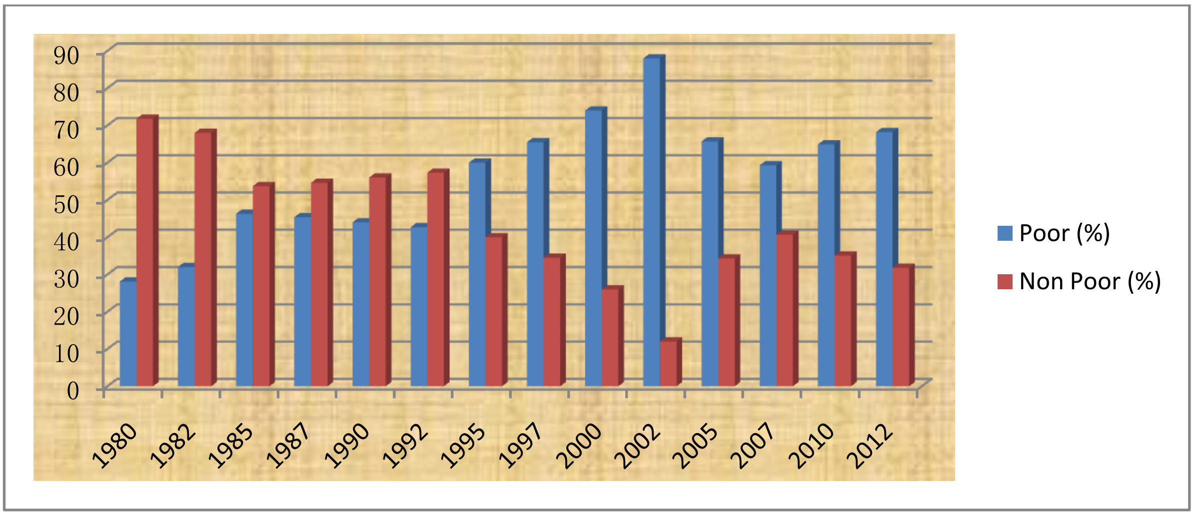


*Figure A1: Trends of Absolute Poverty in Nigeria (1980-2012)*

The poverty situation in Nigeria also depicts regional variation. For example, According to the former Senior Special Adviser (SSA) to a former President on Poverty Alleviation, Dr. Magnus Kpakol, in a paper titled, "NAPEP Programmes As Enabler For Rapid Economic Development in the South-South Region", presented at the South-South Economic Summit in Calabar, the Cross River State Capital, 84 million Nigerians were poor. The figure, which was so as at December 2008, dropped by one million from 85 million in 2007. He said the population of the nation's poor people was 80 million in 1999, when Nigeria returned to democracy. According to him, the poverty rate was higher in the northern part of the country. His analysis showed the following percentage of the poor in all the six geopolitical zones of the country: North West – 72.2%; North East – 71.2%; North Central – 67%; South East – 26.7%; South-South – 35.1% and South West – 43.1% of their respective populations [22].

According to the report, the North-West and North-East recorded the highest poverty rates in the country in 2010 with 77.7 per cent and 76.3 per cent respectively. The South-West geo-political zone recorded the lowest at 59.1 per cent [22].

**2.5 Causes and Consequences of Poverty in Nigeria**

Poverty has various manifestations which include among others: lack of income and productive resources sufficient to ensure sustainable livelihood, hunger and malnutrition, ill health, limited or lack of access to education and other basic services, increased morbidity and mortality from illness, homelessness and inadequate, unsafe and degraded environment and social discrimination and exclusion. It is also characterized by lack of participation in decision making in civil, social and cultural life [23 & 24]. Yahie (1993) reiterates that the factor that causes poverty include; (i) structural causes that are more permanent and depend on a host of (exogenous) factors such as limited resources, lack of skill, location disadvantage and other factors that are inherent in the social and political set-up. The disables, orphans, landless farmers, household headed by females fall into this category; (ii) the transitional causes that are mainly due to structural adjustment reforms and changes in domestic economic policies that may result in price changes, increased unemployment and so on. Natural calamities such as wars, environmental degradation and so on also induce transitory poverty [25].

The main causes of poverty as observed by Obadan (1997) include among others: inadequate access to employment opportunities; inadequate physical assets such as land and capital and minimal access by the poor to credit even on a small scale; inadequate access to the means of supporting rural development in poor regions; inadequate access to market where the poor can sell goods and services; low endowment of human capital; destruction of natural resources leading to environmental degradation and reduced productivity; inadequate access to assistance for those living at the margin and those victimized by transitory poverty and lack of participation. That is, the failure to draw the poor into the design of development programmes that affect their lives [26].

The consequences of this increase in poverty include among others; increase in the number of destitutes, beggars, prostitutes, and paupers. Poverty appears to have also led to increase in the rate of crime in the society, increase in child labour, child abandonment and abuse, increase in infant, child and maternal mortality rates and reduction in life expectancy of most adult. For instance, the rate of crime in the country has been on the increase with cases of crimes and offences reported to the police increasing from 253,098 in 1995 to 258,655 in 1996, while in 1998 the infant mortality rate was 114 per 1000 live birth and maternal mortality rate was 10 per 1000 live births in the same year. An under-five mortality rate of close to 190 per 100,000 live births and 54 years as life expectancy at births were all registered in 1998. These figures when compared with other developing countries like Malaysia provide a pathetic situation [17].

Von Hauff and Kruse (1994) stated three major consequences of poverty as: (i) consequences for those affected. That is, for the people affected, poverty leads to physical and psychological misery caused *inter-alia* by inadequate nourishment, lack of medical care, a lack of basic and job related education and marginalisation in the labour market; (ii) consequences for the national economies of countries affected arising through the formation of slums in cities, a worsening of ecological problems particularly as a result of predatory exploitation in the agricultural sector and through the failure to use the available human resources; and (iii) consequences for the political and social development of the countries affected. That is, mass poverty tends to preserve or re-enforce the existing power structures and thus also the privileges of a minority of the population. In some cases, this involves corrupt elites. These privileged minorities in the population are not generally interested in structural changes for the benefit of the poor population. As a consequence, mass poverty tends to inhibit the development of democratic structure and a higher level of participation [27]. Aku *et al.* (1997) observed that with mass poverty there tend to be a general loss of confidence in the constituted authority there by generating disrespect and rendering government policies ineffective; political apathy among contending forces; and social disillusion with respect to what the societal objectives are and people's responsibilities towards the attainment of these objectives [8].

**2.6 Anti Poverty Initiatives in Nigeria**

Several attempts have been made by past Nigerian governments to reduce the rate of poverty in the country. Notably among them are National Accelerated Food Production Programme (NAFPP) and the Nigerian Agricultural and Co-operative Bank (NACB) introduced in 1972, Operation Feed the Nation (OFN) of 1976 whose main objective was to teach the rural farmers how to use modern farming tools in order to increase food production in the country; Green Revolution Programme introduced in 1979 to reduce food importation and increase local food production; Directorate of Food, Roads and Rural Infrastructure (DFRRI) of (1986); Structural Adjustment Programme (SAP) also introduced in 1986; Family Support Programme and the Family Economic Advancement Programme all in 1993; Poverty Alleviation Programme of 1999 and National Poverty Eradication Programme (NAPEP) introduced in 2001 to replace the defunt Poverty Alleviation Programme among others. All these programmes were initiated by various governments at one time or the other with good policies that were geared towards poverty reduction but unfortunate little or no successes were achieved from these measures since the rate of poverty continues to increase. The regime of President Olusegun Obasanjo even over dramatized its effort to reduce poverty. The government set up a Poverty Alleviation programme in 1999 without any measure of success the government abandoned the alleviation programme and set up National Poverty Eradication Programme in 2001. The poser is how can a government which failed blatantly in its effort to alleviate poverty will succeed in eradicating it? In trying to answer this question and similar others will only lead to the smart mark of insincerity on the part of our past leaders. The lack of a stringent regulatory and monitoring system has allowed for rampant corruption. This has hindered past poverty alleviation efforts, and will continue to do so since resources which could have been paid for public goods or be directed towards investment and so create employment and other opportunities for citizens are being misappropriated. The government of Nigeria has enough money to lift millions of Nigerians out of poverty without the need for foreign aid. If waste and corruption were overcome, money could finally go to the country's infrastructure: hospitals, running water, good education system, electricity, good roads and so on.

## 3. Materials and Methods

### 3.1. Autoregressive Integrated Moving Average Model

The theoretical model which serves as a basic framework of our analysis is the general autoregressive integrated moving average (ARIMA) which is a generalization of an ARMA model. These models are fitted to time series data either to better understand the data or to predict future points in the series (forecasting). The model is generally referred to as an ARIMA (p, d, q) model, where p, d, and q are non-negative integers that refer to the order of the autoregressive, integrated, and moving average parts of the model respectively. ARIMA models form an important part of Box-Jenkins approach to time series modeling.

Given a time series of data $\{y_t\}$ where $t$ is an integer index and the $y_t$ are real numbers, then an ARMA (p, q) model is given by:

$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \cdots + \alpha_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$ or

$$\left(1 - \sum_{i=1}^{p} \alpha_i L^i\right) y_t = \left(1 + \sum_{i=1}^{q} \theta_i L^i\right) \varepsilon_t \qquad (1)$$

Where L is the lag operator, the $\alpha_i$ are the parameters of the autoregressive part of the model, the $\theta_i$ are the parameters of the moving average part and $\varepsilon_t$ are error terms which are generally assumed to be independent, identically distributed variables sampled from a normal distribution with zero mean.

Assuming that the polynomial $\left(1 - \sum_{i=1}^{p} \alpha_i L^i\right)$ has a unitary root of multiplicity $d$, then it can be written as:

$$\left(1 - \sum_{i=1}^{p} \alpha_i L^i\right) = \left(1 + \sum_{i=1}^{p-d} \theta_i L^i\right)(1 - L)^d \qquad (2)$$

An ARIMA (p,d,q) process expresses this polynomial factorization property, and is given by:

$$\left(1 - \sum_{i=1}^{p} \Phi_i L^i\right)(1 - L)^d y_t = \left(1 + \sum_{i=1}^{q} \theta_i L^i\right) \varepsilon_t \qquad (3)$$

ARIMA models are used for observable non-stationary processes $y_t$ that have some clearly identifiable trends:

(i) A constant trend (i.e. zero average) is modeled by $d = 0$

(ii) A linear trend (i.e. linear growth behaviour) is modeled by $d = 1$

(ii) A quadratic trend (i.e. quadratic growth behaviour) is modeled by $d = 2$

We shall now examine the properties of an AR (p) process

### 3.2 Autoregressive Model of Order p

A stationary time series process $\{y_t\}$ is said to be an autoregressive process of order $p$, denoted by AR (p) if it satisfies the difference equation:

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \cdots + \alpha_p y_{t-p} + \varepsilon_t \qquad (4)$$

where $\varepsilon_t$ is a white noise process with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = \sigma_\varepsilon^2; \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

The parameters $\alpha_1, \alpha_2, \ldots, \alpha_p$ must satisfy certain conditions for the process to be stationary. Using lag operators;

$L^j y_t = y_{t-j}$ (4) becomes

$$\left(1 - \alpha_1 L - \alpha_2 L^2 - \cdots - \alpha_p L^p\right) y_t = \varepsilon_t \qquad (5)$$

or $\Phi(L) y_t = \varepsilon_t \Rightarrow \quad y_t = \Phi^{-1}(L) \varepsilon_t \qquad (6)$

where $\Phi(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \cdots - \alpha_p L^p$,

$\Phi(L)$ Can be factored as

$$\Phi(L) = (1 - G_1 L)(1 - G_2 L) \dots (1 - G_p L) \quad (7)$$

Expanding (6) using partial fraction gives:

$$y_t = \Phi^{-1}(L)\varepsilon_t = \sum_{i=1}^{p} \left( \frac{k_i}{1 - G_i L} \right) \varepsilon_t \quad (8)$$

For $\Phi^{-1}(L)$ to converge, $|G_i| < 1$ which implies that the roots of the characteristic equation $\Phi(L) = 0$ should lie outside the unit circle.

**3.2.1 Autocovariance and Autocorrelation of an AR(p)**

Multiplying (4) by $y_{t-k}$ and taking expectation, we have

$E(y_{t-k}y_t) = \alpha_1 E(y_{t-k}y_{t-1}) + \cdots + \alpha_p E(y_{t-k}y_{t-p}) + E(y_{t-k}\varepsilon_t)$ so that

$$E\{(y_{t-k}y_t)\} = \alpha_1\gamma_{k-1} + \alpha_2\gamma_{k-2} + \cdots + \alpha_p\gamma_{k-p} + \begin{cases} \sigma^2 \; if \; k = 0 \\ 0 \; if \; k \neq 0 \end{cases}$$

dividing through by $\gamma_0$, we have

$$\rho_k = \alpha_1\rho_{k-1} + \alpha_2\rho_{k-2} + \cdots + \alpha_p\rho_{k-p}, k > 0 \quad (9)$$

Setting $k = 1,2, \dots, p$, gives

$$\begin{aligned} \rho_1 &= \alpha_1 + \alpha_2\rho_1 + \cdots + \alpha_p\rho_{p-1} \\ \rho_2 &= \alpha_1\rho_1 + \alpha_2 + \cdots + \alpha_p\rho_{p-2} \\ &\vdots \\ \rho_p &= \alpha_1\rho_{p-1} + \alpha_2\rho_{p-2} + \cdots + \alpha_p \end{aligned} \quad (10)$$

Equation (10) is called Yule-Walker equations and are used in estimating $\alpha_1, \alpha_2, \dots, \alpha_p$.

**3.2.2 Partial Autocorrelation of an AR(p)**

The partial correlation between two variables is the correlation that remains if the possible impact of all other random variables has been eliminated. To define the partial autocorrelation coefficient, we use the new notation,

$$y_t = \Phi_{k1}y_{t-1} + \Phi_{k2}y_{t-2} + \cdots + \Phi_{kk}y_{t-k} + \varepsilon_t \quad (11)$$

Where $\Phi_{ki}$ is the coefficient of the variables with lag $i$ if the process has order $k$. According to (4) it holds that $\alpha_i = \Phi_{ki}, i = 1,2, \dots., k$.

The coefficients $\Phi_{kk}$ are the partial autocorrelation coefficients of order $k$, $k = 1,2, \dots$.

The partial autocorrelation measures the correlation between $y_t$ and $y_{t-k}$ which remains when the influences of the $y_{t-1}, \; y_{t-2}, \dots, y_{t-k+1}$ on $y_t$ and $y_{t-k}$ have been eliminated.

In general, the partial autocorrelation coefficients $\Phi_{kk} = 0$ for $k > 1$ is an AR(p) process. All partial autocorrelation coefficients of order higher than p are zero. Thus, for finite order autoregressive processes, the partial autocorrelation function provides the possibility of identifying the order of the process by the order of the last non-zero partial autocorrelation coefficient.

**3.3 Moving Average MA (q) Model**

A Moving Average process of order q is given by

$$y_t = u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \cdots \theta_p u_{t-q} = u_t + \sum_{i=1}^{q} \theta_j u_{t-j} \quad (12)$$

Or using lag operator:

$$\begin{aligned} y_t &= (1 - \theta_1 L - \theta_2 L^2 - \cdots - \theta_q L^q \quad (13) \\ &= \Theta(L)u_t \quad (14) \end{aligned}$$

**3.3.1 Inevitability in MA models**

A time series $y_t$ is invertible if it can be represented by a finite order MA or convergent autoregressive process. Invertibility is important because the use of the ACF and PACF for identification implicitly assume that the $y_t$ sequence can be well approximated by an autoregressive model.

**3.3.2 Autocovariance and Autocorrelation Function of an MA(q) Process**

The autocovariance of an MA(q) process is given by:

$$\gamma_k = \begin{cases} \sigma_a^2(-\theta_k + \theta_1\theta_{k+1} + \cdots + \theta_{q-k}\theta_q), & k = 1, 2, \dots, q \\ 0, & k > q \end{cases} \quad (15)$$

and the autocorrelation function is given by:

$$\rho_k = \begin{cases} \dfrac{-\theta_k + \theta_1\theta_{k+1} + \cdots + \theta_{q-k}\theta_q}{1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2}, & k = 1, 2, \ldots, q \\ 0, & k > q \end{cases} \tag{16}$$

The autocorrelation function of an MA(q) process cut off after lag q. This important property enables us to identify whether a given time series is generated by a moving average process [28].

**3.4 Strategy for Finding an Adequate Model**

Figure A2 outlines the general strategy for finding an adequate ARIMA model. This strategy involves six steps: Data collection and examination, determination of stationary series, model identification, model estimation, diagnostic checking, forecasting and forecast evaluation.

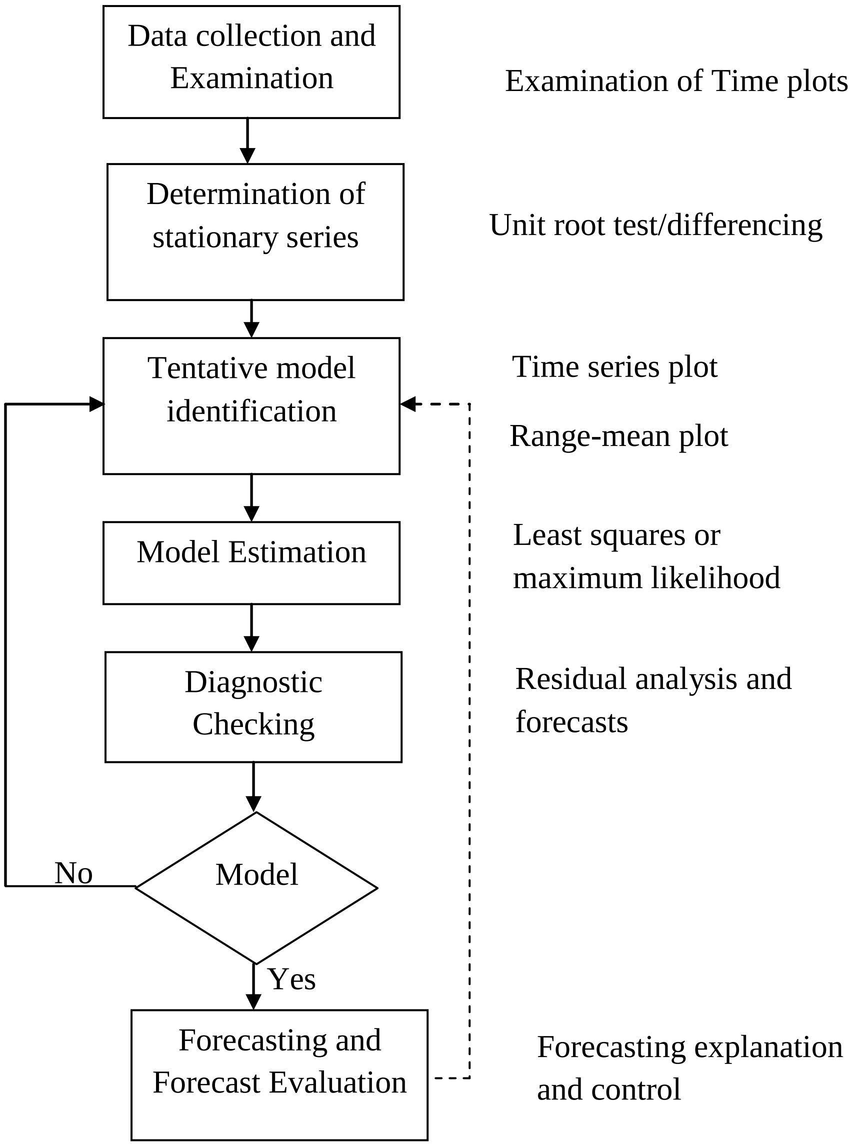


*Figure A2: Flow Diagram of Iterative Model-Building Steps. Aidan, et al. (1998) with Authors' modifications.*

**3.4.1 Graphical Examination of the Data**

Graphically examining the data is important. The data should be examined in levels, logs and differences. The series should be plotted against time to assess whether any structural breaks, outliers or data errors occur. If so one may need to consider use of intervention or dummy variables. This step may also reveal whether there is a significant seasonal pattern in the time series.

Another way to examine the properties of a time series is to plot its autocorrelogram. The autocorrelogram plots the autocorrelation between differing lag lengths of the time series. Plotting the autocorrelogram is a useful aid for determining the stationarity of a time series, and is also an important input into Box-Jenkins model identification. If a time series is stationary then its autocorrelogram should decay quite rapidly from its initial value of unity at zero lag. If the time series is nonstationary then the autocorrelogram will only die out gradually over time.

**3.4.2 Testing for Unit Root and Stationarity**

The Time series under consideration must be stationary before one can attempt to identify a suitable ARMA model. For AR or ARMA models to be stationary it is necessary that the modulus of the roots of the AR polynomial be greater than unity, and for the MA part to be invertible it is also necessary that the roots of the MA polynomials be lie outside the unit circle.

A time series can be non-stationary because of a deterministic trend (a stationary trend) or a stochastic trend (a difference stationary trend) or both. Unit root tests are used to detect stochastic trend and persistence of shocks in a series. There are many unit root tests for testing the stationarity of time series data but we shall use the Ng-Perron modified unit root test and KPSS stationarity test in this work.

**3.4.3 Ng and Perron Modified Unit Root Test**

Phillips (1987) proposed the non parametric test statistics for the unit root null by using consistent estimates of variance as follows [30]:

(i) **AR** (1) Without a drift

$$Z_\rho = T(\hat{\rho}-1) - \frac{1}{2}\frac{(S^2 - S_e^2)}{T^{-2}\sum_1^T y_{t-1}^2}; \quad Z_t = \frac{S_e}{s} t_{\hat{\rho}} - \frac{1}{2}\ \frac{S^2 - S_e^2}{S(T^{-2}\sum_1^T y_{t-1}^2)^{1/2}} \qquad (17)$$

(ii) **AR (1)** with a drift

$$Z_\rho = T(\hat{\rho}-1) - \frac{1}{2}\frac{(S^2 - S_e^2)}{T^{-2}\sum_1^T (y_{t-1} - \bar{y}_{-1})^2}; \quad Z_t = \frac{S_e}{s} t_{\hat{\rho}} - \frac{1}{2}\ \frac{S^2 - S_e^2}{S[T^{-2}\sum_1^T (y_{t-1} - \bar{y}_{-1})^2]^{1/2}} \qquad (18)$$

$$\text{where} \qquad \bar{y}_{-1} = \sum_1^{T-1} {y_t}/{(T-1)}$$

(iii) **AR (1)** with a drift and a linear trend

$$Z_\rho = T(\hat{\rho}-1) - \frac{T^6}{24DX}(S^2 - S_e^2); \quad Z_t = \frac{S_e}{s} t_{\hat{\rho}} - \frac{T^3(S^2 - S_e^2)}{4\sqrt{3}DX^{1/2}S} \qquad (19)$$

Where $DX = \det(X'X)$ and the regressors are $X = (1, t, y_{t-1})$

The PP tests suffered from serious size distortions when there are negative MA errors. Perron and Ng (1996) suggest modifications of the PP test to correct this problem. They used methods suggested by Stock (1990) to derive modifications of $Z_\rho$ and $Z_t$ statistics [31].

The Ng and Perron modified $Z_\rho$ and $Z_t$ statistics are:

$$MZ_\rho = Z_\rho + \frac{T}{2}(\hat{\rho}-1)^2 \qquad (20)$$

Convergence of $\hat{\rho}$ at rate $T$ ensures that $Z_\rho$ and $MZ_\rho$ are asymptotically equivalent. Defining

$$MSB = \left(T^{-2} \sum y_{t-1}^2 / S^2\right)^{1/2}$$

They note that $Z_t = MSB.Z_\rho$. Hence they define the modified $Z_t$ statistic by

$$MZ_t = MSB.MZ_\rho \qquad (21)$$

If we write the model as

$$y_t = \rho y_{t-1} + u_t$$

Then for the computation of the PP test statistics we need estimates of two error variances $\sigma_u^2 = Var(u_t)$ and $\sigma^2 = \lim_{T\to\infty} T^{-1}E(S_T^2)$ where $S_T = \sum_{j=1}^T u_j$. For an estimate of $\sigma_u^2$ they use $S_u^2 = T^{-1}\sum u_t^2$.

For the estimate of $\sigma^2$ they suggest using an autoregressive estimator defined as $S_{AR}^2 = \frac{S_{ek}^2}{1-\hat{b}(1)^2}$

where $S_{ek}^2 = T^{-1}\sum_{t=k+1}^T \hat{e}_{tk}$, $\hat{b}(1) = \sum_{j=1}^k \hat{b}_j$ and $\hat{b}_j$ and $\{\hat{e}_{tk}\}$ are obtained from the autoregression

$$\Delta y_t = b_0 y_{t-1} + \sum_{j=1}^{k} b_j \Delta y_{t-j} + e_{tk} \tag{22}$$

In addition to the $MZ_\rho$ and $MZ_t$ statistics, Ng and Perron also investigated the size and power properties of the $MSB$ statistic defined earlier, but with the above estimate of $\sigma^2$. Critical values for the demeaned and detrended case of this statistic were taken from [32].

**Decision Rule:** Reject $H_0$ (The series has a unit root) if the test statistic is less than the asymptotic critical values at the conventional test sizes; otherwise accept the alternative hypothesis (the series is stationary) if the test statistic is greater than the critical values.

### 3.4.4 Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) Stationarity Test

KPSS tests both the unit root hypothesis and the stationarity hypothesis.

$$S_t = \sum_{j=1}^{t} e_j \tag{23}$$

and $\sigma^2$ be the long-run variance of $e_t$, which is defined as:

$$\sigma^2 = Lim \frac{1}{N} E[S_N^2], \tag{24}$$

Then the consistent estimator of $\sigma^2$ can be constructed from the residuals $e_t$ by [33]:

$$\hat{\sigma}^2(P) = \frac{1}{N}\sum_{t=1}^{N} e_t^2 + \frac{2}{N}\sum_{j=1}^{P} w_j(P) \sum_{t=j+1}^{N} e_t e_{t-j} \tag{25}$$

where $P$ is the truncation lag, $w_j(P)$ is an optional weighting function that corresponds to the choice of a special window, e.g. Barlett window [34]:

$$w_j(P) = 1 - \frac{j}{(P+1)} \tag{26}$$

Then the KPSS Lagrangian multiplier (LM) test statistic is given by [35]:

$$KPSS = \frac{1}{N^2}\sum_{t=1}^{N} \frac{S_t^2}{\hat{\sigma}_{(P)}^2}. \tag{27}$$

Under the null hypothesis of level stationary,

$$KPSS \approx \int_0^1 V_2(r)^2 dr \tag{28}$$

where $V_2(r)$ is the second level Brownian bridge, given by

$$V_2(r) = B(r) + (2r - 3r^2)B(1) + (-6r + 6r^2)\int_0^1 B(s)ds \tag{29}$$

**Decision Rule:** Reject $H_0$ (The series is stationary) if the test statistic is less than the asymptotic critical values at the conventional test sizes; otherwise accept the alternative hypothesis (the series has a unit root) if the test statistic is greater than the critical values.

### 3.4.5 Model Identification

We shall use the Box-Jenkins procedure for identification of ARMA models.

#### 3.4.5.1 Box-Jenkins Methodology

The Box-Jenkins methodology essentially involves examining plots of the sample autocorrelogram, partial autocorrelogram and inferring from patterns observed in these functions the correct form of ARMA model to select. The Box-Jenkins methodology is not only about model identification but is, in fact, an iterative approach incorporating model estimation and diagnostic checking in addition to model identification. To find a reasonably good match and tentatively select one or more ARIMA models, the general characteristics of theoretical ACFs and PACFs are presented in Table 3.1

**Table 3.1:** Characteristics of Theoretical ACFs and PACFs

| Model | ACF | PACF |
|---|---|---|
| AR | Spikes decay towards zero | Spikes cut off to zero |
| MA | Spikes cut off to zero | Spikes decay towards zero |
| ARMA | Spikes decay to zero | Spikes decay to zero |

Spike represents the line at various lags in the plot with length equal to magnitude of autocorrelations.

### 3.6 Model Order Selection

Parametric ARIMA model fitting really involves only one parameter: The model order. The most common approach for model order selection involves selecting a model order that minimizes one or more information criteria evaluated over a range of model orders. The information criteria used in this work include: Akaike Information criterion (AIC), [36], Bayesian information Criterion (SIC), [37], and Hannan-Quinn Criterion (HQC), [38]. Each criterion is a sum of two terms, one that characterizes the entropy rate or prediction error of the model, and a second term that characterizes the number of freely estimated parameters in the model (which increases with increasing model order). By minimizing both terms, we seek to identify a model that is both parsimonious (does not over-fit the data with too many parameters) while also accurately modeling the data. The information criteria are given below:

$$AIC = log\left(\frac{RSS}{n}\right) + \left(2 \times \frac{k}{n}\right) \quad (30)$$

$$SBIC = \log\left(\frac{RSS}{n}\right) + \left(\log(n) \times \frac{k}{n}\right) \quad (31)$$

$$HQC = \log\left(\frac{RSS}{n}\right) + \left(2 \times \log(\log(n)) \times \frac{k}{n}\right) \text{ and} \quad (32)$$

Where n is the number of observations; $k$ is the number of free parameters to be estimated, RSS is the residual sum of squares. Assuming there is a true ARMA model for the time series, the SBIC and HQC have the best theoretical properties. The SBIC is strongly consistent whereas AIC will usually result in an over parameterized model; that is a model with too many AR or MA terms, [39].

**3.7 Estimation of the Model**

At the identification stage one or more models are tentatively chosen that seem to provide statistically adequate representation of the available data. Then we attempt to obtained precise estimates of parameters of the model by the method of least squares as advocated by Box and Jenkins by minimizing the error sum of squares, $\sum \varepsilon_t^2$. For MA models, we write down the covariance matrix of the moving average error and, assuming normality, we use the maximum likelihood method of estimation.

**3.8 Model Diagnostic Checking**

When an AR, MA or ARMA model has been fitted to a given time series, it is advisable to check that the model does really give an adequate description of the data. In doing so, the following diagnostic check is used:

**3.8.1 Plot of residual ACF and PACF:**

Once the appropriate ARIMA model has been fitted, one can examine the goodness of fit by means of plotting the ACF of residuals of the fitted model. If must of the sample autocorrelation coefficients of the residuals are within the limits $\pm 1.96/\sqrt{T}$ where T is the number of observations upon which the model is based then the residuals are white noise indicating that a model is a good fit.

**3.8.2 Jarque-Bera Test for Normality**

The Jarque-Bera test is goodness-of fit test of whether, sample data have the skewness and kurtosis matching a normal distribution. Given a series $\{y_t\}$ the test statistic JB is defined as:

$$JB = \frac{n}{6}\left(S_k^2 + \frac{1}{4}(K-3)^2\right) \quad (33)$$

where $n$ is the number of observations; $S_k$ is the sample skewness, and $K$ is the sample kurtosis;

$$\left.\begin{aligned} S_k &= \frac{\hat{\mu}^3}{\hat{\sigma}^3} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2\right)^{3/2}} \\ K &= \frac{\hat{\mu}^4}{\hat{\sigma}^4} = \frac{\frac{1}{2}\sum_{i=1}^{n}(x_i-\bar{x})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2\right)^2} \end{aligned}\right\} \quad (34)$$

where $\hat{\mu}^3$ and $\hat{\mu}^4$ are the estimates of the third and fourth central moments respectively, $\bar{x}$ is the sample mean, and $\hat{\sigma}^2$ is the estimate of the second central moment, the variance. We test the following hypothesis

$H_0: \hat{\mu}^3 = 0$ and $\widehat{\mu}^4 = 0$ (i.e. $y_t$ is from a normal distribution)

$H_1: \hat{\mu}^3 \neq 0$ and $\widehat{\mu}^4 \neq 0$ (i.e. $y_t$ is not from a normal distribution)

If the data come from a normal distribution, the JB statistic asymptotically has a chi-square distribution with two degrees of freedom, so the statistic can be used to test the hypothesis that the data are from a normal distribution. The null hypothesis is a joint hypothesis of the skewness being zero and the excess kurtosis being zero. Samples from a normal distribution have an expected skewness of 0 and an expected excess kurtosis of 0 (which is the same as a kurtosis of 3). As the definition of JB shows, any deviation from this increases the JB statistic [40].

**Decision Rule:** Accept $H_0$ if $S_k = 0$ and $k = 3$ otherwise reject $H_0$ and conclude that $y_t$ is not from a normal distribution.

**3.8.3 Portmanteau Test for Autocorrelation**

A Portmanteau test is a test used for investigating the presence of autocorrelation in time series. The number of lags k and h are predetermined. The test checks the following pairs of hypotheses:

$H_0$: $\rho_{k,1} = \rho_{k,2} = \cdots = \rho_{k,h} = 0$ (all lags correlations are zero)

$H_1$: $\rho_{k,1} \neq \rho_{k,2} \neq \cdots \neq \rho_{k,h} \neq 0$ (there is at least one lag with non-zero correlation)

The test statistic is given by:

$$Q = n(n+2)\sum_{k=1}^{h}\frac{\hat{\rho}_k^2}{n-k} \qquad (35)$$

where n is the sample size, Q is the sample autocorrelation at lag k and h is the number of lags being tested. Under the statistic Q follows a chi-squared distribution with h degree of freedom, $\chi^2(h)$. We reject $H_0$ if p-value is less than $\alpha = 0.05$ level of significance.

**3.9 Forecasting and Forecast Evaluation**

If the univariate modeling procedure is being utilized for forecasting purposes then this step can also form an important part of the diagnostic checking. Using ARIMA models for forecasting is relatively straightforward.

Suppose that we have estimated the model with n observations. We want to forecast $y_{n+k}$. This is called a k-periods ahead forecast. First we need to write out the expression for $y_{n+k}$. And then replace all future values $y_{n+j}\,(0 < j < k)$ by their forecasts and $\varepsilon_{n+j}\,(j > 0)$ by zero (since its expected value is 0). We also replace all $\varepsilon_{n-j}\,(j \geq 0)$ by the predicted residuals.

**3.9.1 Forecast Evaluation**

Suppose the forecast sample is $j = T+1, T+2, \dots, T+h$, and denote the actual and forecasted value in period $t$ as $y_t$ and $\hat{y}_t$, respectively. The reported forecast error statistics are computed as follows:

$$\text{Root Mean Square Error (RMSE)} = \sqrt{\sum_{t=T+1}^{T+h}(\hat{y}_t - y_t)^2/h}$$

$$\text{Mean Absolute Error(MAE)} = \sum_{t=T+1}^{T+h}|\hat{y}_t - y_t|/h$$

$$\text{Mean Absolute Percentage Error(MAPE)} = 100 \times \sum_{t=T+1}^{T+h}\left|\frac{\hat{y}_t - y_t}{y_t}\right|/h$$

$$\text{Theil Inequality Coefficient(TIC)} = \frac{\sqrt{\sum_{t=T+1}^{T+h}(\hat{y}_t - y_t)^2/h}}{\sqrt{\sum_{t=T+1}^{T+h}\hat{y}_t^{\,2}/h} + \sqrt{\sum_{t=T+1}^{T+h}y_t^{\,2}/h}}$$

The smaller the error, the better the forecasting ability of the model according the criterion. The Theil Inequality Coefficient always lies between zero and one, the zero value indicates a perfect fit.

## 4. Results and Discussion

**4.1 Statistical Properties of the Data**

Descriptive statistics of poverty level in Nigeria is presented in Table 4.1 to aid our understanding of the nature and distributional characteristics of the series. The computed statistics include yearly mean, median, maximum and minimum levels, range, skewness, kurtosis, and Jarque-Bera statistic.

**Table 4.1:** Descriptive Statistics of Poverty Incidence in Nigeria

| Mean | Median | Max. | Min. | Range | Std. Dev. | Skewness | Kurtosis | JB | P-value |
|---|---|---|---|---|---|---|---|---|---|
| 56.15 | 59.30 | 88.00 | 28.10 | 59.90 | 14.57 | -0.012 | 2.36 | 0.6057 | 0.7387 |

The descriptive statistics shows that poverty level in Nigeria is generally higher and much volatile (in absolute terms) in the early 2000's. The maximum and minimum incidences being recorded in 2002 and 1980 respectively. The wide gap between the maximum and minimum poverty level of 59.90% gives a high level of variability of poverty in Nigeria over the period under investigation. The descriptive statistics indicates a yearly mean poverty level of 56.15% with a standard deviation of 14.57% over the period investigated. The poverty distribution which is negatively skewed has a kurtosis value of 2.36. This implies that the distribution has a long left tail, platykurtic and flat relative to a normal distribution. The kurtosis of a normal distribution is 3 while the skewness is zero. The null hypotheses of zero (0) skewness and kurtosis coefficient of 3 are rejected at 1% significance level suggesting that absolute poverty distribution in Nigeria do not follow normal distribution. However, the Jarque-Bera statistic of 0.6057 with its associated p-value of 0.7387 do not reject the null hypothesis of normality of the series.

**4.2 Graphical Properties of the Series**

We also consider the data generating process of the series by examining the time plots of the data as represented by Figure 4.1, Figure 4.2 and Figure 4.3.

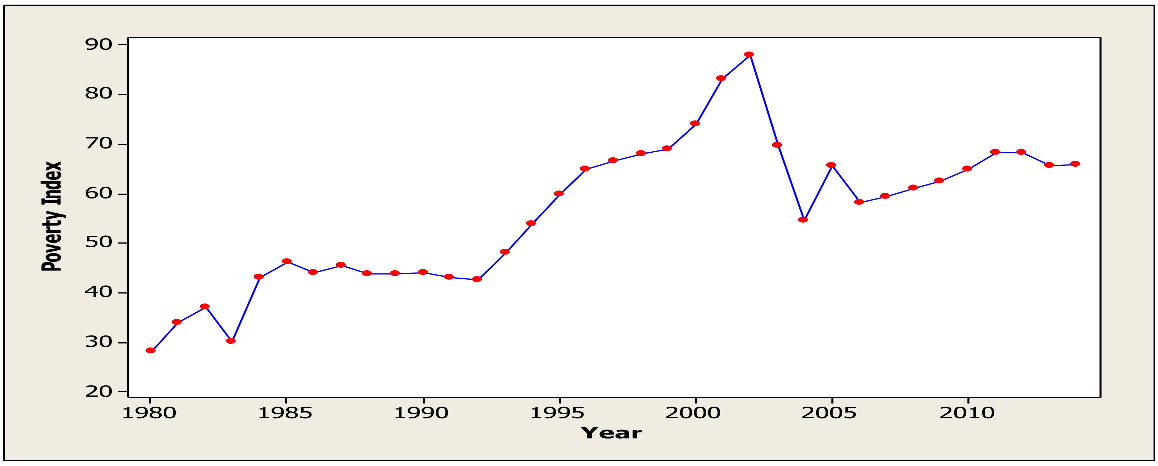


*Figure 4.1: Time Plot of Poverty Incidence in Nigeria in Level*

Figure 4.1 shows the plot of the original data against time. In order to stabilize the variance and mean of the series, we transform the original data to natural logarithms. The time plot below depicts the natural log transform of the series.

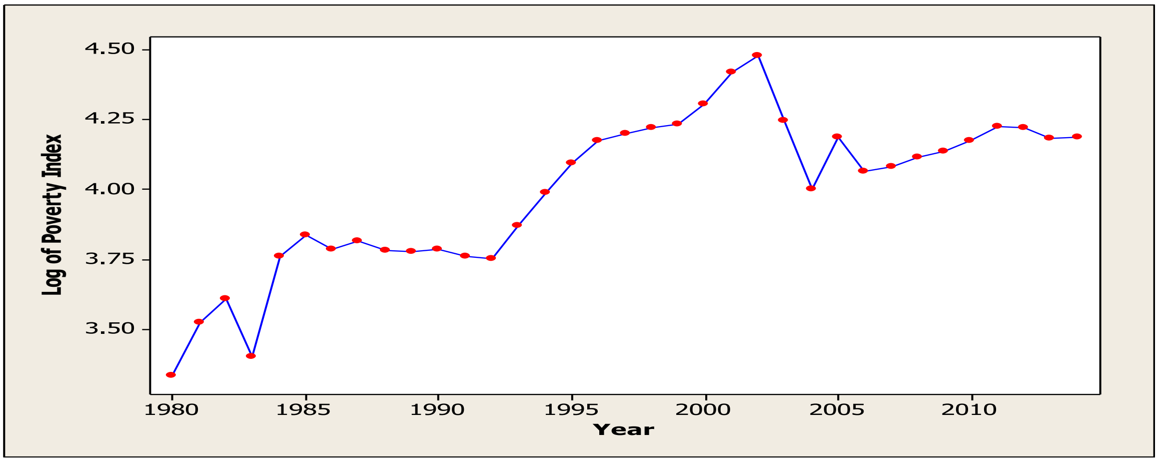


*Figure 4.2: Time Plot of Natural Log of Poverty Incidence in Nigeria*

We observe from Figure 4.2 that the trend in the series is not linear which indicates that the series do not have constant mean and variance. The variability in the series does not appear to be uniform which raises the possibility that the variance is changing with time. These observations suggest that the series is not covariance stationary; hence we take the first difference of the series. The figure below shows time plot of the first difference.

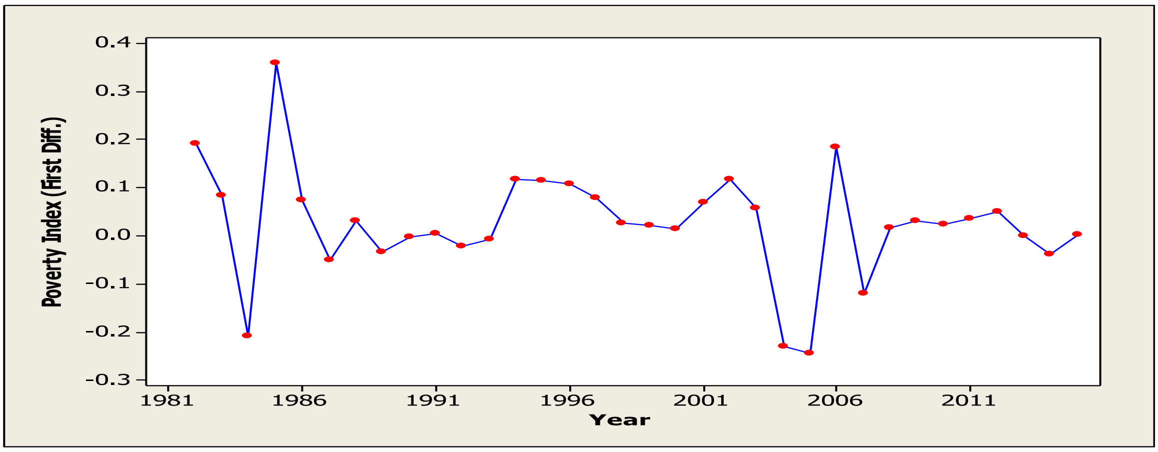


*Figure 4.3: Time Plot of First Difference of Poverty Incidence in Nigeria*

Figure 4.3 depicts the time plot of the first difference of poverty incidence in Nigeria. The plot shows some evidence of stable mean and variance as its values lie within a confidence bound of $\pm 17$. This means that the mean and variance are constant over time, (i.e. homoskedastic). The series also exhibits some gradual rise and fall, which indicates the presence of some degree of autocorrelations.

**4.3 Plots of Correlograms of the Series**

We also examine the plots of the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the series both in level and first difference. The ACF and PACF are the approximate two standard error bounds (the upper confidence limit and the lower confidence limit) computed as $\pm 1.96\sqrt{T}$, where T is the number of observations. If the autocorrelation and partial autocorrelation are within these bounds, it is not significantly different from zero at approximately 5% significance level. And if all values of the data or most of the values fall within these confidence limits, then, the data are independent of time and dependent if otherwise.

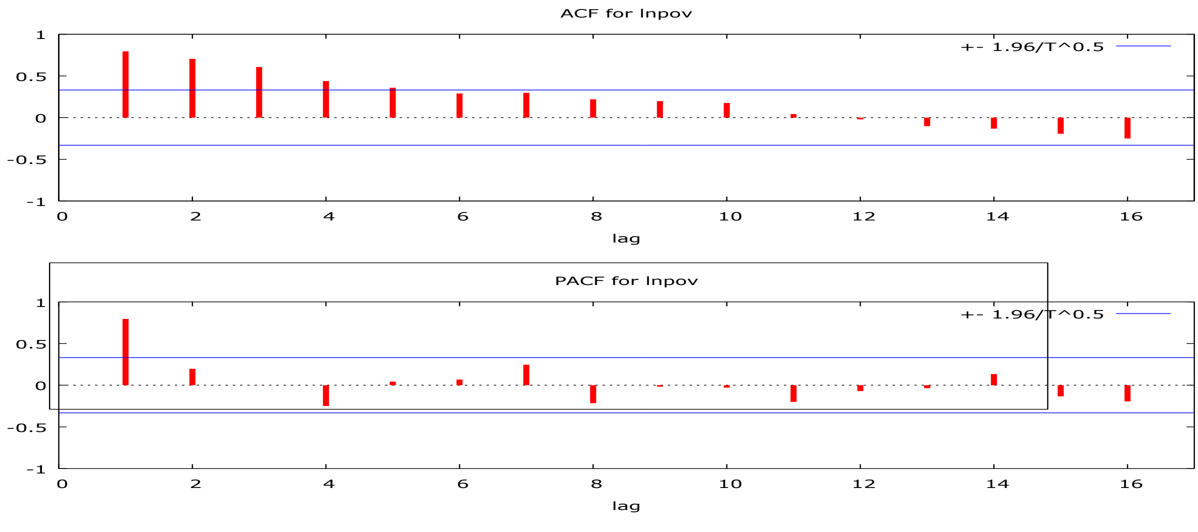


*Figure 4.4: Plot of ACF and PACF of Poverty Incidence in Nigeria (Natural Log)*

From the ACF and PACF plots of figure 4.4, we observe that poverty level in Nigeria is independent of time, i.e. poverty level in a particular year does not depend on the previous year and vice versa. There is also an evidence of non stationarity of the series in level as the correlograms die out only gradually over time. However, the correlograms of the first difference of the series reported in Figure 5 shows some evidence of stationarity as the correlograms decay quite rapidly from its initial value of unity at zero lags.

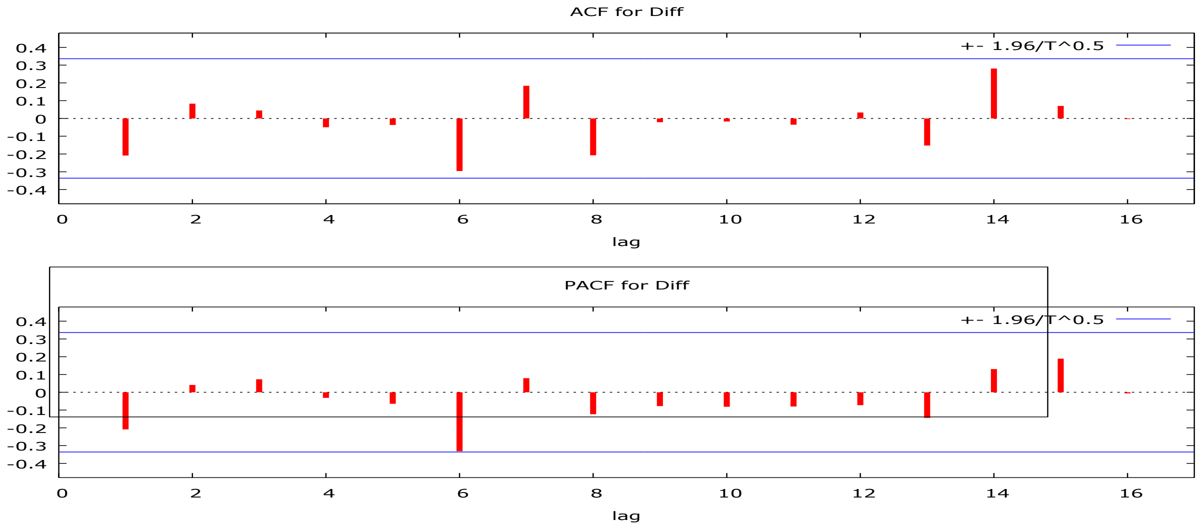


*Figure 4.5: Plot of ACF and PACF of First Diff. of Poverty Incidence in Nigeria*

The Ljung-Box statistics in level of the series reported in Table 4.2 are highly significant at all lags indicating that the series under consideration is non-stationary in level and the residuals are not white noise. The significance of the series also shows how strong the null hypothesis of time dependency at level of the series is rejected. However, the Ljung-Box statistics of the first difference of the series are highly insignificant at all lags indicating that the first difference of the series is stationary and that the residuals are purely random process.

**Table 4.2:** Autocorrelation Functions of Poverty Incidence in Nigeria

| Level Series | | | | First Difference | | |
|---|---|---|---|---|---|---|
| **Lag** | **ACF** | **Std. Error** | **LBQ** | **ACF** | **Std. Error** | **LBQ** |
| 1 | 0.795 | 0.162 | 24.09* | -0.209 | 0.164 | 1.61 |
| 2 | 0.705 | 0.160 | 43.58* | 0.083 | 0.162 | 1.88 |
| 3 | 0.607 | 0.157 | 58.50* | 0.045 | 0.159 | 1.96 |
| 4 | 0.439 | 0.155 | 66.54* | -0.049 | 0.157 | 2.06 |
| 5 | 0.358 | 0.152 | 72.07* | -0.037 | 0.154 | 2.11 |
| 6 | 0.290 | 0.150 | 75.82* | -0.296 | 0.151 | 5.94 |
| 7 | 0.297 | 0.147 | 79.90* | 0.184 | 0.149 | 7.47 |
| 8 | 0.220 | 0.144 | 82.22* | -0.207 | 0.146 | 9.49 |
| 9 | 0.198 | 0.142 | 84.17* | -0.021 | 0.143 | 9.51 |

**Note:** * denotes the significant of LBQ test statistic at 1% marginal significance level

### 4.4 Unit Root and Stationarity Test Results of Poverty Incidence in Nigeria

As a pre-condition for estimation of a model describing the level of poverty in Nigeria and because the order of integration of the series is of great importance for the analysis, we perform the Ng & Perron modified unit root test to determine the order of ordinary integration of the series. The result of the test is reported in Table 4.3.

**Table 4.3:** Summary of Ng & Perron Modified Unit Root Test Result

| Variable | Option | Lag | Ng-Perron Test Statistics | | | |
|---|---|---|---|---|---|---|
| | | | **MZa** | **MZt** | **MSB** | **MPT** |
| $Y$ | C | 2 | -2.4314 | -0.9592 | 0.3945 | 9.2797 |
| | C + T | 4 | -9.9170 | -2.1821 | 0.1850 | 9.3835 |
| $\Delta Y$ | C | 1 | -15.9223*** | -2.8174*** | 0.1723** | 1.5541*** |
| | C + T | 3 | -25.1888** | -3.7555*** | 0.1214*** | 3.0014** |

Asymptotic Critical Values

| Levels | Constant only | | | | Constant + Linear trend | | | |
|---|---|---|---|---|---|---|---|---|
| | MZa | MZt | MSB | MPT | MZa | MZt | MSB | MPT |
| 1% | -13.800 | -2.580 | 0.174 | 1.780 | -23.800 | -3.420 | 0.143 | 4.030 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5% | -8.100 | -1.980 | 0.233 | 3.170 | -17.300 | -2.910 | 0.168 | 5.480 |
| 10% | -5.700 | -1.620 | 0.275 | 4.450 | -14.200 | -2.620 | 0.185 | 6.670 |

**Note:** ** and *** imply significance of the test statistics at 1%, 5% and 1%, 5%, 10% levels for constant only and constant + linear trend respectively for both level and first difference. $C$ denotes constant only and $C+T$ implies constant and linear trend.

The Ng-Perron test was conducted both in level and first difference of the series. The result of the test shows that the series is non stationary in level with both constant only and with constant and linear trend at all conventional test sizes. This is a clear evidence that the series contains a unit root. However, the Ng-Perron modified unit root test of the first difference of the series shows that the series is stationary in its first difference both with constant only and with constant and linear trend at 1%, 5% and 10% significance levels.

We also conducted KPSS stationarity test as a useful confirmatory analysis for Ng-Perron modified unit root test. The KPSS Lagrange Multiplier test evaluates the following hypothesis using equations (27) and (28):

$H_0: \rho < 1$ ( the series is stationary) against $H_1: \rho = 1$ ( the series has a unit root).

The result of KPSS stationarity test is presented in Table 4.4.

**Table 4.4:** KPSS Stationarity Test Results

| Variable | Option | Lag | KPSS Test Statistic | Critical Values | | |
|---|---|---|---|---|---|---|
| | | | | 1% | 5% | 10% |
| Y | Intercept only | 2 | 0.7730 | 0.739 | 0.463 | 0.347 |
| | Intercept & Trend | 5 | 0.3252 | 0.216 | 0.146 | 0.119 |
| ΔY | Intercept only | 1 | 0.1273*** | 0.739 | 0.463 | 0.347 |
| | Intercept & Trend | 3 | 0.0522*** | 0.216 | 0.146 | 0.119 |

**Note:** *** denotes significant of the KPSS test statistic at 1%, 5% and 10% significance levels. ΔY denotes first difference of $Y \cdot C$ denotes constant and $C+T$ denotes constant and trend.

The KPSS test results both with constant only and with constant and linear trend rejects the null hypothesis of level stationarity at all levels of significance. However, in the KPSS stationarity test result of the first difference, we do not reject the null hypothesis at all significance levels because the test statistic $t_\alpha$ here are (0.1273) with constant only and (0.0522) with constant and linear trend which are all less than the critical values for KPSS test at 1%, 5% and 10% significance levels. This confirms the result of Ng-Perron modified unit root test. We therefore conclude that the series under investigation is non-stationary in level but stationary in the first difference and hence integrated of order one, I(1).

### 4.5 Identification of the Model

Having determined the correct order of integration require to render the series stationary, the next step is to find an appropriate process to model the stationary series. By Box-Jenkins procedure of model identification, we observe that the correlogram (ACF and PACF) of the stationary series (first difference of the series) suggests a mixed ARMA process, since the spikes of ACF and PACF both decay to zero as reported in figure 4.5.

### 4.6 Model Order Selection

Having identified an ARMA process for the series, we use the information criteria to determine the optimal values for the ARIMA specification (p,d,q). From the previous results of Ng-Perron modified unit root test in Table 4.3 and KPSS stationarity test in Table 4.4, we discover that $d = 1$. We systematically specify the lag lengths for the AR and MA of the ordinary first integration of the series, we then execute the modeling and record the values of the Akaike information criterion, Schwarz-Bayesian information criterion, and Hannan-Quinn criterion, R-squared, R-squared (adjusted) and Durbin Watson statistic. The execution of the model is repeated for different number of lags following this procedure; we choose using parsimony the model with the least information criteria and highest R-squared. The result is reported in Appendix A.

### 4.7 Estimating the Model

Following the result of Appendix A, ARIMA (4,1,4) seems to provide statistically adequate representation of the given data. After the best model has been chosen, the next thing to do is to estimate the parameters of the model. The result of the parameter estimates of the optimal model is presented in Table 4.5.

**Table 4.5:** OLS Parameter Estimates of ARIMA (4,1,4)

| Variable | Coefficient | Std. Error | t-Statistic | P-value |
|---|---|---|---|---|
| $\alpha_0$ | 0.7702 | 1.1183 | 0.6888 | 0.4985 |
| $\alpha_1$ | -0.2713 | 0.1315 | -2.0632 | 0.0517 |
| $\alpha_2$ | 0.4585 | 0.1308 | 3.5055 | 0.0021 |
| $\alpha_3$ | -0.3504 | 0.1412 | -2.4813 | 0.0216 |
| $\alpha_4$ | -0.5744 | 0.1250 | -4.5933 | 0.0002 |
| $\theta_1$ | 0.4948 | 0.0711 | 6.9643 | 0.0000 |
| $\theta_2$ | -0.5995 | 0.1308 | -4.5821 | 0.0002 |
| $\theta_3$ | 0.4766 | 0.0827 | 5.7598 | 0.0000 |
| $\theta_4$ | 0.9621 | 0.0283 | 34.0462 | 0.0000 |
| R-squared | | | | 0.506937 |
| R-squared Adjusted | | | | 0.319104 |
| Durbin Watson Statistic | | | | 2.061465 |
| F-statistic | 2.698864 | Probability (F-statistic) | | 0.032565 |

From the result of the parameter estimates of Table 5, our data fits an ARIMA (4,1,4) model which is presented thus:

$$\begin{aligned}(1-L)y_t = 0.7702 - 0.2713y_{t-1} + 0.4585y_{t-2} - 0.3504y_{t-3} - 0.5744y_{t-4} + \varepsilon_t \\ +0.4948\varepsilon_{t-1} - 0.5995\varepsilon_{t-2} + 0.4766\varepsilon_{t-3} + 0.9621\varepsilon_{t-4} \qquad (4.1)\end{aligned}$$

where $y_t$ = Poverty response (dependent) variable at time $t$;

$y_{t-1}, y_{t-2}, y_{t-3}, y_{t-4}$ = Poverty response variables at time $t-1, t-2, t-3, t-4$ respectively;

$\varepsilon_t$ = Error term at time $t$;

$\varepsilon_{t-1}, \varepsilon_{t-2}, \varepsilon_{t-3}, \varepsilon_{t-4}$ = Error terms in the previous time periods that are incorporated in the response variable $y_t$.

The result of Table 4.5 shows that the constant parameter $\alpha_0$ is positively related with poverty level and statistically insignificant implying that the predicted value of poverty will be 77.02% if all the explanatory variables are held constant. All the AR and MA coefficients of model are significant at 5 percent levels.

The coefficient of determination ($R^2$) of the regression model is 0.506937 indicating that about 50.69% of the total variations in poverty level have been explained by the regression model while the remaining 49.31% unexplained variations is captured by the error term or by factors not included in the model. The F-statistic is a goodness of fit test which measures the overall significance of the regression parameters. F=2.698864 with a p-value of 0.032565 indicates that the regression model is a good fit. The Durbin Watson statistic value of 2.061465 which is greater than $R^2$ and $R^2$ adjusted means that the model is not spurious. The following subsection contains residual diagnostic check of the estimated ARIMA (4,1,4) model.

**4.8 ARIMA (4,1,4) Model Diagnostic Check**

After fitting the model, we check the model for adequacy. Here we examine the goodness of fit by means of plotting the ACF and PACF of residuals of the fitted model. If most of the sample autocorrelation coefficients of the residuals are within the limits $\pm 1.96/\sqrt{T}$ where T is the number of observations upon which the model is based, then the residuals are white noise indicating that the model is a good fit.

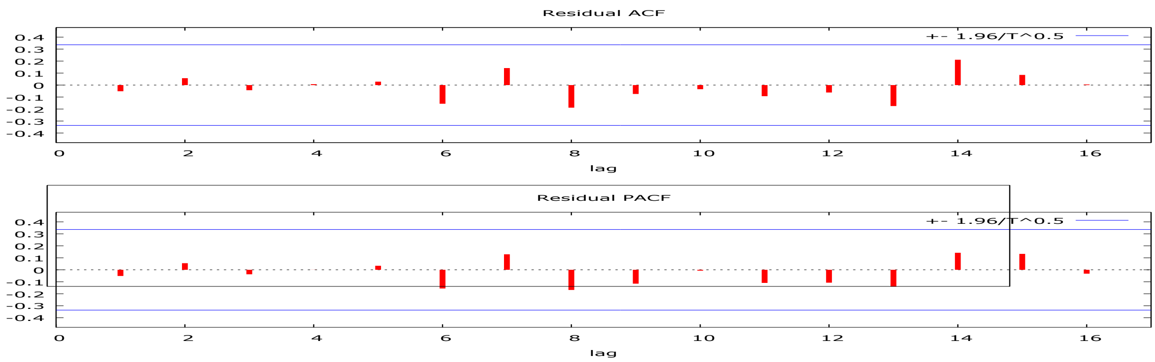


*Figure 4.6: Plot of ACF and PACF of Residuals of ARIMA (4,1,4) against Time*

From Figure 4.6, we observe that the residual ACF and PACF plots of the model are all stationary (all the lag values lying within $\pm 1.96/\sqrt{T}$(i.e. $\pm 0.3578$). That is, the entire lag values lie within the 95% confidence interval. The residual plot against time of the model is approximately white noise indicating that our model is a good fit.

**4.9 Residual Tests of ARIMA (4,1,4)**

We also conducted some tests on the residuals of the fitted ARIMA (4,1,4) model. The result of the test in Table 4.6, reveal that the residuals of the estimated model have satisfied the Jarque Bera test for normality of residuals because the p-value $= 0.1670 > 0.05$. The null hypothesis is that the errors are normally distributed and the decision is to accept the null hypothesis whenever the p-value of the Jarque-Bera test is greater than *0.05*. The residuals have also passed the Bruesch-Godfrey serial correlation lagrange multiplier test because the p-values of F-statistic and $nR^2$ are 0.6414 and 0.5056, which are both greater than 0.05. The null Hypothesis of no serial correlation in the residuals at all lags is accepted since the p-values are greater than 0.05.

**Table 4.6:** Jarque-Bera Test of Normality and LM Serial Correlation Test of Residuals

| Test | Test statistic | P-value |
|---|---|---|
| Jarque-Bera | 3.543956 | 0.1670 |
| F-statistic | 0.454678 | 0.6414 |
| $nR^2$ | 1.364162 | 0.5056 |

**4.10 Portmanteau Test for Autocorrelation of Residuals**

We also conduct Portmanteau test for autocorrelation of residuals of the estimated ARIMA (4,1,4) model and the result is reported in Table 4.7.

**Table 4.7:** Portmanteau Test for Autocorrelation of Residuals

| Lag | ACF | PACF | Q-stat. | P-value |
|---|---|---|---|---|
| 1 | -0.0514 | -0.0514 | 0.0978 | 0.754 |
| 2 | 0.0575 | 0.0550 | 0.2245 | 0.894 |
| 3 | -0.0428 | -0.0374 | 0.2968 | 0.961 |
| 4 | 0.0075 | 0.0006 | 0.2991 | 0.990 |
| 5 | 0.0285 | 0.0336 | 0.3334 | 0.997 |
| 6 | -0.1556 | -0.1560 | 1.3915 | 0.966 |
| 7 | 0.1419 | 0.1293 | 2.3039 | 0.941 |
| 8 | -0.1882 | -0.1680 | 3.9716 | 0.860 |
| 9 | 0.1927 | 0.1652 | 3.9937 | 0.765 |

Since the p-values of Q-statistic reported in Table 4.7 are insignificant at all lags, we accept the null hypothesis that all lags correlations are zero. This indicates the absence of autocorrelation in the estimated model. And since the residuals of our model have passed the diagnostic tests, we validate it as being an adequate and good model.

An adequate, valid and good model should be able to forecast future values of the relevant series. In the following subsection, we will consider the ability of the series to forecast future values.

**4.11 ARIMA (4,1,4) Forecast Evaluation**

After a good ARIMA model has been fitted, we want to see its ability to forecast the relevant time series. The ability to do this will further testify the validity of this model. We are going to use four benchmarks to evaluate this forecast ability.

**Table 4.8:** Result of Forecast Comparison of ARIMA (4,1,4) Model Using Accuracy Measures

| Mode of forecast | RMSE | MAE | MAPE | TIC |
|---|---|---|---|---|
| In-sample | 5.7739 | 3.6951 | 257.0665 | 0.6812 |
| Out-of sample | 3.7540 | 2.6802 | 487.8871 | 0.3993 |

From Table 4.8, we consider the following measures of accuracy: Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Theil Inequality Coefficient (TIC) to compare the In-sample and Out –of sample forecasts performance of the estimated ARIMA (4,1,4) model to evaluate its forecast ability and to decide on which mode of forecast that is better for the model.

We observe that the RMSE, MAE and the MAPE of the out –of sample forecasts are smaller than those of the In-sample forecasts, and the decision is that the smaller the forecast errors, the better the forecasting ability of that model, according to the criterion, our model is good for future forecast. Also, the Theil Inequality Coefficient always lies between 0 and 1, the 0 value indicates a perfect fit. Comparing our In-sample and Out-of sample forecasts using the theil inequality coefficient, the Out-of sample forecast fits more perfectly than that of the In-sample forecast. We therefore conclude that Out-of sample forecast is the best forecast method for this model.

**4.12 Forecast of Absolute Poverty Incidence in Nigeria**

Having selected the out-of sample forecast mode for the series, we use the estimated ARIMA (4,1,4) model to forecast future values of absolute poverty in Nigeria for the period of 7 years. The result of the forecast is presented in Table 4.9.

**Table 4.9:** Forecast of Absolute Poverty Incidence in Nigeria Using ARIMA (4,1,4) Model

| Year | Forecast (in log of 1st diff.) | Actual Forecast (%) | Std. Error (in log of 1st diff.) | 95% Interval (in log of 1st diff.) |
|---|---|---|---|---|
| 2015 | 4.20037 | 66.71 | 0.111101 | [3.98262, 4.41812] |
| 2016 | 4.25671 | 70.58 | 0.127472 | [4.00687, 4.50655] |
| 2017 | 4.30436 | 74.01 | 0.145211 | [4.01975, 4.58897] |
| 2018 | 4.35710 | 78.03 | 0.163625 | [4.03640, 4.67779] |
| 2019 | 4.35707 | 78.03 | 0.165738 | [4.03223, 4.68191] |
| 2020 | 4.40556 | 81.90 | 0.166757 | [4.07872, 4.73240] |
| 2021 | 4.43418 | 84.28 | 0.169989 | [4.10101, 4.76735] |

**Note:** For 95% confidence intervals, $Z_{0.025} = 1.96$

Table 4.9 shows the forecasts of absolute poverty in Nigeria from 2015 to 2021 using the fitted ARIMA (4,1,4) Model. The forecast shows a linear growth of absolute poverty in Nigeria over the forecast period. According to our forecast 66.71% of Nigeria's population will be in absolute poverty by the year 2015. While 70.58%, 74.01%, 78.03%, 78.03% and 81.90% of the country's population is predicted to be in absolute poverty by the year 2016, 2017, 2018, 2019 and 2020. The highest poverty level is predicted to occur in 2021 which is 84.28%. This shows a steady increase in the level of absolute poverty in Nigeria, (See Figure 4.7).

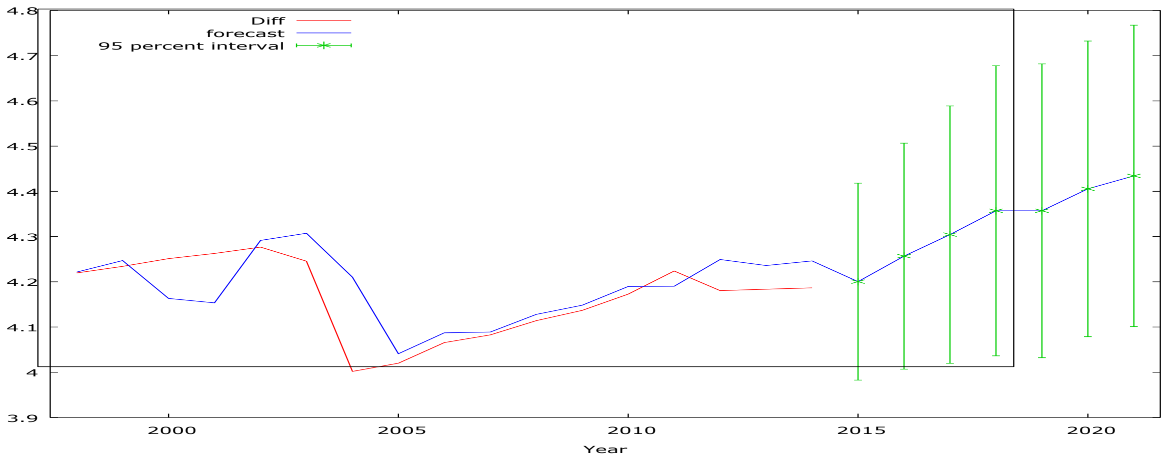


*Figure 4.7: Plot of Forecast Values of Poverty Level in Nigeria in Log of First Difference.*

**4.13 Economic Implication of the Model**

The model equation represented by equation (4.1) has some economic implications. The order of integration which led to the estimation of the equation indicates linear growth behaviour in the series. This suggests that if Nigeria does not intensify effort in her fight against poverty, poverty will continue to grow over time. The estimate of the coefficient of $\varepsilon_{t-1}$ in the model equation which represents the error term in the previous time period incorporated in the poverty response variable $y_t$, is positive and significant at 99 percent level. This implies that the poverty response variable of $\varepsilon_{t-1}$ associated with the error term of the previous year (i.e. at time

$t-1$) is positive. The statistical significant coefficient of $\varepsilon_{t-1}$ confirms a bi-directional relationship between $\varepsilon_{t-1}$ and $y_t$. All the estimated coefficients of the AR and MA terms of the model are highly statistically significant.

The positive (negative) and significant relationship between the AR and MA terms shows that they are interdependent. We, conclude, therefore that over the period 1980-2014, poverty had a negative effect on the people of Nigeria. The linear equation is justified by the significant reverse relationship between the dependent and some explanatory variables in the model.

## 5. Conclusion

Poverty in Nigeria is explained by the combined factors of inadequate food supply, low incomes and the inability to acquire the basic necessities of life such as shelter, cloths, good health care, quality education, clean water and sanitation, inadequate physical security, lack of voice, and insufficient capacity and opportunity to better one's life. It means susceptibility to violence, and implies living in marginal or fragile environments. In this paper, we focused on the aspect of poverty that relate to the basic necessities of life (absolute poverty).

This paper is an attempt to search for an optimal Autoregressive Integrated Moving Average model that best forecast absolute poverty incidence in Nigeria. The study employed absolute poverty data in Nigeria for 35 years from January, 1980 to December, 2014. The data was obtained as secondary data from Central Bank of Nigeria, Federal Office of Statistics, National Bureau of Statistics and International Monetary Fund World Economic Outlook. Time series plots, Ng & Perron modified unit root test and KPSS stationarity test were used to investigate the graphical and statistical properties of the series. The results indicate that the series is non-stationary in level but stationary in the first difference implying that it is integrated of order one, I(1). The ACF and PACF plots of the stationary series suggest a mix ARMA (p,q) model for the series. ARIMA (p,d,q) model in line with Box-Jenkins procedure were then employed to model the poverty time series data. The result shows that ARIMA (4,1,4) was the best candidate to model poverty incidence in Nigeria. It was generally observed from the tests of residuals of the modeled equation that, the model was good, valid and adequate in describing absolute poverty situation in Nigeria. Accuracy measures such as Root Mean Square Error, Mean Absolute Error, Mean Absolute Percentage Error and Theil Inequality Coefficient were used to evaluate the forecast ability of the model and an out-of sample forecast mode was best for the model. The modeled ARIMA (4,1,4) was then used to forecast future poverty values in Nigeria. The forecast indicates a linear growth in poverty level in Nigeria meaning that poverty in Nigeria will continue to grow over time.

## 6. Recommendations

Based on the findings of this paper, the following recommendations are suggested to help in reducing poverty level in Nigeria:

1. The order of integration which led to the estimation of the model equation indicates linear growth behaviour in poverty level in Nigeria. This indicates that poverty level will continue to increase in Nigeria if nothing is done to reduce it. Therefore, government of Nigeria should intensify her fight against poverty in order to reduce the trend.
2. Corruption has to be reduced to the barest minimum and government's anti-poverty programmes /Campaigns should basically target the poor in rural areas in order to reduce the menace and aversion of poverty.
3. Educational system and programmes in Nigeria should be redirected towards functionality, entrepreneurial, vocational and technical to enhance self-employed, self-reliance and reduce poverty in the country.
4. To reduce poverty in Nigeria, economic growth has to be favourable for the poor class of the society. Resources should be directed to those sectors like agriculture where majority of the poor lives.
5. The poor should be actively involved in the formulation, implementation and monitoring of poverty alleviation or reduction programmes in Nigeria. Why government anti-poverty initiatives fail in Nigeria is because the target people (the poor) are not involved in the implementation and monitoring of the programmes.
6. Government should increase the number of financial institutions like commercial banks and moderate their lending and borrowing interest rates to avail the poor access to loan and credit facilities in order to alleviate poverty and help them engage in productive ventures.

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**APPENDIX**

**APPENDIX A: Model Order Selection Using Information Criteria**

| S/N | Model | Information Criterion | | | $R^2$% | $\bar{R}^2$% | DW Stat |
|---|---|---|---|---|---|---|---|
| | | AIC | SBIC | HQC | | | |
| 1 | ARIMA (0,1,1) | 6.3744 | 6.4641 | 6.4050 | 3.96 | 0.83 | 2.019 |
| 2 | ARIMA (0,1,2) | 6.4203 | 6.5550 | 6.4662 | 5.13 | -1.00 | 1.964 |
| 3 | ARIMA (0,1,3) | 6.4705 | 6.6501 | 6.5318 | 5.91 | -3.52 | 2.002 |
| 4 | ARIMA (0,1,4) | 6.4439 | 6.6684 | 6.5204 | 13.6 | 1.73 | 1.842 |
| 5 | ARIMA (1,1,0) | 6.3942 | 6.4849 | 6.4247 | 4.47 | 1.38 | 1.991 |
| 6 | ARIMA (1,1,1) | 6.4548 | 6.5909 | 6.5006 | 4.42 | -0.20 | 1.991 |
| 7 | ARIMA (1,1,2) | 6.5040 | 6.6854 | 6.5650 | 5.58 | -4.32 | 1.988 |
| 7 | ARIMA (1,1,3) | 6.5363 | 6.7630 | 6.6126 | 8.17 | -5.00 | 2.008 |
| 8 | ARIMA (1,1,4) | 6.5629 | 6.7924 | 6.6221 | 9.33 | -6.13 | 2.014 |
| 9 | ARIMA (2,1,0) | 6.4825 | 6.6199 | 6.5281 | 4.84 | -1.71 | 1.908 |
| 10 | ARIMA (2,1,1) | 6.4696 | 6.6529 | 6.5304 | 11.72 | 2.33 | 1.905 |
| 11 | ARIMA (2,1,2) | 6.0912 | 6.3202 | 6.2187 | 43.36 | 34.87 | 1.772 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 12 | ARIMA (2,1,3) | 6.2308 | 6.5056 | 6.3219 | 38.71 | 26.93 | 1.985 |
| 13 | ARIMA (2,1,4) | 6.2328 | 6.5535 | 6.3391 | 42.30 | 28.42 | 1.997 |
| 14 | ARIMA (3,1,0) | 6.5169 | 6.7019 | 6.5772 | 5.48 | -5.11 | 1.721 |
| 15 | ARIMA (3,1,1) | 6.5167 | 6.7479 | 6.5920 | 11.37 | -2.37 | 1.922 |
| 16 | ARIMA (3,1,2) | 6.5632 | 6.8407 | 6.6536 | 12.96 | -4.51 | 1.988 |
| 17 | ARIMA (3,1,3) | 6.2492 | 6.5731 | 6.3548 | 40.37 | 25.40 | 1.843 |
| 18 | ARIMA (3,1,4) | 6.3751 | 6.7453 | 6.4959 | 36.62 | 17.25 | 1.728 |
| 19 | ARIMA (4,1,0) | 6.4986 | 6.7322 | 6.5734 | 2.52 | -13.17 | 2.057 |
| 20 | ARIMA (4,1,1) | 6.5566 | 6.8369 | 6.6463 | 3.31 | -16.84 | 2.006 |
| 21 | ARIMA (4,1,2) | 6.6218 | 6.9488 | 6.7264 | 3.58 | -21.70 | 1.992 |
| 22 | ARIMA (4,1,3) | 6.4753 | 6.8489 | 6.5948 | 22.00 | -2.72 | 1.992 |
| 23 | ARIMA (4,1,4)** | 6.0835 | 6.3039 | 6.2160 | 50.69 | 31.93 | 2.061 |
| 24 | ARIMA (5,1,0) | 6.6012 | 6.8841 | 6.6898 | 3.61 | -17.38 | 2.041 |
| 25 | ARIMA (5,1,1) | 6.5410 | 6.8710 | 6.6444 | 15.37 | -7.81 | 2.011 |
| 26 | ARIMA (5,1,2) | 6.3129 | 6.6901 | 6.4310 | 37.11 | 16.18 | 2.280 |
| 27 | ARIMA (5,1,3) | 6.4986 | 6.9229 | 6.6315 | 29.34 | 1.01 | 2.074 |
| 28 | ARIMA (5,1,4) | 6.6438 | 7.1152 | 6.7914 | 23.70 | -12.53 | 2.059 |
| 29 | ARIMA (5,1,5) | 6.3755 | 6.8941 | 6.5379 | 45.55 | 15.38 | 2.454 |

**Note:** ** implies the model with the least information criteria, and highest R-squared, $\bar{R}^2$ means R-squared adjusted, DW Stat means Durbin-Watson Statistic.