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Probabilistic Settlement Analysis of Rafts using First Order Reliability Method

BENNET KURIAKOSE, ANJU KRISHNAN, G R DODAGOUDAR AND B NAGESWARA RAO
Indian Institute of Technology Madras, Chennai, 600036, Tamil Nadu, INDIA
Email: shemmashan@gmail.com, anju.krishnan08@gmail.com, goudar@iitm.ac.in, bnrao@iitm.ac.in

Abstract: Settlement of foundations is of crucial concern as it affects the strength and durability of the superstructure as well as the foundation itself. Traditional settlement analysis methodologies cease to incorporate uncertainties associated with design parameters of the foundation. Probability-based settlement analysis of raft foundations accounting for the uncertainties in design variables is presented in this paper. Probabilistic soil-structure interaction model of the raft is developed duly considering the variability involved in various design parameters. The first order reliability method (FORM) is used to evaluate the probability of failure and reliability index associated with the settlement of the raft foundations. Parametric sensitivity analysis is also carried out to identify the important design variables that affect the performance of the raft foundation in connection with the settlement limit state. The reliability analysis of the raft employing the finite element method is also presented.

Keywords: *Raft foundation, Soil-structure interaction, Uncertainties, FORM, Probability of failure, Reliability index*

Introduction

Uncertainties are unavoidable in civil engineering problems and the scatter of associated structural and geotechnical parameters from their nominal ideal values is ineluctable. Traditionally, raft design has been carried out with pressure distributions from the soil evaluated based on the load, accounting for soil-structure interaction. The factor of safety value based on experience and engineering judgment is normally assumed to be reasonable for accounting the uncertainties in the design variables. However, the factor of safety is usually kept as of single value, without considering the varying degree of uncertainty in colligation with the various design parameters of the structure. These uncertainties in the design variables can be rationally accounted using a probabilistic approach, known as reliability analysis. The results of reliability analysis are used in reliability-based design (RBD) codes to deal rationally, the uncertainties linked with the loads, material properties, computational models, etc.

Attempts have been made by various researchers to account for the uncertainties in soil parameters in a more rational manner using probability theory (1, 2). Reliability analysis of a circular raft was carried out by Melerski (3) considering the randomness of the raft material as well as the soil medium. Chang (4) attempted statistical analysis of a circular plate supported on a random Winkler soil. Probabilistic settlement analysis of shallow foundations resting on layered subsoil was performed by Brząkała and Puła (5), incorporating the shape of the subsoil, the material parameters and loads as random variables. Bauer and Puła (6) carried out the reliability analysis of strip foundation based on settlements considering Young's modulus and Poisson's ratio of the soil as random parameters. Babu and Srivastava (7) incorporated variability in different soil parameters in reliability assessment of shallow strip foundation considering the limit states of bearing capacity and settlement using response surface method. The effects of different types of probability distributions of the variables in shallow foundation settlements were investigated by Jimenez and Sitar (8). Probability analysis of strip foundations at the ultimate limit state subjected to inclined loading was studied by Soubra and Mao (9). Probability based settlement analysis of strip and circular footings resting on granular soils were presented by Dodagoudar and Shyamala (10). Sudret and Der Kiureghian (11) presented different reliability approaches using finite element method (FEM). Similiarly, many other researchers have used the FEM accounting the probability theory to analyse the response of shallow foundations (8, 10, 12, 13, 14, 15, 16).

Although many studies have been attempted in the field of probabilistic analysis of different types of foundations, very few studies have been reported on the reliability analysis of raft foundations. The objective of this study is to present an approach for assessing the reliability of raft foundation against settlement using first order reliability method (FORM), considering soil-structure interaction (SSI). The soil properties are considered as uniformly random with mean values, standard deviations and their distributions. The modulus of subgrade reaction of the soil, Young's modulus of the raft and the load are taken as random variables in the probabilistic

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analysis of the raft. Sensitivity analysis is also carried out to identify the important design parameters and their influence on the settlement reliability of the raft foundation. Finite element reliability analysis is also attempted considering the elastic modulus and Poisson's ratio of the soil as well as the load on the raft as random variables. The information obtained from the reliability analysis can be used in the RBD of rafts.

Interaction Analysis of Raft

Structural design of foundations is usually carried out by determining the forces acting on the foundations without considering their deformations. In SSI, the combined action of the structural foundation members and the supporting soil are taken into account. In the analysis of structural elements resting on a continuous medium such as footings, raft foundation, concrete pavements, floor systems of industrial yards, etc., the problem is usually simplified as a plate supported on an elastic foundation. The behaviour of plates on elastic foundation represents a complex SSI problem.

An analytical description of the interaction problem related to the finite plate should take into account the following factors: the type of plate, type of soil medium, type of boundary conditions and type of external loading. Voyiadjis and Kattan (17) presented the analysis of simply supported thick rectangular plate on elastic foundation under uniformly distributed load. A refined theory for moderately thick plates which incorporates the transverse normal strain effect in addition to the transverse shear and normal stress effects is used in their study. The governing differential equation of the plate on elastic foundation is given as (17).

$$\left\{\left(\frac{h^2}{10}D\right)\nabla^6-\left[D+\left(\frac{h^2}{10}\right)^2k\left(\frac{2-\nu_p}{1-\nu_p}\right)\right]\nabla^4+\left[\left(\frac{h^2}{10}\right)k\left(\frac{3-2\nu_p}{1-\nu_p}\right)\right]\nabla^2-k\right\}w=$$
$$-\left\{\left[\left(\frac{h^2}{10}\right)^2\left(\frac{2-\nu_p}{1-\nu_p}\right)\right]\nabla^4-\left[\left(\frac{h^2}{10}\right)\left(\frac{3-2\nu_p}{1-\nu_p}\right)\right]\nabla^2+1\right\}p \quad (1)$$

where, k is the modulus of subgrade reaction, w is the transverse displacement of the plate and h is the thickness of the plate, ν_p is the Poisson's ratio of the plate material, p is the load and the plate rigidity is defined as $D=E_p\,h^3/12\left(1-\nu_p{}^2\right)$ with E_p being the Young's modulus of the plate material.

Assuming a uniformly distributed load and expanding it in a double Fourier series:

$$p(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}a_{mn}\sin\alpha_m x\,\sin\beta_n y \quad (2)$$

where, $\alpha_m=\frac{m\pi}{a}$; $\beta_n=\frac{n\pi}{b}$; and $a_{mn}=\frac{16p_0}{\pi^2 mn}$ in which a and b are the length and width of the plate respectively. Assuming a Navier-type solution for rectangular plates simply supported at all the edges, the transverse displacement can be of the form:

$$w(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}w_{mn}\sin\alpha_m x\sin\beta_n y \quad (3)$$

The assumed form for w given by Eq. (3) satisfies the boundary conditions of simply supported edge conditions at all the four sides. Substituting Eqs. (2) and (3) in Eq. (1), the expression for w_{mn} becomes

$$w_{mn}=\frac{\alpha}{\beta} \quad (4)$$

where,

$$\alpha=a_{mn}\left[\begin{array}{l}-\frac{(2-\nu_p)h^4}{100(1-\nu_p)}(\alpha_m{}^4+2\alpha_m{}^2\beta_n{}^2+\beta_n{}^4)+\\ \frac{(3-2\nu_p)h^2}{10(1-\nu_p)}(-\alpha_m{}^2-\beta_n{}^2)-1\end{array}\right] \quad (5)$$

and,

$$\beta=\frac{Dh^2}{10}(-\alpha_m{}^6-3\alpha_m{}^4\beta_n{}^2-3\alpha_m{}^2\beta_n{}^4-\beta_n{}^6)-\left(D+\frac{kh^4}{100}\frac{2-\nu_p}{1-\nu_p}\right)(\alpha_m{}^4+2\alpha_m{}^2\beta_n{}^2+\beta_n{}^4)+\frac{kh^2}{10}\left(\frac{3-2\nu_p}{1-\nu_p}\right)(-\alpha_m{}^2-\beta_n{}^2)-k \quad (6)$$

Reliability Analysis

Uncertainty and reliability have a long history in geotechnical engineering practice (18, 19). The main contributions of uncertainty in the performance of the structure can arise from the soil properties, geometry and magnitude of the loads. Apart from these, the errors in in-situ tests and errors in laboratory tests also contribute to the overall uncertainty in the analysis. In order to deal rationally with these uncertainties in analysis and design, several reliability-based analysis and design approaches have been developed for geotechnical structures.

The term 'reliability' of a structural system may be defined as the probability of satisfactory performance under the given environmental conditions. The reliability analysis consists of two main steps: (i) identification and analysis of uncertainties of each of the contributing factors; and (ii) combining the uncertainties of the random variables to determine the overall reliability of the system. The probabilistic reliability analysis provides more meaningful results than a deterministic analysis since the former incorporates uncertainties explicitly in the analysis. This enhances the ability of the geotechnical engineer to make informed decisions regarding the acceptability of designs.

The performance function associated with settlement failure of the raft can be expressed as function of several random variables, $\mathbf{x}=(X_1, X_2, \ldots, X_n)$ as

$$g(\mathbf{x})=u_{all}-u(\mathbf{x}) \quad (7)$$

where, u_{all} stands for the maximum allowable settlement and u is the evaluated settlement at any point of the raft. The condition $g(\mathbf{x}) < 0$ implies failure, while $g(\mathbf{x}) > 0$ connotes acceptable performance. The hypersurface defined by $g(\mathbf{x}) = 0$ separating the stable and failure states is called the limit state.

Mathematically, the probability of failure (P_f) can be determined by constructing a probability density function (PDF) on the performance function $g(\mathbf{x})$ and calculating the area under the limit state surface. The probability of failureis defined as

$$P_f = \int_{g(\mathbf{x})<0} f_{\mathbf{x}}(\mathbf{x})d\mathbf{x} \quad (8)$$

where, $f_X(\mathbf{x})$ is the multidimensional joint probability density function of all the basic random variables. Reliability is the complement of the probability of failure associated with a particular performance function, $f_X(\mathbf{x})$. The statistical data for the basic variables is generally limited in practice to second order statistics (i.e., mean, μ and standard deviation, σ), and the correlations among the variables are also not well known. The reliability of a system is contemplated with 'reliability index' (β) which is defined as

$$\beta = \frac{\mu_g}{\sigma_g} \quad (9)$$

If the performance function is normally distributed and is a linear function of the random variables that are normally distributed, whereby the probability of failure is related to reliability index as

$$P_f = \Phi(-\beta) \quad (10)$$

Baecher and Christian (19) and Phoon (20) recommended a target reliability index varying from 2.5 to 4 for various geotechnical structures based on extensive reliability calibrations with existing designs for different limit states.

First Order Reliability Method (FORM)

Even though the reliability theory has potential value, it has not been used in routine geotechnical engineering practice. In this study, an attempt has been made to present the practical geotechnical reliability analysis methodology for settlement analysis of the raft foundation using First Order Reliability Method (FORM). The FORM can be applied to linear or nonlinear limit state functions of correlated or uncorrelated, normal or nonnormal random variables (18). In the FORM, the limit state is linearised at the most probable failure point (MPP) rather than at the mean values of the random variables. The Taylor series expansion of a general nonlinear function $g(X_1, X_2, \ldots, X_n)$ at the most probable failure points ($x_1^*, x_2^*, \ldots, x_n^*$) is

$$g(X_1, X_2, ..., X_n) = g(x_1^*, x_2^*, ..., x_n^*) + \sum_{i=1}^{n}(x_i - x_i^*)\frac{\partial g}{\partial X_i} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(x_i - x_i^*)(x_j - x_j^*)\frac{\partial^2 g}{\partial X_i X_j} + ... \quad (11)$$

The problem of lack of invariance associated with other reliability methods like First Order Second Moment (FOSM) method is resolved in FORM which is accomplished by transforming the X variables into an equivalent set of uncorrelated standard unit normal (U) variables. In this transformed "U – space", the reliability index β is given by the shortest distance from the origin to the surface defining the failure function, $g(U)$ as depicted in Fig. 1. The point of intersection of this line with the failure surface is called as design point. The nonlinear failure function can be conveniently approximated by its tangent plane at the design point. For the detailed Hasofer-Lind procedure, the reader may refer to Ditlevsen and Madsen (21) and Choi et al. (18).

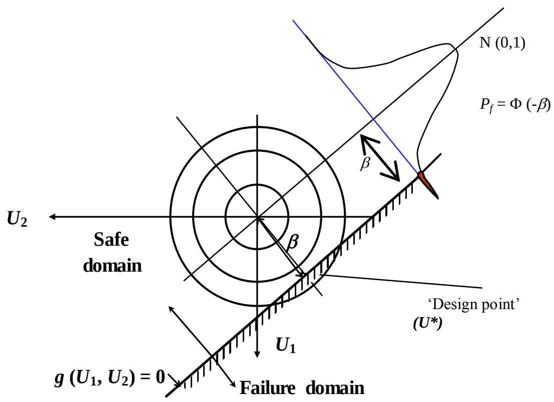

***Fig. 1**. Depiction of Hasofer-Lind reliability index for two-variable case*

In some of the cases the derivatives of the performance function $g(\mathbf{x})$ with respect to the random variables $\mathbf{x}$ required to search the minimum distance point on the limit state are not readily available. In such cases, response surface method (RSM) is used to approximate the performance function by a polynomial expression. In RSM, a polynomial is constructed to obtain an approximate performance function $g'(\mathbf{x})$ through a few selected simulations in the neighbourhood of the most likely failure point. Quadratic polynomials are shown to be suitable for the localised approximation of the response variables in geotechnical engineering and the form of the polynomial is given as

$$g'(\mathbf{x}) = a + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n} c_i X_i^{\ 2} \quad (12)$$

where, a, b and c are the coefficients to be determined and n is the number of random variables considered in the analysis.

Numerical Examples

In order to demonstrate the applicability of the FORM, a computer program in MATLAB environment has been developed and implemented using a numerical example. Probability based settlement analysis methodology of raft is also developed and demonstrated employing the FEM. Results of the present study have highlighted the practical applicability of the reliability method in geotechnical engineering practice.

Interaction Analysis of Raft Foundation

A square raft foundation of plan dimensions 20 m × 20 m subjected to a uniform load (p) of 200 kN/m^2 resting on a soil layer of modulus of subgrade reaction (k) of 24 × 10^3 kN/m^3, is considered. It is assumed that the edges of the raft have firm contact with the soil. The Young's modulus of the raft (E_p) is taken as 20 ×10^6 kN/m^2 and the Poisson's ratio (ν_p) as 0.2. A computer implementation of the analytical formulation given by Voyiadjis and Kattan (17) is made in MATLAB. The settlement of the plate at the centre is evaluated for different values of plate thickness to width ratio (h/a) and is depicted in Fig. 2.

Reliability Analysis of Raft

A raft of dimensions 20 m × 20 m × 1 m with all the other properties retained as in the preceding section is considered for reliability analysis. Foundation settlement, if excessive, can lead to cracking of the structural and nonstructural elements of the supported building. For this reason, most geotechnical design codes limit the settlement of footings, typically 25 to 50 mm. The allowable settlement (u_{all}) is taken as 20 mm in the present study. It is considered that, the settlement u is a function of three random variables (i.e., k, E_p and p) along with the other parameters which will take single values and the correlation between the random variables is neglected. The statistical properties of the random variables considered are given in Table 1. The reliability analysis of the raft foundation has been carried out for the settlement limit state using the FORM.

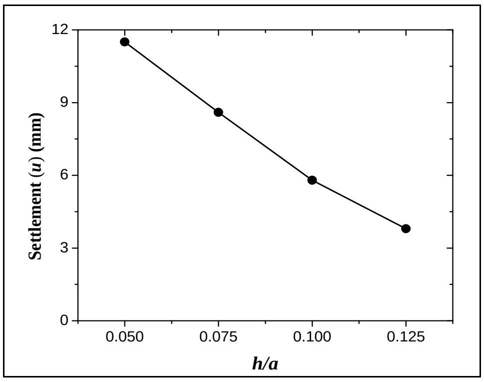

Fig. 2. *Central deflection of the raft (20 m× 20 m)*

The regression analysis is carried out to obtain the approximate performance function (i.e., response surface) for the settlement limit state and is given as

$$g'(\mathbf{x}) = 0.02015745 - 0.00000111011\,X_1 - 0.00000000010888\,X_2 + 0.000057952\,X_3 + 0.00000000001456\,X_1^2 - 0.0000000011\,X_3^2 \quad (13)$$

Table-1. *Statistical properties of design variables of the raft (20 m× 20 m× 1 m)*

Parameter	Distribution	Mean value(μ)	COV (%)
Modulus of subgrade reaction, k (kN/m^3)	Normal	24 × 10^3	10
Modulus of elasticity of raft, E_p (kN/m^2)	Normal	20 × 10^6	10
Load, p (kN/m^2)	Normal	200	10

where, [X_1, X_2, X_3] correspond to [k, E_p, p]. The reliability index is obtained as 4.96, which is more than the target reliability index 4. The corresponding probability of failure is 3.476 × 10^{-7}. In order to have satisfactory performance of the raft foundation, a target reliability index of 4 is usually considered in geotechnical engineering practice. Based on the present reliability analysis, it is noted that the obtained reliability index value is higher than the target reliability index and the performance of the raft chosen for the analysis is safe and satisfactory.

Parametric Sensitivity Analysis

Parametric sensitivity analysis is an important aspect of modern reliability analysis. By carrying out sensitivity analysis, one can calculate the susceptibility of the estimated failure probability to the changes in the input random variables.

Sensitivity of reliability with k

The coefficients of variation of the modulus of elasticity of the raft and the load are taken as COV(E_p) = 0.1 and COV(p) = 0.1, respectively. In order to perform the sensitivity analysis, different values of k are selected by taking COV(k) as 0.1, 0.2 and 0.3. The settlement reliability analysis of the raft is carried out considering the allowable settlements of 20 mm, 30 mm and 40 mm and the results are shown in Fig. 3. These allowable settlements specify the stringent requirements as given in code provisions for different structures which house precision instruments for control applications.

From Fig. 3, it can be observed that the variation in the modulus of subgrade reaction affects drastically the performance of the raft. For a lower variation of COV(k) i.e., 10 – 20%, the performance of the raft is affected in terms of the rapid reduction in the value of the reliability index, i.e., the rate of decrease of reliability index is more. It is obvious that the reliability index is much low for the higher values of COV(k).

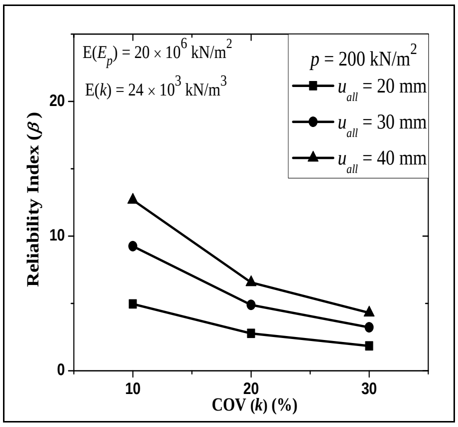

Fig. 3*. Variation of reliability index with modulus of subgrade reaction (k)*

Sensitivity of reliability with E_p

The parametric sensitivity analysis has also been carried out considering variation in E_p. The coefficients of variation of the modulus of subgrade reaction of the soil and the load are taken as constants [i.e., COV(k) = 0.1 and COV(p) = 0.1]. The variation considered for the COV(E_p) as 0.1, 0.2 and 0.3. In reality, a higher variation in E_p, i.e., COV(E_p) = 0.3 is rarely encountered, however, this value is chosen just for academic interest. Such a high value is usually not acceptable in practice because of the stringent codal requirements on the quality of the concrete. The results of the reliability analysis are shown in Fig. 4. From the results, it is evident that there is a slight variation in the reliability index with the variation in the value of the modulus of elasticity of the raft. For the given thickness of the raft, it can be concluded that the settlement limit state is less sensitive to the variation in the values of the modulus of elasticity of the raft. It can be noted that the modulus of elasticity of the raft can be treated as deterministic one for the purpose of further analyses, if any.

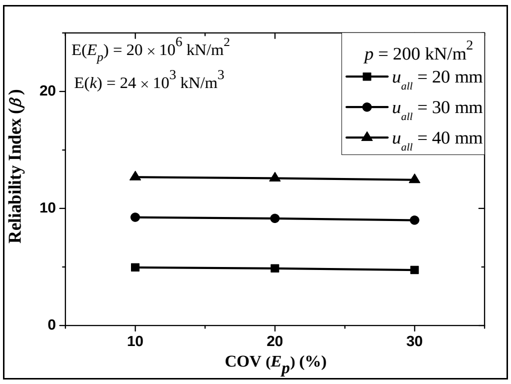

Fig.4*.Variation of reliability index with Young's modulus of the raft (E_p)*

Sensitivity of reliability with *p*

The applied loading (p) has also been considered as one of the random variables affecting the overall performance of the raft. The modulus of elasticity of the raft and modulus of subgrade reaction of the soil are taken as constants [i.e., COV (k) = 0.1 and COV (E_p) = 0.1]. The COV(p) is taken as 0.1, 0.2 and 0.3. The results of the reliability analyses are shown in Fig. 5. It is noted from the figure that the variation in the value of the applied loading affects the performance of the raft as expected, especially when the COV(p) ranges from 10% - 20%. Whenever the COV(p) is more than 20%, it eventually affects the performance of the raft. For satisfactory performance of the raft foundations, correct evaluation of the variation of the applied loading is mandatory.

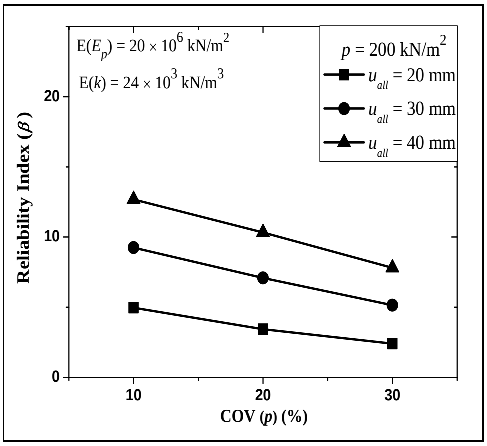

Fig.5*. Variation of reliability index with intensity of the applied load (p)*

Finite Element Reliability Analysis

The finite element method is a powerful tool for the numerical solution of a wide range of engineering mechanics and structural engineering problems. The conventional FEM ignores uncertainty in the variables and the analysis is carried out in a deterministic manner to evaluate the response of the system. This difficulty is overcome by the use of probabilistic finite element analysis where the uncertainty associated with the input variables is treated within the framework of probability theory to evaluate the resulting uncertainty of the response variable.

The finite element reliability analysis of the raft example is carried out using PLAXIS software. A 6-noded triangular plate element formulated based on the Mindlin's plate theory, equipped with six degrees of freedom per node, viz., three translational degrees of freedom and three rotational degrees of freedom, is used to decretise the raft. The soil volume is meshed by means of 15-noded wedge elements. The raft and the soil are modelled based on Hooke's law of isotropic linear elasticity.

A square raft foundation of dimensions 10 m × 10 m × 0.5 m subjected to a uniformly distributed load of 100 kN/m^2 resting on a soil layer of thickness 40 m is considered. The Young's modulus of the raft, E_p is taken as 15000 MPa and Poisson's ratio, v_p as 0.2; and for the soil (i.e., stiff clay) the values are E_s= 81.9 MPa and v_s = 0.3. The finite element model of the raft generated using PLAXIS is depicted in Fig.6. The lateral boundaries of the model are considered to be five times the breadth (i.e., b = 10 m) units away from the edges of the raft. The boundaries of the soil block are considered to be fixed.

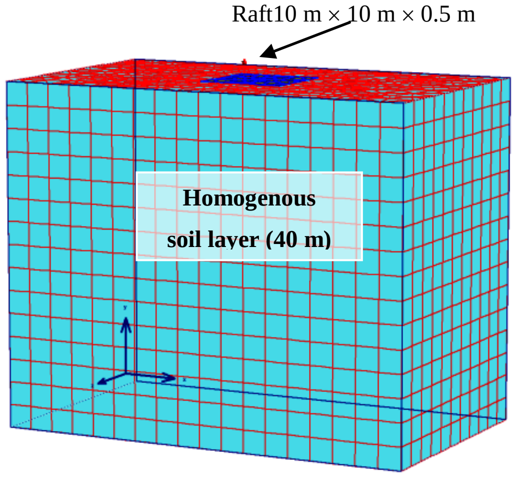

***Fig. 6.** Finite element model of the raft*

The deterministic finite element analysis of the raft foundation is carried out using the mean values of the material properties and load. The predicted central deflection of the raft is 12 mm which is comparable with the result available in the literature (22) and is given in Table 2. The spatial variation of the settlement of the raft is shown in Fig. 7.

***Table-2.** Settlement of the raft (10 m × 10 m × 0.5 m)*

Study	Central deflection (u)
Closed-form solution (22)	10.7 mm
Finite element method (present study)	12.0 mm

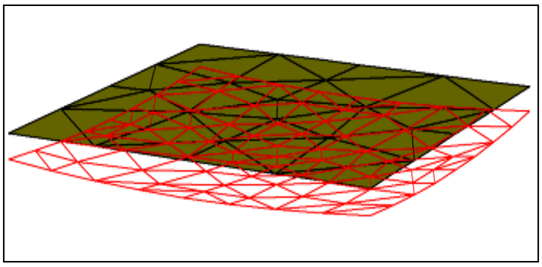

***Fig.7.** Spatial variation of settlement of the raft (10 m × 10 m × 0.5 m)*

In order to demonstrate the procedure, the finite element reliability analysis of the 10 m × 10 m × 0.5 m raft foundation has been carried out for the settlement limit state using the FORM. Due to the lack of true limit state function, the reliability analysis can be performed by means of the analytical expression developed using the response surface method. The FORM is used to evaluate the reliability index and the corresponding probability of failure using the limit state expression. The statistical properties of the random variables considered for the analysis are given in Table 3. The developed performance function is expressed as

$$g'(\mathbf{x}) = 24.735 - 0.431149\,X_1 + 114.7777\,X_3 + 0.0017227\,X_1^2 - 34.722\,X_2^2 - 22.222\,X_3^2 \qquad (14)$$

***Table-3**. Statistical properties of design variables of the raft (10 m × 10 m × 0.5 m)*

Parameter	Distribution	Mean value(μ)	COV (%)
Modulus of elasticity of soil, E_s (kN/m^2)	Normal	8.19 × 10^4	15
Poisson's ratio of soil, v_s	Normal	0.3	8
Load, p (kN/m^2)	Normal	100	15

The allowable settlement is taken as 40 mm and the reliability index (β) obtained is 7.06, which is well above the target reliability index, 4. The corresponding probability of failure is 8.92 × 10^{-13}.

Conclusions

This study has highlighted the importance of considering the uncertainties associated with the design variables in the settlement performance assessment of the raft foundation. Evaluating reliability provides a means of assessing the degree of uncertainty associated with the geotechnical performance. The results of simple reliability analyses are more rational than the conventional deterministic analyses even though they use the same type of data, judgment and approximation. The reliability analysis of the raft foundation using analytical formulation is carried out for the settlement limit state using the FORM, duly considering the soil-structure interaction. As the limit state function is not available in an explicit form, the FORM is difficult to apply and hence the response surface approach is used to construct the approximate settlement limit state function for the raft foundation.

The parametric sensitivity analysis is carried out considering variations in the modulus of subgrade reaction, the modulus of elasticity of the raft and the applied load. The variations in the modulus of subgrade reaction and loading are found to have a greater influence on the settlement reliability of the raft. The settlement limit state is found to be insensitive to the changes in the modulus of elasticity of the raft. For lower coefficients of variation of modulus of subgrade reaction, there is a rapid reduction in the reliability index. The reliability analysis of another raft is also presented to throw light on the reliability-based finite element procedure. The process of calculating the probability of failure or reliability index reveals which sources of uncertainty are most important, and which are unimportant. This understanding provides an effective guide to what improvements in knowledge will reduce the overall uncertainty the most, in real-life applications associated with the raft foundations.

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