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PROBABILITY DISTRIBUTION OF ENERGY DEMAND FORECASTS FOR PAKISTAN

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Energy sector is relatively more sensitive. Its effects almost instantaneously reach out to the rest of the economy. High population growth coupled with rapid urbanization has put tremendous pressures on demand management of this sector. The planning process and appropriate pricing policies need to be based on more objective information. In this study, energy demand projections for the residential sector in Pakistan have been made and the extent of uncertainty of these projections have been assessed on the basis of a bootstrapped probability distribution of the projected demand. Price and income elasticities of aggregate energy demand and of its components have also been estimated.

I. Introduction

Although energy is one of the top priority concerns in Pakistan, yet energy economics is a relatively neglected field. Significant structural changes have taken place in the residential sector with regard to consumption patterns, prices and energy modes. For instance kerosene oil which used to account for 74 per cent of the total commercial energy demand in 1969 has dropped to 37 per cent [Energy Year Book, (1990)]. The share of electricity has increased significantly from 24 per cent to 51 per cent during the same period. The yearly consumption growth in kerosene, gas and electricity during the last 25 years has been 5.7, 22.0 and 14.3 per cent respectively. Both rationing and pricing policies have played important roles in the management of demand in the sector. However, since the impact of energy pricing policies is not confined to its demand management but also reaches out to the whole system with economic, social and environmental implications, these policies need to be determined on a more objective basis. Energy sector also happens to be import intensive and therefore has serious implications on the balance of payments. The increasing emphasis of the donor agencies to provide funds subject to appropriate pricing policies also dictates the importance of research in this area. Demand projections is a pre-requisite for planning and policy intervention in the sector. The main purpose of this study is to econometrically model the behavior of the residential sector of Pakistan with regard to energy consumption, and make demand projections for the future. We have assigned an appropriate degree of uncertainty to the projections by simulating a probability distribution of the projections, which is essential for the planning process of the sector. It may be mentioned here that the study is confined to commercial energy only. A large part of the residential energy demand is met with non-commercial energy sources like bio-mass including fuelwood, specially in the rural area, but, the data on non-commercial energy use are not available.

The conventional approach to model the aggregate energy demand consists of two steps. In the first step input share equations are derived from some flexible cost function such as the Translog. The estimated parameters obtained from these share equations are used to construct an aggregate energy price index. In the second stage, this aggregate index is used to estimate an aggregate energy demand function. In earlier studies aggregate energy was measured usually in terms of thermal equivalents. However, if the relative shares of various energy modes change not in line with the change in their relative prices, the thermal equivalence approach introduces a systematic bias in the elasticity estimates. In our sample period the shares have actually changed differently as compared to changes in relative prices. This approach is therefore not suitable to model energy demand. The alternative approach is to measure the aggregate energy demand in terms of the constant monetary unit [Bernard et al., (1987)]. We have preferred to use this approach in the study.

The paper is organized as follows. Section II describes the model and the data. Results are reported and discussed in section III.

II. The Model and Data Sources

The aggregate energy demand model is specified as follows

$$LHEXP = A_0 + A_1 * LPENCPI + A_2 * LHINCOM \qquad (1)$$

where

LHEXP = total annual expenditure on energy per household expressed in constant energy prices. The procedure used to obtain the aggregate energy price index to convert current values into constant values is explained later in this section.

LPENCPI = the log of the ratio of the aggregate energy price index to the overall consumer price index.

LHINCOM = the log of the annual household income expressed in constant Rupees.

The model specified in equation (1) is a conventional demand function, the only difference being that aggregate demand is measured by total expenditure on energy.

The aggregate energy price index is constructed following the Fuss approach [Fuss, (1977)], which is based on the concept of a unit cost function. The aggregate energy price may be considered as the cost per unit to the optimizing agent. This unit energy cost function may be represented by some standard cost function. We have used the following translog cost function for the purpose:

$$\ln(PE) = \ln \beta_o + \sum \beta_i \ln P_i + 1/2 \sum \sum \beta_{ij} \ln(P_i) \ln(P_j) \qquad (2)$$

where i, j = E(electricity), G(Gas), O(Kerosene oil); PE being the aggregate energy price index.

Partially differentiating equation (2), invoking Shepard's Lemma, and imposing homogeneity adding up and symmetry restrictions yields the following input demand equations:

$$S_i = \beta_i + \sum_j \beta_{ij} l_n (P_j) \qquad (3)$$

subject to the constraints $\sum \beta_i = 1$; $\sum_i \beta_{ij} = \sum_j \beta_{ij} = 0$; $\beta_{ij} = \beta_{ji}$;

where S_i is the share of the ith input. Corresponding to equation (3) the Allen-Uzawa partial elasticities are given by:

$$\theta_{ii} = \frac{B_{ii} + S_{ii} - S_i}{S_{ii}}$$

$$\theta_{ij} = \frac{B_{ij} + S_i \bullet S_j}{S_i \bullet S_j} \qquad (4)$$

and the price elasticities are given by:

$$E_{ii} = S_i \theta_{ii}$$

$$E_{ij} = S_j \theta_{ij} \qquad (5)$$

Equation (5) gives gross (partial) elasticities. They account for substitution among energy components under the constraint that the total quantity of energy consumed remains constant. However, a relative change in the price of one type of energy component will affect the price of aggregate energy. This in turn will affect the use of energy relative to other consumption expenditures. To account for this, net elasticities are computed as follows:

$$E^{*i}_{j} = E_{ij} + S_j * E_{ee} \tag{6}$$

where E_{ee} is the own price elasticity of demand for aggregate energy.

Equation (3) has been estimated using Zellner's iterative estimation procedure. The invariance of the estimates of the parameters has been ensured by estimating the model three times, dropping one equation each time. These estimated parameters are substituted in equation (2) to obtain an aggregate price index which in turn has been used in equation (1) to estimate the aggregate demand function.

This estimated aggregate demand function may be used to make forecasts of future energy demand. However, meaningful planning process needs greater sophistication than simple point projections. The degree of uncertainty attached to the projections is important in policy analysis. Corresponding to regression based forecasts there is a standard formula to develop confidence intervals for these forecasts. However, this approach is based on asymptotic and parametric assumptions and is likely to give somewhat optimistic standard errors [Freedman and Peters (1984)]. An alternative way, known as bootstrapping is better then the asymptotic approach, specially when applied to the finite-sample situation [Efron (1979), Freedman and Peters (1984)]. Bootstrapping is a computer-based technique, which replaces theoretical assumptions and algebraic formulas with a large number of stochastic simulations. The basic idea of bootstrapping is to draw random samples (with replacement) from the least squares residual to create an artificial residual set [Freedman and Peters (1984), Veall (1987)]. Simulate this procedure for a large number of times. With these sets of residuals an empirical approximation of the theoretical distribution of the disturbance term can be generated. This procedure of re-sampling residuals, adding them to the original forecasted value can be used to generate a probability distribution of the forecast. This is basically the procedure used in this study.[1]

The main data sources are the various issues of 'Power System Statistics', Karachi Electric Supply Corporation; 'Energy Year Book', Ministry of Petroleum and Natural Resources; and 'Power System Statistics', Water and Power Development Authority. The present study covers the period from 1964 to 1989.

[1] Bootstrapping is a semi-parametric approach. More recently, pure non-parametric approach has also been adopted using 'Nadaraya-Watson Kernal Density Estimator' [Amanullah, (1988); Hardle, (1990)]. However, it could not be used here because of the non-availability of appropriate software.

III. Results

The parameter estimates of equation (3) are presented in Table 1 and the corresponding price elasticity estimates in Table 2. All the own price elasticity estimates are negative and have been high t-ratios. The estimates are all less then unity, indicating price inelasticity for all components of energy.

TABLE 1

Parameter Estimates of the Share Equation

Parameter	Estimate	T-Statistics
BE	– 0.55	11.3
BEE	0.03	1.9
BEG	– 0.08	– 6.3
BG	– 0.10	– 2.8
BGG	– 0.01	1.8

TABLE 2

Own and Cross Price Elasticities and Other Asymptotic Standard Errors

Elasticities	Electricity	Gas	Kerosene
Electricity	– 0.58	– 0.19	0.77
	(0.08)	(0.04)	(0.10)
Gas	– 0.09	– 0.74	1.73
	(0.24)	(0.16)	(0.22)
Kerosene	0.41	0.18	– 0.59
	(0.054)	(0.02)	(0.07)

Figures in the parentheses are asymptotic standard errors.

All the cross price elasticities are positive, the only exception being the elasticity between electricity and gas. However, both EGE and EEG are very small in magnitude and EGE is statistically insignificant as well. The overall impression which emerges from these cross price elasticities is that there is not much substitutability between different modes of energy because all of them are less than unity. The only exception is EGO which indicates gas consumption response to a change in the price of kerosene oil. Intuitively it is quite a plausible result in view of the fact that currently kerosene is far more expensive than gas. The net elasticities are presented in Table 3.

In the next step the above estimated parameters are substituted in equation (2) to obtain an aggregate price index of energy. This aggregate indexis used as an instrument in equation (1) to estimate the aggregate energy demand equation. The results are presented in Table 4.

TABLE 3

Net Elasticities

	Electricity	Gas	Kerosene
Electricity	– 0.82	– 0.23	0.33
Gas	– 0.32	– 0.78	1.29
Kerosene	0.17	0.13	– 1.02

TABLE 4

Estimated Aggregate Demand for Energy

Parameter	Estimated Coefficient (Elasticities)	t- Ratio
Constant	– 15.26	– 12.7
LHINCOM	2.24	15.7
LPENCPI	– 0.72	– 5.6

Adjusted R^2 = 0.98; D.W. = 1.97; D.F. = 22.

Although the specification of the demand function is simple it yields a fit which has a high precision as indicated by t-ratios and the R^2 value.[2] This result indicates that energy expenditure is quite responsive to income but is relatively inelastic to relative price changes.

We wish to get a bootstrapped forecast of energy demand in the residential sector of Pakistan. Before bootstrapping, however, a number of diagnostic tests have been applied to the above equation. The Durbin-Watson statistic of 1.96 indicates that we cannot outright reject the hypothesis that there is no autocorrelation. The goodness of fit test for normality of residuals yields a chi-square value of 1.57. Similarly the Jarque-Berra asymptotic LM test of normality of residuals yields a chi-square value of 1.49. The critical chi-square values for the two tests at 5 per cent significant level are 3.84 and 7.814 respectively. Therefore the hypothesis of normality of residuals cannot be rejected by any of the two tests. The hypothesis of homoscedasticity cannot be rejected on the basis of the Breusch-Pagan (1979) test which yields a chi-square value of 1.23 and the Glegser test which gives a chi-square of 0.47, the critical value at 5 per cent being 7.84.

The above tests indicate that the estimated equation is suitable to apply bootstrapping. Using Efron's methodology, 1,000 samples of 25 observations, each with replacement have been drawn from the residuals. Adding these residuals to the forecasted value, artificial data of 1,000 samples has been generated and for each of these samples an OLS is estimated. The bootstrapped coefficients along with their standard deviations are presented in Table 5.

TABLE 5

Bootstrapped Coefficients and Standard Deviations

Parameter	Number	Mean	Standard Deviation
Constant	1,000	– 15.23	1.2
LHINCOM	1,000	2.24	0.14
LPENCPI	1,000	– 0.72	0.13

[2] It may be mentioned that we estimated several alternative specifications, including the one with a one year lagged term on the RHS. This type of specification has been found to satisfy several goodness of fit criteria more frequently than other specifications in an earlier study by Welsch (1989). However, in our case, the simple version and the lagged version were put to several diagnostic tests and the simple version satisfied all of them and therefore we preferred to work with that version.

The bootstrapped coefficients are virtually the same as the estimated coefficients, which shows that the estimated coefficients are unbiased.

In the next step, a forecast value is obtained using the estimated aggregate demand equation. However, in order to do so, one needs the forecast values of the two exogenous variables, LHINCOM and LPENCPI. We have estimated the following rather simple equations which are then used to obtain the forecast of the two exogenous variables.[3]

$$\begin{aligned} \text{LHINCOM} &= 7.85 + \underset{(0.0009)}{0.0385} * \text{time} \\ R^2 &= 0.99 \end{aligned} \tag{7}$$

$$\begin{aligned} \text{LPENCPI} &= -0.001 + \underset{(0.17)}{0.63} * D + \underset{(0.034)}{0.085} * T1 - \underset{(0.13)}{0.46} * T2 \\ R^2 &= 0.82 \end{aligned} \tag{8}$$

where D = 0 for the first six observations, T1 = (1-D)* time; T2 = D*time.

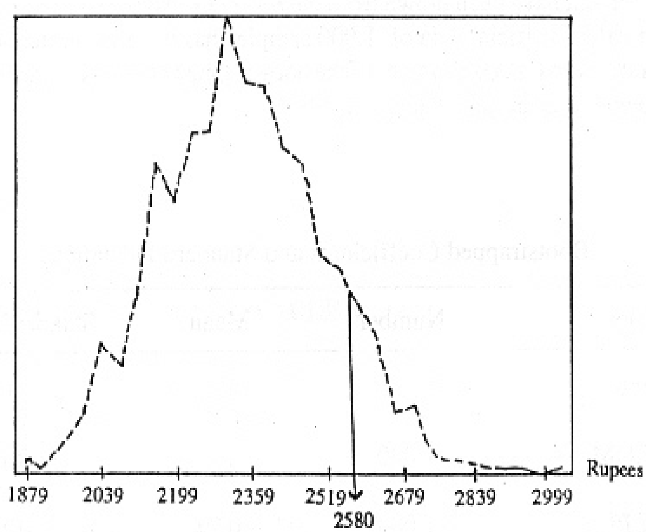

FIGURE 1

Bootstrapped Probability Distribution of Projected Per Household Energy Demand in 1997-98

[3] Equation (2) has not been used to make LPENCPI projections because it involves projected prices of electricity, gas and kerosene oil, and is liable to a larger margin of error with the simple approach adopted here.

The rationale for the specification of equation (8) is derived on the basis of observing a small kink at the seventh year in the LPENCPI series. The forecast values obtained from these two equations are used to obtain the forecast of aggregate demand using equation (1).

Finally, the bootstrapping of this forecast is made using the methodology explained in the above section. Figure 1, presents the probability distribution of the forecast.

According to the forecast the yearly average expenditure per household on energy in 1997-98 expressed in 1959-60 prices comes to Rs.437.11, which in 1990-91 prices comes to about Rs.2,335. The degree of uncertainty of the projections is reflected in Figure 1 above. The 90 per cent confidence upper limit of the average expenditure on energy as shown in the diagram is Rs.2,580 per household.

Using the above estimates the total energy demand of the residential sector in 1997-98 is estimated at about 46 billion rupees. This is based on the assumption that the past trend of population growth and energy prices would continue during the forecast period.[4] The 95 per cent confidence limit of total energy demand in the residential sector is 50.7 billion rupees.

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[4] The trend forecast for the number of households has been obtained using data for the past 25 years whereas that for energy prices is based ob data pertaining to the last 5 years only.

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